B Online Appendix: Not For Publication

B.1 National Accounts in the Model

GDP in constant prices (steady state prices) is measured by

$$L_D(P_d d + P_f f + P_g g) + \sum_{i=F,W} x_i p_i^d L_i d_i - p_D^f L_D f - p_D^g L_D g + v_D(a_D^f + a_D^g) - x_F v_F a_F^d - x_W v_W a_W^d,$$

consumption and investment in constant prices is measured by[43]

$$L_D(P_{d,t} d_t + P_{f,t} f + P_{g,t} g_t) \frac{c_t}{G(d_t, f_t, g_t)}, \quad (34)$$

$$L_D(P_{d,t} d_t + P_{f,t} f + P_{g,t} g_t) \frac{i_t}{G(d_t, f_t, g_t)}, \quad (35)$$

and population size is L_i (population appears in the equation whenever the values are per capital values). Notice that investment in marketing does not show up explicitly in the expenditure side measurement of GDP. This assumption is consistent with the methodology of national income accounting, in which expenses on R&D, marketing, advertising are all treated as intermediate inputs – see (1993) Par. 1.49, 6.149, 6.163, 6.165.

B.2 Data Sources

Bilateral trade statistics were taken from International Monetary Fund, Direction Of Trade Statistics, 2005. From Source OECD, Quarterly National Accounts: Gross Fixed Capital Formation ("P51: Gross fixed capital formation", "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"), GDP in constant prices ("B1_GE: Gross domestic product - expenditure approach", "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"). Our measure of labor is civilian employment or employment from Quarterly National Accounts or the International Labor Organization (based on data availability). GDP is available from 1980Q1 to 2011Q4 for all countries in our sample. Employment data is missing for some countries for some years (see column 2 in Table 8 for details). Since labor data is often not seasonally adjusted, we apply the X-12-ARIMA Seasonal Adjustment Program from census.gov.

Nominal GDP from World Development Indicators, World Bank. Gross Fixed Capital Formation, GDP in constant prices and Civil Employment from Source OECD.org, Quarterly National Accounts. Series for physical capital have been constructed using the perpetual inventory method with a constant depreciation of 2.5%. Aggregate GDP for blocks of countries has been computed from growth rates of GDP in constant prices (recent years, varies by country) weighted by the nominal GDP of each country in 2004 (we applied the growth rates backwards).

[43] Consumption and investment in period zero prices are not equal to c and i. The reason is that the Euler's Law does not apply for period zero (steady state) prices. However, quantitatively the difference is essentially zero.

Table 8: Data Sources and Years.

Country	Labor available	Measure	Source
Australia	1980:Q1	Employed Population Quantity	OECD
Austria	1980:Q1	Civilian Employment in % of pop.	OECD
Belgium	1983:Q2	Civilian Employment Quantity	OECD
Canada	1980:Q1	Employed Population Quantity	OECD
Denmark	1990	Employed Population Quantity	OECD
Finland	1980:Q1	Tototal Employment Quantity	ILO
France	1980:Q1	Civilian Employment in % of pop.	OECD
Germany	1980:Q1	Civilian Employment in % of pop.	OECD
Ireland	1989	Employed Population Quantity	OECD
Italy	1980:Q1	Civilian Employment in % of pop.	OECD
Japan	1980:Q1	Employed Population Quantity	OECD
Korea	1983	Employed Population Quantity	OECD
Netherlands	1998	Employed Population Quantity	OECD
Norway	1980:Q1	Civilian Employment in % of pop.	OECD
Portugal	1999	Employed Population Quantity	OECD
Spain	1980:Q1	Civilian Employment Quantity	OECD
Sweden	1980:Q1	Total Employment Quantity	ILO
Switzerland	1980:Q1	Civilian Employment in % of pop.	OECD
UK	1980:Q1	Civilian Employment Quantity	OECD
USA	1980:Q1	Employed Population Quantity	OECD

B.3 Volatility Ratio in the Cross Section

Table 9 presents estimates of the volatility ratio in our sample.

B.4 TFP correlations

In our empirical exercise, we also document the relationship between trade and TFP correlations. We use data on read GDP, capital and labor in order to construct measures of TFP for our 190 country pairs. We then run the same cross-country regression as for real GDP in the main text. The results are presented in Table 10. Both OLS and IV specifications give highly significant positive coefficients on bilateral trade, implying that moving from the 10th to the 90th percentile of the bilateral trade distribution increases TFP correlations by 0.19 (IV) and 0.15 (OLS), which is high relative to a median TFP correlation of 0.44 in our sample. As with real GDP, the trade-comovement relationship is stronger in the high trade subsample.

Results from model-based regressions are presented in Table 11. Our benchmark model implies an endogenous relationship between trade and TFP correlations. It accounts for 50-60% of the empirical slope in the overall sample and 60-70% in the high trade sample. In the next section, we discuss the sources of endogenous TFP movements in our model, and the source of the trade-comovement relationship for TFP.

Table 9: Volatility ratio in a cross-section of major industrialized countries

	Detrending method	
Country	Hodrick-Prescott filter (1600)	Linearly detrended
Australia	0.88	0.78
Austria	2.76	2.31
Belgium	1.21	1.27
Canada	1.27	1.24
Denmark	1.17	1.52
Finland	1.67	1.31
France	0.77	0.86
Germany	1.38	1.36
Italy	1.07	1.12
Japan	0.68	0.63
Korea	0.59	0.65
the Netherlands	0.99	0.77
Norway	1.18	1.21
Portugal	1.07	1.04
Spain	1.89	1.21
Sweden	1.59	2.14
Switzerland	1.05	0.87
United Kingdom	0.90	0.67
United States	1.20	0.88
Median	1.17	1.12

B.4.1 Discussion

The effect of trade on TFP is more subtle in the model, and comes solely from the specifics of the national accounting procedures. In the data the TFP residuals are measured by subtracting from the log of final output (real GDP) the log of payments to labor and capital. Since final output excludes intermediate inputs, and marketing expenditures are classified this way by the national accounting procedures ((1993)), TFP goes up when less marketing is needed for a given level of production/sales, as is the case of a foreign positive productivity shock.

Real GDP in the model is given by (25), and it does not include domestic marketing expenditures a_D^d and search effort of the retailer h. Both of these are intermediate inputs, and hence do not directly enter GDP in accordance with NIPA. From the resource constraint:

$$zf(K,L) = a_f^D + a_g^D + a_d^D + \sum_{i=D,F,W} L_i d_i + \chi h,$$

we can calculate the Solow residual, $TFP = \frac{rGDP}{f(k,l)}$. Then, the ratio of (measured productivity

Table 10: Regression results: trade-comovement in the data.

Dependent Variable: TFP correlation			
	OLS	OLS bottom 50%	OLS top 50%
$trade_{ij}$	0.029**	−0.021	0.062***
	(0.013)	(0.032)	(0.018)
E_{ij}	0.011	0.170	−0.119
	(0.087)	(0.187)	(0.104)
R-squared	0.697	0.678	0.755
	IV	IV bottom 50%	IV top 50%
$trade_{ij}$	0.044**	0.180	0.053**
	(0.020)	(0.240)	(0.026)
E_{ij}	−0.033	−0.232	−0.104
	(0.103)	(0.537)	(0.109)

,* denote significance at 5% and 1% level. Numbers in parentheses are standard errors.

TFP)/(exogenous shock z) is

$$\Lambda \equiv \frac{rGDP}{a_f^D + a_g^D + a_d^D + \sum_{i=D,F,W} L_i d_i + \chi h}$$

where $rGDP$ is given by (25).

In the frictionless model, $P - p$, $h = 0$ and $a = 0$ so that Λ is a constant. In the benchmark model, Λ moves around for three reasons: (i) shifts between d_D and d_W, d_F and the terms coming from net exports move Λ, because of differences in steady state markups between retail and producer prices; (ii) movements of production between marketing investment a_i^j and goods' production d_i; and (iii) shifts between search h and physical production d_i.

Table 12, quantifies the contribution of each of these components to the endogenous movements of the TFP defined by deviations of Λ_t from 1. In column 1, we report the overall equilibrium variance of Λ_t for the benchmark parameterization, of around 1%. We then compute the counter-factual variance of Λ_t when we shut off all marketing investment (a_i^j) and search channels (h). The variance in this case comes solely due to the existence of retail-producer markups and amounts to only 0.2% (column 2). If we then turn on the marketing investment channel (a_i^j), but not the search channel (h), the variance of Λ goes up to 0.9% (column 3). The difference between column 3 and 1 is the contribution of the search channel.

The analysis in Table 12 implies that an overwhelming majority of endogenous TFP movements in our model comes from movements in investment in our form of intangible capital - the marketing capital. This channel of TFP movement, due to mis-measurement, has also been identified as important for measuring productivity in previous literature (in Carol Corrado, Charles Hulten & Daniel Sichel (2005), Carol Corrado, Charles Hulten & Daniel Sichel (2009), Ellen R McGrattan & Edward C Prescott (2010a) and Ellen R McGrattan & Edward C Prescott (2010b), among others).

Table 11: Regression results: Data versus Model.

Dependent Variable: TFP correlation			
Coefficient β_{TFP}	OLS	OLS bottom 50%	OLS top 50%
Data	0.029^{**}	-0.021	0.062^{***}
Model	0.018	0.003	0.036
Model/Data	62%	–	59%
Frictionless Model/Data	–	–	–
	IV	IV bottom 50%	IV top 50%
Data	0.044^{**}	0.18	0.053^{**}
Model	0.023	0.003	0.039
Model/Data	52%	–	74%
Frictionless Model/Data	–	–	–

,* denote significance at 5% and 1% level for the data regression.

Table 12: Sources of Endogenous TFP Movements.

	Benchmark	Markups Only	Markups + Marketing a_i^j
Volatility of Λ_t	1%	0.2%	0.91%

B.5 Detailed Proofs

B.5.1 Proof of Proposition 2

By welfare theorems, first period allocation solves:

$$\max_{d,f,d^*,f^*,l,l^*} Log(G_{\phi,\rho}(d,f)) - l + Log(G_{\phi,\rho}(f^*,d^*)) - l^*$$

subject to

$$\begin{aligned} d(s) + d(s)^* + \tau(x)d(s)^* &= Al \\ f(s) + f(s)^* + \tau(x)f(s) &= A^*l^*, \end{aligned}$$

where $\tau(x)$ is as defined in text.

The second period planning problem is similar but with $A = A^* = 1$ and $G_{\omega,\gamma}$ in place of $G_{\phi,\rho}$.

The first order conditions characterizing the solution are:

$$\begin{aligned}
\lambda - \frac{\partial G_{\phi,\rho}(d,f)/\partial d}{G_{\phi,\rho}(d,f)} &= 0 \\
\mu - \frac{\partial G_{\phi,\rho}(f^*,d^*)/\partial f^*}{G_{\phi,\rho}(f^*,d^*)} &= 0 \\
\mu\tau(x) - \frac{\partial G_{\phi,\rho}(d,f)/\partial f}{G_{\phi,\rho}(d,f)} &= 0 \\
\lambda\tau(x) - \frac{\partial G_{\phi,\rho}(f^*,d^*)/\partial d^*}{G_{\phi,\rho}(f^*,d^*)} &= 0 \\
\lambda A - 1 &= 0 \\
\mu A^* - 1 &= 0 \\
d + d^* + \tau(x)d^* &= Al \\
f + f^* + \tau(x)f &= A^*l^*,
\end{aligned}$$

where λ, μ are the Lagrange multipliers imposed on the feasibility constraints. In addition, we define:

$$\begin{aligned}
y &= Al, \\
y^* &= A^*l^*.
\end{aligned}$$

We guess and verify that the symmetric (deterministic) solution ($A = 1$, $A^* = 1$) of the above system is given by:

$$l = 1 \tag{36}$$

$$f = \frac{1}{-\frac{\phi(\tau+1)^\rho}{\phi-1} + \tau + 1} \tag{37}$$

$$d = \frac{1}{1 - \frac{(\phi-1)(\tau+1)^{1-\rho}}{\phi}}. \tag{38}$$

Solution of the second period can be obtained after appropriate modifications (e.g. ρ replaced by γ).

By definition of trade intensity stated in text, the symmetric solution applied to the second period gives:

$$x := \tau^{-1}(x) = \frac{1}{1 + \tau - \frac{\omega(\tau+1)^\gamma}{\omega-1}}.$$

Implicitly differentiating this expression, we derive

$$\frac{d\tau}{dx} = -\frac{1}{x\left((1-\gamma)x + \frac{\gamma}{\tau+1}\right)}.$$

Next, we define ϕ to assure that the trade intensity (as defined in text) is equal to x for any fixed value of τ. This way we obtain

$$\phi = (1 + \frac{x(1+\tau)^\rho}{1 - x - \tau x})^{-1}.$$

We use this formula to substitute out for ϕ whenever applicable. We also note that for this value of ϕ the symmetric (deterministic) solution of the first order conditions stated above is given by $d = 1 - (1+\tau)x$, $f = x$ and $l = 1$.

We next log-linearize the first order conditions at the symmetric solution. The linearized system can be inverted using conventional methods. The first order conditions used to log-linearize the system before plugging in for ϕ are given by:

$$\begin{aligned}
d^*\phi\left(f^*\left(d^*\phi\right)^{\frac{1}{\rho}} + d^*\left(f^* - f^*\phi\right)^{\frac{1}{\rho}}\right)^{-\rho} + d(\phi-1)\left((\tau+1)\left(d(f\phi)^{\frac{1}{\rho}} + f(d-d\phi)^{\frac{1}{\rho}}\right)\right)^{-\rho} &= 0,\\
f^*(1-\phi)\left(f^*\left(d^*\phi\right)^{\frac{1}{\rho}} + d^*\left(f^* - f^*\phi\right)^{\frac{1}{\rho}}\right)^{-\rho} - f\phi\left(\frac{\tau+1}{d(f\phi)^{\frac{1}{\rho}} + f(d-d\phi)^{\frac{1}{\rho}}}\right)^{\rho} &= 0,\\
\frac{A(f\phi)^{\frac{1}{\rho}}}{d(f\phi)^{\frac{1}{\rho}} + f(d-d\phi)^{\frac{1}{\rho}}} - 1 &= 0,\\
\frac{A^*}{d\left(\frac{f\phi}{d-d\phi}\right)^{\frac{1}{\rho}} + f} - \tau - 1 &= 0,\\
-A^*l^* + f^* + f\tau + f &= 0,\\
-Al + d^*\tau + d^* + d &= 0,\\
y - Al &= 0,\\
y^* - A^*l^* &= 0
\end{aligned}$$

The log-linearization gives:

$$\begin{aligned}
(\tau+1)(\phi-1)\left(d^*(\rho-1) + f^* - f\rho\right) + \phi(\tau+1)^\rho\left(d(\rho-1) - f^*\rho + f\right) &= 0,\\
\phi(\tau+1)^\rho\left(d^* - d\rho + f^*(\rho-1)\right) + (\tau+1)(\phi-1)\left(-d^*\rho + d + f(\rho-1)\right) &= 0,\\
(\tau+1)(\phi-1)(-A\rho + d + f(\rho-1)) + \rho\phi(A-d)(\tau+1)^\rho &= 0,\\
\phi(\tau+1)^\rho\left(-A^*\rho + d(\rho-1) + f\right) + \rho(\tau+1)(\phi-1)\left(A^* - f\right) &= 0,\\
-A^* + \frac{f(\tau+1)(\phi-1) - f^*\phi(\tau+1)^\rho}{(\tau+1)(\phi-1) - \phi(\tau+1)^\rho} - l^* &= 0,\\
-A + \frac{d^*(\tau+1)(\phi-1) - d\phi(\tau+1)^\rho}{(\tau+1)(\phi-1) - \phi(\tau+1)^\rho} - l &= 0,\\
A + l - y &= 0,\\
A^* + l^* - y^* &= 0,
\end{aligned}$$

where, abusing notation, each variable represents a log deviation from the symmetric solution[44].

Inverting the system, we derive the solution for the domestic output y:

$$y = A\left[1 - 2x(1-\rho)(1+\tau)(1-x-\tau x)\right] + A^*\left[2x(1-\rho)(1+\tau)(1-x-\tau x)\right].$$

The complete solution can found in the Mathematica notebook posted online.

B.5.2 Proof of Proposition 5

Proof of Lemma 1. In equilibrium prices are such that the corresponding dynamic budget constraint of the domestic household must boil down to the static budget constraint stated above. This follows from the fact that in this environment resources can only be physically transferred between countries and not across time. As a result, prices must assure that the aggregate resources saved in the world in the first period must always be zero (i.e. for each s). The remaining conditions straightforwardly follow from the first order conditions and market clearing/feasibility conditions discussed in text. For example, conditions ii) can be obtained by solving the Lagrangian pertaining to household problem discussed in text. In the case of the domestic country, we obtain the Lagrangian

$$L = G_{\phi,\rho}(d,f) - \lambda(d + pf(1+\tau) - Al(s) - B(s)),$$

which gives the following first order conditions:

$$\begin{aligned} \partial G_{\phi,\rho}/\partial d &= \lambda \\ \partial G_{\phi,\rho}/\partial f &= \lambda\, p(1+\tau) \end{aligned}$$

These conditions imply:

$$\partial G_{\phi,\rho}/\partial f = p(1+\tau)\partial G_{\phi,\rho}/\partial d.$$

Finally, to obtain the risk sharing condition (last equation), we solve for the demand equations for bonds $B(s)$ as a function of the price Q both in the home and in the foreign country. The solution gives the risk sharing condition vi. ■

The above system yields the following conditions characterizing the equilibrium in the first period:

[44] More precisely, we redefine each variable as follows: $z \equiv Log(\frac{z}{z^s})$, where z^s is the value of any generic variable z at the approximation point.

$$p-\frac{(d(1-\phi))^{\frac{1}{\rho}}(f\phi)^{-1/\rho}}{\tau+1} = 0, \quad (39)$$

$$\frac{A(f\phi)^{\frac{1}{\rho}}}{f(d(1-\phi))^{\frac{1}{\rho}}+d(f\phi)^{\frac{1}{\rho}}}-1 = 0, \quad (40)$$

$$-Al+d+fp\tau+f-R = 0, \quad (41)$$

$$y-Al = 0, \quad (42)$$

$$\frac{1}{p}-\frac{(d^*\phi)^{-1/\rho}\left(f^*(1-\phi)\right)^{\frac{1}{\rho}}}{\tau+1} = 0, \quad (43)$$

$$\frac{A^*\left(d^*\phi\right)^{\frac{1}{\rho}}}{f^*\left(d^*\phi\right)^{\frac{1}{\rho}}+d^*\left(f^*(1-\phi)\right)^{\frac{1}{\rho}}}-1 = 0, \quad (44)$$

$$y^*-A^*l^* = 0, \quad (45)$$

$$-Al+d^*\tau+d^*+d = 0, \quad (46)$$

$$-A^*l^*+f^*+f\tau+f = 0, \quad (47)$$

$$\frac{(\tau+1)(1-\phi)\phi(d(1-\phi))^{\frac{\rho-1}{\rho}-1}}{\phi(d(1-\phi))^{\frac{\rho-1}{\rho}}+(1-\phi)(f\phi)^{\frac{\rho-1}{\rho}}}-\frac{(1-\phi)\phi\left(d^*\phi\right)^{\frac{\rho-1}{\rho}-1}}{(1-\phi)\left(d^*\phi\right)^{\frac{\rho-1}{\rho}}+\phi\left(f^*(1-\phi)\right)^{\frac{\rho-1}{\rho}}} = 0. \quad (48)$$

We next log-linearize the equilibrium system with respect to all variables except R. Since R is zero at the symmetric (deterministic) solution, we linearize the system with respect to R. This way we obtain:

$$d-f-p\rho = 0, \quad (49)$$

$$(\tau+1)(\phi-1)(-A\rho+d+f(\rho-1))+\rho\phi(A-d)(\tau+1)^{\rho} = 0, \quad (50)$$

$$-A+\frac{\phi(R-d)(\tau+1)^{\rho}+(\phi-1)(\tau(f+p-R)+f-R)}{(\tau+1)(\phi-1)-\phi(\tau+1)^{\rho}}-l = 0, \quad (51)$$

$$A+l-y = 0, \quad (52)$$

$$d^*-f^*-p\rho = 0, \quad (53)$$

$$(\tau+1)(\phi-1)\left(A^*(-\rho)+d^*(\rho-1)+f^*\right)+\rho\phi\left(A^*-f^*\right)(\tau+1)^{\rho} = 0, \quad (54)$$

$$A^*+l^*-y^* = 0, \quad (55)$$

$$-A+\frac{d^*(\tau+1)(\phi-1)-d\phi(\tau+1)^{\rho}}{(\tau+1)(\phi-1)-\phi(\tau+1)^{\rho}}-l = 0, \quad (56)$$

$$-A^*+\frac{f(\tau+1)(\phi-1)-f^*\phi(\tau+1)^{\rho}}{(\tau+1)(\phi-1)-\phi(\tau+1)^{\rho}}-l^* = 0, \quad (57)$$

$$\phi(\tau+1)^{\rho}\left(d^*-d\rho+f^*(\rho-1)\right)+(\tau+1)(\phi-1)\left(-d^*\rho+d+f(\rho-1)\right) = 0, \quad (58)$$

where all variables except for R represent a log deviation from the approximation point (symmetric deterministic solution stated in the previous proposition).

To derive the decomposition discussed in text, we proceed as follows. In the first step, we linearize the above equilibrium system by treating p and R as exogenous shocks and by dropping equation (33) and the foreign demand equation (29) (equivalently, we could drop equations (43),

(48) or (53) and (58)). This first step yields:

$$y = A - R - xp.$$

We next include equation (29) (equivalently, equation (43) or (48)) to obtain the equation characterizing the endogenous dynamics of the terms of trade p as function of state variables A, A^* and the still exogenous stochastic process for R (after plugging in for ϕ from the proof of the previous proposition)

$$p = \frac{R - x(1+\tau)(A - A^*)}{x\left(-2\rho(\tau+1) + \tau + 2(\rho-1)(\tau+1)^2 x\right)}.$$

Finally, we include all equilibrium conditions to obtain the equation for R:

$$R = (A - A^*)x\left(1 + 2\tau - 2(1+\tau)(\rho + x(1-\rho)(1+\tau))\right)$$

The latter equation closes the system. The coefficients in text are derived from the above set of equations by evaluating appropriate derivatives with respect to A^*.