

# Long-Run Price Elasticity of Trade and the Trade-Comovement Puzzle\*

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## Abstract

In this paper we argue that modeling dynamic properties of trade elasticity in a unified framework is essential for understanding one of the fundamental questions that lies at the intersection of business cycle and trade theory: What role do international trade linkages play in transmitting shocks across borders? Analytically, we show that in a broad class of open economy macroeconomic models shock transmission crucially depends on dynamic properties of trade elasticity. We demonstrate how modeling the dynamics of trade elasticity is thus of critical importance for applications aiming to relate economic models to cross-sectional variation of business cycle moments in the data. In the context of such applications, our paper cautions against drawing conclusions from models relying on a single (static) trade elasticity and advocate the use of models consistent with *dynamic* elasticity.

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# 1 Introduction

A well-established fact of international trade is that trade flows respond sizably to persistent relative price changes but not to transient price shocks that occur on business cycle frequency.<sup>1</sup> This property of trade flows results in vastly different estimates of the price elasticity of trade depending on the time horizon of the analysis, implying low short-run estimates and high long-run estimates (*dynamic elasticity*, henceforth). The two different set of estimates have thus far been used largely in separation in international macroeconomics and trade, depending on the question at hand. In particular, international business cycle theory uses short-run estimates and focuses on high-frequency time-series predictions the models whereas trade theory uses long-run estimates and focuses on cross-sectional implications.

In this paper we argue that modeling dynamic properties of trade elasticity in a unified framework is essential for understanding one of the fundamental questions that lies at the intersection of business cycle and trade theory: What role do international trade linkages play in transmitting shocks across borders? Analytically, we show that in a broad class of open economy macroeconomic models shock transmission crucially depends on dynamic properties of trade elasticity. We demonstrate how modeling the dynamics of trade elasticity is thus of critical importance for applications aiming to relate economic models to cross-sectional variation of business cycle moments in the data. In the context of such applications, our paper cautions against drawing conclusions from models relying on a single (static) trade elasticity and advocate the use of models consistent with *dynamic* elasticity.

To illustrate the relevance of our findings, we focus on the link between trade and business cycle comovement, which has attracted considerable empirical and theoretical attention (recent examples include M. A. Kose & K-M. Yi (2006), Julian diGiovanni & Andrei Levchenko (2010), Wei Liao & Ana Maria Santacreu (2011) and Robert C. Johnson (2013), among others). Empirically, using regression analysis, the literature has provided robust evidence on the existence of a positive link between measures of bilateral average trade intensity and measures of business cycle synchronization, such as business cycle comovement of real GDP.<sup>2</sup>

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<sup>1</sup>For example, see K. J. Ruhl (2008) for an excellent review of the literature.

<sup>2</sup>By running cross-country regressions J. A. Frankel & A. Rose (1998), T. E. Clark & E. van Wincoop

Yet, theoretically, it has been argued that this pattern is inconsistent with micro-founded mechanisms governing international transmission of business cycles in standard models.<sup>3</sup> In this particular context, we argue that modeling of trade elasticity can crucially affect the performance of the model. Specifically, we show that dynamic trade elasticity can bring an otherwise standard business cycle theory in line with the data, which is an illustration of a broader point of the importance of dynamic elasticity in applications relating international business cycle theory to data on cross-country variation of business cycle moments.

To conceptualize our key insight, consider the following decomposition of the marginal effect of the long run trade level  $x$  on a business cycle moment  $b$ , which in the case of our model is the international correlation of business cycles:

$$\frac{db(x, \tau(x))}{dx} = \frac{\partial b(x, \tau(x))}{\partial x} + \frac{\partial b(x, \tau(x))}{\partial \tau} \frac{d\tau(x)}{dx}. \quad (1)$$

The term  $db(x, \tau(x))/dx$  can be related to the theoretical regression coefficient in a cross-sectional study of country (or region) pairs characterized by different levels of bilateral trade in the model. As is clear from equation (1), such a relation potentially depends not only on the effect of trade (first term), but also on the effect of trade costs (second term). The presence of the second term comes from the fact that trade is generally endogenous in economic models, and so it can only be varied by changing some exogenous parameters, here labeled as trade costs.<sup>4</sup> Thus, to the extent that trade costs affect business cycle implications of the model, the model-implied impact of trade crucially depends on both the short-run and the long-run trade elasticity, as the latter approximately corresponds to the inverse of  $d\tau(x)/dx$ .

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(2001), C. A. Calderon, A. E. Chong & E. H. Stein (2002), G. Otto, G. Voss & L. Willard (2001), M. Baxter & M. A. Kouparitsas (2005), Kose & Yi (2006) and Robert Inklaar, Richard Jong-A-Pin & Jakob de Haan (2008) all find that, among bilateral country pairs, more trade is associated with more synchronized business cycle fluctuations. Johnson (2013) confirms these findings using disaggregated industry level data.

<sup>3</sup>M. A. Kose & K-M. Yi (2001) and Kose & Yi (2006) point out difficulties of canonical business cycle models in accounting for these regularities. Johnson (2013) points out difficulties in a model with a rich input-output structure. For a study exploring the potential of technology diffusion to account for the data patterns, see Liao & Santacreu (2011).

<sup>4</sup>In cross-country studies, trade is observable and trade cost is used to fit the observable trade patterns, effectively making it a function of trade.

Motivated by the above observation, our paper provides a complete characterization of the decomposition given by (1) in a prototypical international business cycle model that captures the idea of dynamic elasticity in a stylized fashion. In particular, our analytic framework embeds the static elasticity model as a special case, and otherwise rests on the canonical open economy macroeconomics assumptions, such as country-specific goods, endogenous labor-leisure trade-off, and international borrowing and lending.<sup>5</sup> Within this framework, we express  $\partial b/\partial x$  and  $\partial b/\partial \tau$  as functions of the economic fundamentals and show that they are both positive and economically significant. This implies that both the value and the sign of the trade-comovement relationship in the model is determined by the value of the long-run elasticity, alongside the short-run elasticity that enters directly through the terms  $\partial b/\partial x$  and  $\partial b/\partial \tau$ .

In further analysis of our prototypical model, we relate the terms of the decomposition implied by equation (1) to two broad economic forces responsible for shock transmission across borders, which we label i) the *complementarity channel* and ii) the *risk sharing channel*. The complementarity channel is associated with the fact that goods produced by different countries are imperfectly substitutable. The risk sharing channel comes from the fact that countries share risk by trading assets (and also through terms of trade fluctuations). It turns out that these two forces work in opposite directions in the model, and directly map onto the decomposition terms  $\partial b/\partial x$  and  $\partial b/\partial \tau$ .

To understand the intuition these two channels, consider first the complementarity channel, which comes from the fact that domestic and foreign goods are imperfectly substitutable. For this reason, business cycle shocks move terms of trade, which by determining the price of domestic consumption relative to leisure, transmits business cycle fluctuations through the endogenous labor-leisure choice. In our setup, when the foreign country good becomes more abundant (e.g. after a positive productivity shock abroad), the reduction in the relative price of good  $f$  makes domestic consumption cheaper relative to domestic leisure. Through the usual substitution effect, this increases labor supply and production in the domestic

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<sup>5</sup>For illustrative purposes, the model relies on productivity shocks, though this assumption is not critical for our results. Qualitatively same conclusion applies to models featuring demand shocks.

country. Importantly, since average trade increases the share of good  $f$  in domestic country consumption, the effect is stronger when countries trade more with each other, implying  $\partial b/\partial x > 0$ .

Consider next the risk sharing channel, which is solely responsible for  $\partial b/\partial \tau > 0$ . Intuitively, this channel stems from the desire to share consumption risk associated with business cycle fluctuations. In our setup, after a positive shock in the foreign country, domestic country households desire to finance their consumption by borrowing from the foreign country. Borrowing is used to import the more abundant foreign good, rather than supply more labor and produce more. Since importing is subject to trade costs, higher trade costs suppress this motive. Given it is a source of negative shock transmission,  $\partial b/\partial \tau > 0$  follows.<sup>6</sup>

The above intuition implies that, depending on the values of the short-run and long-run trade elasticity, standard macroeconomic theory can have vastly different prediction for international transmission of business cycle shocks. The demand complementarity channel depends inherently on business cycle frequency responses to shocks, and thus its effect is stronger for a lower short-run trade elasticity and generally positive. The impact of the risk-sharing channel depends on how permanent changes in trade costs affect long-run trade levels and is weaker for a higher long-run elasticity—in fact, it is almost completely eliminated when the long-run elasticity value is set in line with data evidence. Hence, our analysis provides evidence on the importance of modeling dynamic elasticity for international shock transmission.

In order to provide a quantitative assessment of our findings, we consider a quantitative business cycle framework featuring micro-founded model of dynamic trade elasticity. Specifically, we extend the framework proposed by Lukasz A. Drozd & Jaromir B. Nosal (2012) to a multi-country framework with asymmetric country sizes. We evaluate the model’s quantitative predictions in the context of the effects of trade on business cycle synchronization. By fitting our model to the data on both long-run and short-run elasticity, we are able to isolate the quantitative role of trade elasticity.

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<sup>6</sup>As we show, these two effects arise in a similar way in models featuring demand shocks. Hence, our conclusions do not depend on the fact that we assume productivity shocks.

To establish a quantitative goal for the theory, we quantify the trade-comovement relation in a sample of 20 OECD countries over the period 1980-2011. We document a positive and significant<sup>7</sup> link between a measure of bilateral trade intensity and the level of bilateral correlation of real GDP.<sup>8</sup> Additionally, we document that the trade comovement relationship is much stronger in the top half of our bilateral trade intensity distribution, with the relationship in the bottom half statistically insignificant. Hence, we show that the data relationship in the overall sample is essentially driven by the top half of the bilateral trade distribution.

To relate the theory to the data, we use a three country version of our model, and reproduce trade intensities observed in our sample of 190 country pairs, including their trade with the rest of the world (third large country). We perform the same regression analysis on the model-generated data as in the empirical part of the paper. We find that, indeed, dynamic elasticity critically affects the findings. Specifically, for the high trade intensity half of the sample, our dynamic elasticity model accounts for 70-80% of the data relationship, while the relationship becomes insignificant in the lower trade intensity half, just like in the data.

For comparison, an analogously parameterized single (static) elasticity model implies essentially no trade-comovement relationship, or even a negative relation. Furthermore, we show that shutting down dynamic elasticity in the model reverts the predicted trade-comovement coefficient to counterfactually negative values. Finally, we show that in our quantitative model, the complementarity channel is the key driver of positive shock transmission across countries.

The rest of the paper is organized as follows. Section 2 sets up a prototype business cycle economy and derives key theoretical results. Section 3 presents and analyzes the predictions of our quantitative model. Section 4 concludes.

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<sup>7</sup>Both in the data and subsequent model regressions, we include European Union dummies and country fixed effects.

<sup>8</sup>We are not the first to document this regularity. For a list of empirical contributions, see footnote 2.

## 2 Role of Dynamic Trade Elasticity in Shock Transmission

In this section, we set up a prototypical international business cycle model with dynamic elasticity that embeds the canonical static elasticity model as a special case.<sup>9</sup> We show analytically that modeling the properties of trade elasticity in a unified framework is essential to understanding the linkages between trade and the transmission of business cycle shocks across countries. We then provide intuition for our results.

### 2.1 Analytic Framework

There are two periods and two symmetric countries. Goods are differentiated by the country of origin and all goods are tradable. International trade is subject to a positive trade cost<sup>10</sup>  $\tau$ . In what follows, we assume that the allocation and prices in the first period corresponds to theory's business cycle-frequency predictions, while the average of the first period and the second period correspond to the model's cross-sectional predictions. In particular, the second period is interpreted as the allocation that would prevail in the long-run.

Unless otherwise noted, we exploit symmetry and present the setup from the domestic country's perspective. Where appropriate, we differentiate foreign variables from their domestic counterparts by an asterisk. Second period variables are differentiated from their first period counterparts by a prime.

**Dynamic consumption aggregation** The key element of our model is that consumption is aggregated differently in the short-run and in the long-run—giving rise to dynamic elasticity. In particular, when the economy is given enough time to adjust, i.e. after the first

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<sup>9</sup>The model we use captures the core structure of business cycle models used in international economics. It is thus representative of the key forces governing international transmission mechanism in a large class of macroeconomic models used in the literature.

<sup>10</sup>While we will refer to this cost as transportation or trade cost, it should be interpreted more broadly as cost associated with all kinds of frictions involved in trade (including the non-pecuniary ones).

period, it is assumed that the economy is more flexible in switching between consumption of good  $d$  and  $f$  than it is the case in the first period only.

Specifically, we assume that consumption of individual goods  $d$  and  $f$  is aggregated in the second period according to a CES function featuring a potentially high elasticity  $\gamma$ :<sup>11</sup>

$$c' = G_{(\omega, \gamma)}(d', f') = (\omega((1 - \omega)d')^{\frac{\gamma-1}{\gamma}} + (1 - \omega)(\omega f')^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}}. \quad (2)$$

At the same time, preferences in the first period are given by

$$c = G_{(\phi, \rho)}(d, f) = (\phi((1 - \phi)d)^{\frac{\rho-1}{\rho}} + (1 - \phi)(\phi f)^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}, \quad (3)$$

and governed by a potentially lower elasticity  $\rho \leq \gamma$ .

To make our model internally consistent, we assume that preferences across the two periods imply that the average level of trade in the first period (in the short-run) is the same as the level of trade in the second period (in the long-run). This is achieved by assuming that the value of parameter  $\phi$  is determined together with the trade cost  $\tau$  to imply:

$$x := \mathbb{E}\left\{\frac{f}{d + f(1 + \tau)}\right\} = \frac{f'}{d' + f'(1 + \tau)}, \quad (4)$$

where  $x$  is an exogenous bilateral trade intensity level in the model and  $\mathbb{E}$  denotes the expectation operator.<sup>12</sup>

In a simple way, the above specification of preferences captures the basic idea of dynamic trade elasticity. To see this, consider a 1 percentage point decrease in a trade cost paid for importing good  $f$  to the domestic country. Because the law of one price holds in the model, the price of the foreign good at home is the after-tariff price of this good abroad, which implies that the observed price of good  $f$  at home falls by approximately 1 percentage point.<sup>13</sup> At the same time, the response of quantities to such a permanent price shock

<sup>11</sup>Our specification has the desirable property that the consumption basket remains well defined for all values of  $\gamma$ , including the limit when  $\gamma \rightarrow 0$ . In such a case the function converges to a Leontief aggregator (up to a constant):  $c = \lim_{\rho \rightarrow 0} G_{(\omega, \gamma)}(d, f) \propto \min\{(1 - \omega)d, \omega f\}$ .

<sup>12</sup>The denominator is real GDP. See footnote 16 for more details.

<sup>13</sup>Approximately due to possible GE effects. To eliminate general equilibrium effects, assume there is a



unravels gradually: in the first period the ratio  $d/f$  increases by  $\rho$  percent, while after two periods (i.e. in the long-run of the model) it increase by  $\gamma$  percent. Thus, if  $\rho < \gamma$ , the implied measured short-run trade elasticity implied by the model is low, while the long-run trade elasticity is high.<sup>14</sup>

Finally, we assume a log-linear utility function in both periods given by  $u(c, l) = \log(c) - l$ , where  $c$  is aggregate consumption as defined above and  $l$  is labor supply.

**Production and feasibility** Goods are produced using (local) labor, which is the only input into production. The first period production functions are:

$$y = Al, \quad y^* = A^*l^*, \quad (5)$$

where  $A$  and  $A^*$  are country-specific productivity shocks and  $s \equiv (A, A^*)$  is distributed according to  $\pi(s)$ . The second period production function is deterministic and given by:

$$y' = l', \quad y'^* = l'^*. \quad (6)$$

The deterministic nature of the second period production function captures the fact that the second period pertains to the average behavior of the economy in the long-run.

Trade cost  $\tau$  works as an iceberg cost, and so for a unit of a good to be delivered across the border it is assumed that  $1 + \tau$  of the good must be shipped from the origin.<sup>15</sup> For a given  $\tau$ , a feasible allocation in the first and second period must thus satisfy:

$$d + d^* = Al - \tau d^*, \quad f + f^* = A^*l^* - \tau f, \quad (7)$$

$$d' + d'^* = l' - \tau d'^*, \quad f' + f'^* = l'^* - \tau f'. \quad (8)$$

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large set of similarly differentiated varieties  $f$ , and a tariff on only a small subset of differentiated goods  $f$  changes. For example, this is the approach underlying data estimation in K. Head & J. Ries (2001). Since such a setup is otherwise equivalent to our setup, we do not make such a distinction explicit here.

<sup>14</sup>An exercise in this spirit is at the heart of trade elasticity measurement in the data. For example, see Head & Ries (2001).

<sup>15</sup>This is a standard assumption and our results generalize to other specifications of trade costs. We have studied the case of tariffs or cost that is paid 1/2 in domestic good and 1/2 in foreign to assure neutrality with respect to prices. Results are available upon request from the authors.

**Trade and Trade Costs** To set the stage for our analysis of relating the model’s business cycle predictions to cross-sectional data, we treat bilateral trade intensity, here denoted by  $x$ , as an exogenously assumed target, while we make trade costs endogenous so as to sustain targeted level of trade in the long-run (on average in the first period and in the second period). Accordingly, we assume that trade cost  $\tau$  is a function of trade,  $\tau(x)$ , such that:<sup>16</sup>

$$x = \frac{f'}{d' + f'(1 + \tau(x))}, \quad (9)$$

where  $x$  is exogenously assumed trade intensity target. In terms of parameter restrictions, we focus on the case when there is home-bias, implying  $0 < x(1 + \tau) < 1/2$  is assumed throughout.

It should be clear how our assumptions link long-run trade intensity  $x$  to trade costs  $\tau$ . Specifically, the previously defined  $\phi$  ensures that the first period average level of trade is equal to the second period level of trade, while  $\tau(x)$  is a function of the assumed trade target  $x$  in that it is chosen to ensure that the endogenous trade intensity in the model is indeed equal to  $x$ . These two assumptions together make our framework consistent with the implications of most models of dynamic elasticity. In particular, when the elasticity is static, i.e.  $\rho = \gamma$ , our assumptions reduce to  $\phi = \omega$ , and imply that  $\tau$  is solely determined by this common trade elasticity. However, when  $\rho$  is lower than  $\gamma$ , for example in the extreme case  $\rho \rightarrow 0 < \gamma$ , the endogenous choice of the parameter  $\phi$  implies that  $c \propto \min\{xd, (1 - x)f\}$ , and thus the short-run consumption basket is determined by  $x$ . Yet, for the same value of  $\gamma$ ,  $\tau(x)$  is identical across the two cases. Intermediate cases of  $0 < \rho < \gamma$  imply a similar setup: the share of good  $f$  in consumption basket is also on average equal to  $x$ , except that in these cases it responds to relative prices with elasticity  $\rho$  when shocks hit.

**Equilibrium** We assume that households in our model operate a production technology, and by deciding how much to work, directly derive income from production. Households

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<sup>16</sup> Real GDP is measured in period zero prices, which is assumed to be equal to the symmetric solution  $p = 1$ . Real GDP is total consumption expenditures,  $C + G + I = d + f + \tau f$  plus net exports  $NX = d^* - f$ , which gives  $RGDP = d + d^* + \tau f$ . For more details, refer to Robert C Feenstra, Benjamin R Mandel, Marshall B Reinsdorf & Matthew J Slaughter (2009).

trade a complete set of financial assets contingent on the realization of the shock  $A, A^*$ . That is, before the first period starts, domestic households buy a set of claims  $B(s)$  conditional on realization of state  $s$ , entitles them to a payment equivalent to a unit of good  $d$  in state  $s$ . Analogous asset trade takes place before the second period (denoted by  $B'(s)$ ). Prices of assets traded before the first and the second periods are denoted by  $Q(s)$  and  $Q'(s)$ , respectively. In addition, we assume that goods  $d$  and  $f$  are traded in a competitive market, and denote the relative price of good  $f$  in terms of good  $d$  by  $p$ . We refer to this price as the *terms of trade*.

These assumptions imply the following household budget constraint in the first period:

$$\sum_s \pi(s)Q(s)B(s) = 0, \quad (10)$$

$$d(s) + p(s)f(s)(1 + \tau(x)) + Q'(s)B'(s) = Al(s) + B(s), \text{ all } s, \quad (11)$$

where  $B'(s)$  denotes domestic claims to a payment of one unit of good  $d$  in the second period.

The second period budget constraint is analogous and given by:

$$d'(s) + p'(s)f'(s)(1 + \tau(x)) = l(s) + B'(s). \quad (12)$$

The equilibrium in our model consists of foreign and domestic analogs of  $d(s)$ ,  $f(s)$ ,  $l(s)$ ,  $B(s)$ ,  $S(s)$ , and  $p(s)$ ,  $Q(s)$ ,  $Q'(s)$ , such that conditions (7), (8),  $B(s) + B^*(s) = 0$  and  $B'(s) + B'^*(s) = 0$  are satisfied, and given prices the allocation solves the household problem in the domestic country and in the foreign country, where the domestic country household problem is:

$$\sum_s \pi(s)[u(c(s), l(s)) + u(c'(s), l'(s))]$$

subject to (3), (2), (10), (11), and (12).

Given our focus on the business cycle dynamics of the model, it is instructive to note that  $B'(s) = B'^*(s) = 0$ , implying that the equilibrium allocation and prices in the first period can be found by solving an essentially static problem. Proposition 1 summarizes this result

and the problem we focus on in the remainder of this section.

**Proposition 1** *Equilibrium in the first period consists of an allocation, which in the case of the domestic country is given by  $c, d, f, l, B(s)$ , and prices  $Q(s), p(s)$ , such that given prices the allocation satisfies:*

$$\max_{d, f, l, B} \sum_s \pi(s) u[G_{(\phi(x, \tau), \rho)}(d(s), f(s)), l(s)]$$

$$\sum_s \pi(s) Q(s) B(s) = 0,$$

$$d(s) + p(s) f(s) (1 + \tau(x)) = Al(s) + B(s),$$

an analogous problem in the foreign country, and feasibility conditions given by (5), (7) and  $B(s) + B^*(s) = 0$ . Additionally,  $\tau(x)$  solves:

$$x = \frac{1}{1 + \tau(x) - \frac{\omega(\tau(x)+1)^\gamma}{\omega-1}},$$

while, given  $\tau(x)$ ,  $\phi(x, \tau(x))$  satisfies:

$$\phi(x, \tau(x)) = \left(1 + \frac{x(1 + \tau(x))^\rho}{1 - x - \tau(x)x}\right)^{-1}.$$

**Proof.** (Sketch) By welfare theorems, the allocation can be found by solving an appropriate planning problem, which can be represented by a a sequence of separate welfare maximization problems in each state and date (see appendix). This implies that the second period allocation is fully independent from the realization of first period shock and symmetric. Hence, no financial flows take place across the two periods, implying  $B'(s) = B^*(s) = 0$ . ■

## 2.2 International Transmission of Business Cycle Shocks

Below, we characterize how the foreign shock, i.e.  $A^*$ , affects output in the domestic country. We refer to it as the transmission of business cycles across borders. To this end, we approximate the solution of the model by log-linearizing it around the symmetric (deter-

ministic) state  $A = A^* = 1$ , and retain analytic formulas so as to study comparative statics properties of the model.<sup>17</sup> For the most part, in what follows, we focus on the log-linearized policy function for domestic and foreign output,  $y$  and  $y^*$ , given by:<sup>18</sup>

$$\hat{y} = a\hat{A} + b\hat{A}^*, \quad \hat{y}^* = b\hat{A} + a\hat{A}^*, \quad (13)$$

where  $a, b$  are the coefficients implied by the approximating hyperplane and  $\hat{y}, \hat{y}^*, \hat{A}, \hat{A}^*$  pertain to log deviations of the variables from the symmetric solution ( $A = A^* = 1$ ). Such a narrow focus is largely inconsequential, as all business cycle moments depend on model parameters through  $a$  and  $b$ , and so output can serve as a good example of more broader implications of our analysis.<sup>19</sup>

Proposition 2 shows that  $a + b = 1$ , and so  $b$  fully summarizes domestic country business cycle properties, as implied by<sup>20</sup>  $\hat{y} = (1 - b)\hat{A} + b\hat{A}^*$ . Proposition 2 also establishes that international spillover of shocks is positive in the model iff  $\rho < 1$ , which is not surprising as it is well known that a low value of short-run elasticity is needed to account for the positive comovement of business cycles in the data.

**Proposition 2** *The coefficients of the policy function in (13) are*

$$\begin{aligned} b &= 2(1 - \rho)(1 + \tau)x(1 - x(1 + \tau)), \\ a &= 1 - b. \end{aligned} \quad (14)$$

*Moreover, the model implies endogenous positive shock spillovers across countries ( $b > 0$ ) iff  $\rho < 1$ .*

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<sup>17</sup>The point of the approximation is analytic and changes with the parameters, and hence the coefficients of the tangent hyperplane fully preserve local comparative statics properties of the model. We use this ‘near closed-form’ approach to shed light how trade and comovement are linked by the theory.

<sup>18</sup>Since the economy is symmetric, the policy functions are also symmetric across countries.

<sup>19</sup>For example, consider the correlation coefficient between foreign and domestic output. In such a case, as long as the  $(A, A^*)$  process is independent of trade  $x$ , correlation coefficient is a monotone increasing function of  $b$ , assuming as we show below  $b = 1 - a$ . If  $A$  and  $A^*$  are i.i.d., correlation coefficient is:  $\rho_{y, y^*} = \frac{1}{1 - 2(1 - b)b} - 1$ . The presence of such a monotone transformation has no bearing on our conclusions. This is clear from what follows next.

<sup>20</sup>Note that admissible trade levels imply  $b < 0.5$  which makes comparative statics with respect to  $b$  valid only for  $b < 0.5$ .

**Proof.** In Appendix. ■

We next define our object of interest: the total derivative  $db/dx$ , which we label as the *trade-comovement link*. This derivative measures how trade (locally) affects the degree foreign shock spillovers into the domestic country, and it also relates to a regression coefficient between trade and business cycle comovement.

**Definition 1** *Let the **trade-comovement link** be the total derivative of the spillover parameter  $b$  w.r.t. bilateral trade intensity  $x$ , i.e.  $db/dx$ .*

Proposition 3 below derives the main result of this section. It shows that  $db/dx$  is a function of the dynamic properties of trade elasticity, that is, it depends separately on  $\rho$  and  $\gamma$ . Corollary 1 derives the key implication of this result by showing that, qualitatively, dynamic elasticity crucially affects the sign of  $db/dx$ . In particular, in the case of dynamic elasticity ( $\rho < 1 < \gamma$ ), the model exhibits a robust positive trade-comovement link. In contrast, in the case of static elasticity ( $\rho = \gamma$ ), our model implies a strongly negative link. These results are illustrated in Figure 1 for a wide range of model parameters.

The static elasticity case illustrates the known failure of standard models to generate a sufficiently positive link between trade and business cycle synchronization. On the other hand, the dynamic elasticity case points to a natural path for the resolution of this puzzle, i.e. explicit modeling of a high long-run trade elasticity alongside a low short-run elasticity. Data estimates point to  $\rho < 1$  and  $\gamma$  well in excess of 1, which in our model is sufficient for a positive link. We exploit these finding in the next section, which aims to quantify these effects in a context of a specific application.

**Proposition 3** *For any admissible initial trade intensity  $0 < x(1 + \tau) < 1/2$ , the trade-comovement link is given by*

$$\frac{db(x, \tau(x))}{dx} = -\frac{(1 - \gamma)}{\gamma} 2(1 - \rho)(1 + \tau) \frac{(1 - 2x(1 + \tau))(1 - x(1 + \tau))}{1 + x(1 + \tau)^{\frac{1-\gamma}{\gamma}}}. \quad (15)$$

**Proof.** Take the total derivative of the coefficient  $b$  listed in Proposition 2. ■

**Corollary 1** *For any admissible initial trade intensity  $0 < x(1 + \tau) < 1/2$ :*

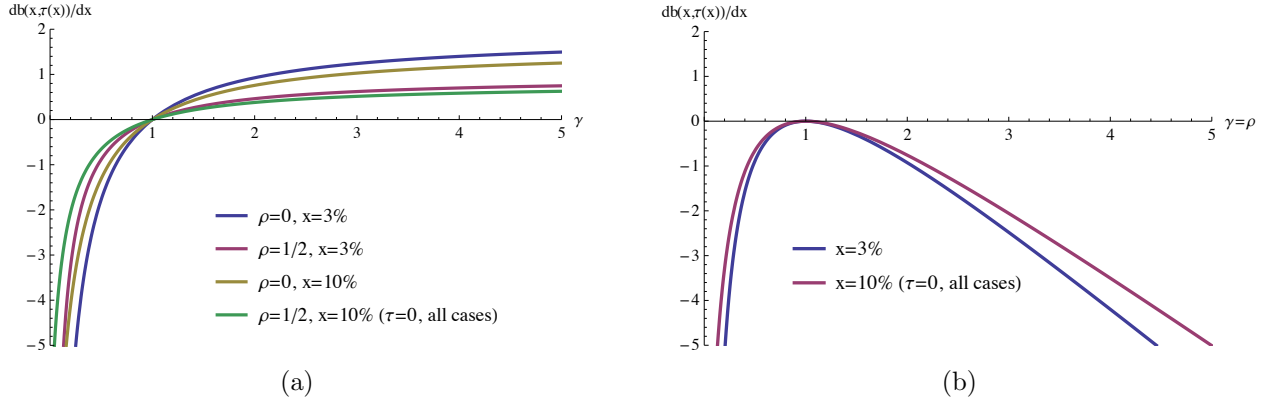


Figure 1: Trade-comovement link in dynamic (a) and static (b) elasticity models as a function of the long-run ( $\gamma$ ) and short-run ( $\rho$ ) trade elasticity.

- i) In the dynamic elasticity economy ( $\rho < 1 < \gamma$ ), the trade-comovement link ( $db/dx$ ) is always positive.
- ii) In the static elasticity economy (for all  $\rho = \gamma > 0$ ), the trade-comovement link ( $db/dx$ ) is always negative.

**Proof.** For (i), observe that (15) is always positive under the stated assumptions (note that  $2x(1 + \tau)$  is less than 1 and  $1 + \tau$ ). (ii) follows from equating  $\gamma = \rho$ , which gives  $\frac{db}{dx} = -2(1 + \tau)(1 - \rho)^2 \frac{(1 - 2x(1 + \tau))(1 - x(1 + \tau))}{\rho + x(1 + \tau)(1 - \rho)} < 0$ . ■

### 2.3 Why Dynamic Trade Elasticity Affects Business Cycle Transmission

As the next proposition shows, the result in Corollary 1 is driven by the fact that the impact of trade on the shock spillover coefficient  $b$  is determined by *both* trade  $x$  and trade cost  $\tau$ , which in this exercise is varied to induce variation of trade.

**Proposition 4** Trade-comovement link derived in the previous proposition can be decomposed as follows:

$$\frac{db(x, \tau(x))}{dx} = \frac{\partial b(\tau(x), x)}{\partial x} + \frac{\partial b(\tau(x), x)}{\partial \tau} \frac{d\tau(x)}{dx},$$

where

$$\begin{aligned}\frac{\partial b}{\partial x} &= 2(1 - \rho)(1 + \tau)(1 - 2(1 + \tau)x), \\ \frac{\partial b}{\partial \tau} &= \frac{x}{1 + \tau} \frac{\partial b}{\partial x}, \\ \frac{d\tau}{dx} &= -\frac{1}{x \left( (1 - \gamma)x + \frac{\gamma}{\tau + 1} \right)}.\end{aligned}$$

**Proof.** Take the total derivative of the coefficient  $b$  in Proposition 2. ■

As is clear from the statement of the proposition, the effect of trade costs is crucially influenced by how much trade costs need to be varied to induce the targeted variation of trade intensity, i.e. on the term  $d\tau/dx$ . This follows from the fact that trade is endogenously determined by trade costs in the model. Since this term approximately equals the inverse of the long-run trade elasticity, and since  $\partial b/\partial \tau > 0$ , the long-run elasticity impacts  $db/dx$  in a significant way. Interestingly, the above result additionally shows that  $sign(\partial b/\partial x) = sign(\partial b/\partial \tau)$ , and so  $\partial b/\partial \tau = 0$  implies  $db/dx = 0$ . This means that in most applications in which a model involves non-trivial relationship between trade and comovement the long-run elasticity will factor in (Corrolary 2 below).

**Corollary 2** *For any positive trade level  $x > 0$ ,  $\partial b/\partial \tau = 0$  implies  $db/dx = 0$ .*

## 2.4 Why Trade Costs Affect Business Cycle Transmission

The key reason why long-run trade elasticity matters is because trade costs influence business cycle properties of the model, i.e.  $\partial b/\partial \tau > 0$ . To understand why, note that in our model there are two channels of shock transmission across the borders. The first channel works through a substitution effect between consumption and leisure induced by endogenous fluctuations in the terms of trade,  $p$ . The second effect is associated with an income effect of terms of trade fluctuations and asset payout  $B(s)$ , which jointly implement risk sharing between countries.

To isolate the two channels of shock transmission, we define a function  $R$  that captures any redistributive (zero-sum) transfers between the two countries over the business cycle.



Using the domestic budget constraint, these transfers are given by:

$$R(s) := B(s) + (1 - p(s))f(s). \quad (16)$$

$R$  includes direct asset payments  $B(s)$ , as well as re-distributive income effect associated with the terms of trade fluctuations. For  $\tau = 0$ , these are the only income effects in the model. For  $\tau \neq 0$ , there are additional income effects associated with the fact that part of imports is lost in transportation, and it is incurred in different good in each respective country. This can be immediately seen from the budget constraint of the domestic household, which can be re-written using  $R$ :

$$d(s) + f(s) = Al(s) + R(s) - \tau(x)p(s)f(s).$$

As is clear from the expression, the income side of the budget constraint includes labor income, re-distributive transfer  $R$ , but also the income lost due to the presence of trade costs.<sup>21</sup>

We next proceed to decompose the equilibrium dynamics of quantities in response to prices into income and substitution effects. That is, we evaluate the first order conditions as functions of terms of trade and other prices, but substitute for  $R$  into budget constraints. We then log-linearize the relevant equilibrium conditions by treating  $p$  and  $R$  as exogenous stochastic processes. Finally, we use all remaining equilibrium conditions to describe the dynamics of  $p$  and  $R$ . We do so by incrementally expressing them as the function of state  $s$ .<sup>22</sup>

$$\begin{aligned} \hat{y} &= \alpha \hat{A} + \eta \hat{p} + \chi \hat{R}, \\ \hat{p} &= \mu(\hat{A}^* - \hat{A}) + \xi \hat{R}, \\ \hat{R} &= \theta(\hat{A}^* - \hat{A}). \end{aligned}$$

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<sup>21</sup>Substituting  $R$  into the foreign budget constraint gives an expression featuring a term that is similarly proportional to  $\tau$ :  $d^*(s) + f^*(s) = A^*l^*(s) - R(s) - \tau \left[ \frac{1-p(s)}{1+\tau} (A^*l^*(s) - f^*(s)) + d^*(s) \right]$ .

<sup>22</sup>For details of the derivation, see the Appendix.

The above system can be used to trace the effect of  $A^*$  on domestic output, and allows us to obtain the following decomposition of the underlying effects:

$$b = \underbrace{\eta(\mu + \xi\theta)}_{SE_{+\tau}} + \underbrace{\chi\theta}_{IE_{-\tau}}. \quad (17)$$

The terms identified by the above decomposition have clear interpretation. For low levels of trade costs, the first effect,  $SE_{+\tau}$ , captures the substitution effect of the terms of trade on domestic labor-leisure choice (and income effect of trade costs). Since the fluctuations of the terms of trade are implied by the built-in complementarity between domestic and foreign goods, we refer to this effect as the *complementarity channel*. The second effect captures the re-distributive income effect associated with risk sharing transfers. It reflects the fact that, due to concave utility function, countries trade assets so as to share the risk of business cycle fluctuations, as well as income transfers implied by terms of trade fluctuations. We refer to this effect as the *risk sharing channel*.

Proposition 5 derives the decomposition laid out above, showing that trade costs only affect the redistributive income effect. Corollary 3 summarizes this key result. It states that trade costs matter for business cycle properties of the model, as they amplify the transmission of shock across borders due to their dampening effect on risk sharing—which by itself is a source of negative shock transmission in the model and negative trade comovement link.

**Proposition 5** *The decomposition defined by equation (17) implies:*

$$\begin{aligned} SE_{+\tau} &= x, \\ IE_{-\tau} &= x(1 + 2\tau - 2(1 + \tau)(\rho + x(1 - \rho)(\tau + 1))). \end{aligned}$$

**Proof.** In Appendix. ■

**Corollary 3** *Trade costs dampen the negative re-distributive income effect associated with endogenous terms of trade fluctuations  $p$  and asset trades  $B(s)$ .*

To see the intuition behind Corollary 3, note that the risk sharing channel is driven

by the fact that, over the business cycle, it is optimal to shift productive resources to the more productive country, and sustain consumption in the less productive country through an appropriate transfer  $R > 0$ . Since higher trade costs make the implementation of such a scheme more costly, as it involves importing goods from the foreign country,  $\partial b/\partial\tau > 0$  follows. <sup>23</sup>

Finally, we should mention that, while we rely on productivity shocks, this assumption does not affect our conclusions. Similar arguments would apply to a model featuring demand shocks. This stems from the fact that risk sharing is a feature of the environment that is independent from the type of shocks that drive the cycle. Risk sharing follows from the curvature of the utility function, country specific shocks, and the ability of household to trade assets that fully or partially allow to disperse business cycle risks.

## 2.5 Why Trade Intensity Affects Business Cycle Transmission

In isolation, trade intensity  $x$  can lead to  $db/dx > 0$ . The primary force behind this is the complementarity channel, which can be seen by differentiating the equations from Proposition 5 with respect to  $x$ . Such differentiation yields:  $\frac{\partial SE_{+\tau}}{\partial x} = 1$  and  $\frac{\partial IE_{-\tau}}{\partial x} = 1 + 2\tau - 2(1 + \tau)(\rho + 2(1 - \rho)(\tau + 1)x)$ .

**Corollary 4** *Higher trade intensity amplifies the substitution effect between consumption and leisure associated with endogenous terms of trade fluctuations  $p$ .*

The intuition behind the complementarity effect ( $\partial SE_{+\tau}/\partial x > 0$ ) is fairly straightforward. The more countries trade with each other, the greater is the share of foreign goods in the domestic country's consumption basket. As a result, when the world supply of the foreign good goes up due to a positive productivity shock there, and the price of the foreign good falls due to imperfect substitutability between the two goods, the price of domestic

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<sup>23</sup>While it may appear that our assumption of complete asset markets is critical for the this effect, this is not entirely the case. The bulk of risk sharing in this environment is carried out by endogenous terms of trade fluctuations rather than active asset trade, and so some effects still stay. This is a known feature of the business cycle models, discussed in detail by Harold L Cole & Maurice Obstfeld (1991). While autarky reduces the term  $\partial b/\partial\tau$ , it does not eliminate it.

consumption falls in terms of domestic good and thus also in terms of domestic labor (i.e. real wage goes up). In response, domestic households' supply more labor, and output goes up. This induces a positive transmission of foreign shock into the domestic country. Since the increase in domestic output correlates positively with the foreign output, and the effect is stronger when the two countries trade more to begin with, more trade implies a stronger effect, which explains  $\partial SE_{+\tau}/\partial x > 0$ .<sup>24</sup>

The complementarity channel is not the only force that is present here. The risk sharing channel also affects the direct effect of trade on business cycle transmission. However, its contribution is smaller in cases when  $\rho$  is close to data estimates (around 1/2). To economize on space, we do not discuss it here.

Having established that modeling of trade elasticity is crucial for understanding how shocks are transmitted across the border, we next study the quantitative implications of these findings. To this end, we develop a model of fully endogenous dynamic elasticity, and relate it to cross-country data. As we show, in such a model the effects identified in this section are quantitatively significant and impact the model's predicted relation in a crucial way. In particular, we find that modeling dynamic elasticity can bring the predictions of an otherwise standard theory in line with the data.

### 3 Quantitative Analysis

In what follows, we lay out the environment of our quantitative model, discuss the empirical findings which provide a quantitative benchmark for the theory, outline the model's parameterization, and then evaluate the quantitative predictions of the parameterized model vis à vis the data.

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<sup>24</sup>In our formulation, the overall strength of the complementarity channel does not explicitly depend on the value of the short-run elasticity. This is due to the dependence of  $p$  on the risk-sharing function  $R$  in (17). When we consider the pure complementarity channel, defined as  $SEP_{+\tau} := \eta\mu$ , and given by  $SEP_{+\tau} = \frac{x}{2\rho - \frac{\tau}{\tau+1} + 2(1-\rho)(\tau+1)x}$ , its strength depends negatively on  $\rho$  for all elasticities bigger than  $x/(1-x)$ , which for typical parameterizations means essentially always.

### 3.1 Quantitative Model

Our setup is based on the model by Drozd & Nosal (2012), extended to include three countries of varying size. Time is discrete,  $t = 0, 1, 2, \dots$ , and horizon infinite. The first two countries, labeled *domestic* (D) and *foreign* (F), are symmetric and of equal size, and the third country, labeled *rest of the world* (W), is allowed to differ in size. The size of each country is determined by the population size of atomless households residing in the country, denoted by  $L_i, i = D, F, W$ .

Labor and capital, supplied by the households, are assumed to be immobile across countries, and are used by local producers to produce goods. Goods are differentiated by the country of origin and are tradable. The good produced in the domestic country is labeled  $d$ , the good produced in the foreign country is labeled  $f$ , and the good produced in the rest of the world is labeled  $g$ . Households in each country use these goods for consumption and investment in physical capital. Their preferences are characterized by imperfect substitutability between each type of good, and a bias towards the locally produced good. Financial markets are assumed to be complete. As before, the presentation of the model will be from the domestic country perspective, with the remaining countries' problems being analogous.

**Technology and Notation** Tradeable goods are country-specific and are produced by a unit measure of atomless competitive producers residing in each country. Producers employ local capital and labor and use the technology available in their country of residence. Production technology is Cobb-Douglas,  $Ak^\alpha l^{1-\alpha}$ , and is subject to country-specific technology shock  $A$ , given by an exogenous AR(1) process:  $\log(A) = \psi \log(A_{-1}) + \varepsilon$ , where  $0 < \psi < 1$  is the shock persistence parameter, and  $\varepsilon$  is a Normally distributed i.i.d. random variable with zero mean.

We summarize production constraints by an economy-wide marginal cost  $v_D$ , which, given competitive factor prices  $w$  and  $r$ , is

$$v_D \equiv \min_{k,l} \{wl + rk \mid Ak^\alpha l^{1-\alpha} = 1\}. \quad (18)$$

The history of technology shocks up to and including period  $t$  is denoted by  $s^t$ . All variables formally depend on  $s^t$ , but we suppress this notation whenever possible.

**Preferences and Consumption** The model features an infinitely lived representative household, which trades a set of complete state-contingent bonds, accumulates physical capital, supplies labor and consumes. The household's preferences for goods  $d, f, g$  are described via a CES aggregator  $G$ :

$$G(d, f, g) = \left( \omega_d d^{\frac{\gamma-1}{\gamma}} + \omega_f f^{\frac{\gamma-1}{\gamma}} + \omega_g g^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

The household sector's problem is a completely standard, 3-country, complete markets maximization problem, and we omit it here.

**Endogenously Dynamic Elasticity** The crucial feature of the model is that producers actively match with the retailers, who then supply the goods to households through a local competitive retail market. In each period, a free-entry-determined mass of retailers  $h$  are searching to meet with producers in the domestic country. Producers, in order to attract the searching retailers, accumulate what we term *marketing capital*  $m$ —separately in each country where the producer wants to sell the goods. The marketing capital accumulated by a given producer relative to the marketing capital held by other producers in that market determines the fraction of searching retailers that meet with this producer. Formed matches are long-lasting, with separation rate  $\delta_H$ , which gives the law of motion for the customer base  $H_D$  of a domestic producer with marketing capital  $m_D^d$  accumulated in the domestic country:

$$H_D = (1 - \delta_H)H_{D,-1} + \frac{m_D^d}{\bar{M}_D}h, \quad (19)$$

where  $\bar{M}_D$  is the average marketing capital of all producers selling at home:<sup>25</sup>  $\bar{M}_D = \bar{m}_D^d + \bar{m}_D^f + \bar{m}_D^g$ .

The size of the customer base is critical for the producer as it puts a limit on the amount

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<sup>25</sup>In equilibrium  $\bar{m}$  and  $m$  coincide as there is measure one of producers in each country.

of good they can sell in each country. Specifically, in each match one unit of the good can be traded per period, which gives the *sales constraint* of the producer of good  $d$  in the domestic country,  $d \leq H_D$ . In equilibrium, for our parameter specification, this condition always binds, i.e.  $d = H_D$ .

The accumulation of marketing capital follows a standard capital-theoretic law of motion with depreciation rate  $\delta_m$  and adjustment cost  $\phi$ . Specifically, given last period's level of marketing capital  $m_{D,-1}^d$  and the current level of instantaneous marketing input  $a_D^d$ , current period's marketing capital is given by

$$m_D^d = (1 - \delta_m)m_{D,-1}^d + a_D^d - \phi m_{D,-1}^d \left( \frac{a_D^d}{m_{D,-1}^d} - \delta_m \right)^2. \quad (20)$$

This specification features decreasing returns from the instantaneous marketing input  $a_D$ , parameterized by the *market expansion friction parameter*  $\phi$ . The cost  $\phi$  captures the fact that the build-up of marketing related assets, like brand awareness, reputation or distribution network takes time and resources. Importantly, Drozd & Nosal (2012) show that this specification, together with the assumption that country specific goods are closely substitutable, generates a high long-run and low short-run price elasticity of trade flows – the dynamic elasticity we identified as crucial in our theoretical analysis.

**Prices** Producers from the domestic country sell goods in countries  $i = D, F, W$  for the wholesale prices  $x_i p_i^d$ , where  $x_i$  is the real exchange rate between country  $i$  and the domestic country. These prices are determined by bargaining with the retailer, who resells the good in a competitive domestic retail market for price  $P_d$ , which is determined by the domestic consumer's valuation of the good. Specifically, we assume that the wholesale prices are set in consistency with a Nash bargaining solution with continual renegotiation over the continuation surplus from the match. As Drozd & Nosal (2012) show, the solution is a

simple static surplus splitting rule which gives:<sup>26</sup>

$$p_D^d = \theta(P_d - v_D) + v_D,$$

where  $\theta$  is the Nash bargaining power of the producers. The competitive retail prices in the above equation come from the household problem, and are given by the partial derivatives of the CES aggregator from the household's problem:  $P_d = G_d(d, f, w)$ .

**Profit maximization** Given a customer base of a domestic producer in each country,  $H_D, H_F, H_G$ , the instantaneous profit function  $\Pi$  of the producer is given by the difference between the profit from sales in each market and the total cost of marketing the goods:

$$\Pi = \sum_{i=D,F,W} (x_i p_i^d - v_D) H_i - \sum_{i=D,F,W} x_i v_i a_i^d. \quad (21)$$

Dynamically, a representative producer from the domestic country, who enters period  $t$  in state  $s^t$  with the customer base  $(H_D, H_F, H_W)$  and marketing capital  $m_D^d, m_F^d, m_W^d$  chooses the allocation of marketing expenditures  $a_D^d, a_F^d, a_W^d$ , period- $t$  marketing capitals and customer bases, to satisfy the Bellman equation

$$V = \max \{ \Pi + EQV_{+1} \}$$

where  $Q$  is the stochastic discount factor implied by the household problem and the optimization is subject to the marketing technology constraints (20) and the laws of motion for customer base (19).

**Free Entry of Retailers** In each country there is a sector of atomless retailers, who purchase goods from producers and resell them to local households. Retailers who enter into the sector must incur an initial search cost  $\chi v_d$  in order to find a producer with whom they can match and trade. The matching probabilities are taken as given by entering retailers,

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<sup>26</sup>For the case with tariffs, we maintain the static surplus splitting rule with the same proportion  $\theta$ .



but in equilibrium are determined in consistency with (19).

Industry dynamics are governed by a free entry and exit condition, which endogenously determines the measure  $h$  of searching retailers at each date and state:

$$\frac{\bar{m}_D^d}{\bar{M}_D} J_D + \frac{\bar{m}_D^f}{\bar{M}_D} J_F + \frac{\bar{m}_D^g}{\bar{M}_D} J_W = \chi v_D, \quad (22)$$

where  $J_i$  is the value function associated with being matched with an  $i$  country producer, which satisfies the Bellman equation:

$$J_i = P_i - x_i p_i + (1 - \delta_H) E Q J_{i,+1}.$$

The equilibrium is defined in the usual way and its definition is omitted here.

## 3.2 Empirical Evidence

This section discusses the empirical link between bilateral trade and comovement of business cycles in a sample of 20 industrialized countries over the period 1980Q1-2011Q4. Specifically, we use regression analysis to quantify the effect of bilateral trade intensity on the bilateral correlation of GDP. We find strong and significant effect of trade. Additionally, we identify a nonlinearity in the data: the effects seem to be much stronger in part of our sample with above-median bilateral trade. We use our estimates to quantitatively evaluate the performance of our model.

For our empirical exercise, we use data on real GDP<sup>27</sup> in order to construct measures of bilateral correlations of GDP for 190 country pairs.<sup>28</sup> Countries in our sample constitute about 59% of world GDP and 53% of world trade (as of year 2011). We run a cross-sectional

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<sup>27</sup>In Online Appendix, we also provide evidence and model results for regressions using TFP correlations. For a list of data sources, see the Online Appendix.

<sup>28</sup>Our country list includes: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

regression of the correlations on a measure of bilateral trade, of the form:

$$\text{corr}(x_i, x_j) = \alpha + \beta_x \text{trade}_{ij} + X_i + X_j + E_{ij} + \varepsilon_{ij}. \quad (23)$$

In the regression,  $\text{corr}(x_i, x_j)$  is the correlation between countries  $i$  and  $j$  of the logged and HP-filtered series of real GDP.  $X_i$  and  $X_j$  are country dummies, and  $E_{ij}$  is the European dummy, which takes the value of 1 if both countries in the pair are European countries. The variable  $\text{trade}_{ij}$  is a symmetric measure of bilateral trade intensity of countries  $i$  and  $j$ , measured at the beginning of the sample<sup>29</sup> (in 1980), and given by the log of

$$\max\left\{\frac{IM_{ij}}{GDP_i}, \frac{IM_{ji}}{GDP_j}\right\}, \quad (24)$$

where  $IM_{ij}$  are nominal imports (in US dollars) by country  $i$  from country  $j$  and  $GDP_i$  is the nominal GDP (in US dollars) of country  $i$ , both measured in year 1980.

The measure of trade defined in (24) varies in our sample from 0.03% (Korea with Portugal) to 27% (Ireland with United Kingdom). It is symmetric, and immune to having trade partners of very different size. In particular, it is able to capture relationships of small countries with large countries—if the United States is an important trading partner to Canada but less so vice versa, the equation in (24) will capture a significant number.<sup>30</sup>

Table 1 reports the results from our regression analysis. We include OLS results as well as results from an IV regression in which the instruments are common border, common language and distance. The first column of Table 1 gives the results for the whole sample of 190 pairs. Both OLS and IV regressions give highly significant positive coefficients, which suggests a strong effect of bilateral trade on comovement of GDP. The estimated numbers imply that moving from the 10th to the 90th percentile of the bilateral trade spectrum increases the GDP correlations by 0.21 (IV) or 0.11 (OLS). Relative to median GDP correlation of 0.52

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<sup>29</sup>All of the results are robust to picking other years as base year for the bilateral trade measure.

<sup>30</sup>This is in contrast to measures expressed as averages, for example  $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$ . Such measure gives small numbers when trade partners have asymmetric sizes, i.e. small countries trade with big countries. For example, our measure is 8 times higher than  $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$  for Germany-Austria pair, 6 times bigger for the US-Canada pair, and 15 times bigger for the UK-Ireland pair. Our empirical results are robust to using this alternative measure – the results are available from the authors upon request.

in our sample, this is an economically significant effect.

Columns 2 and 3 of Table 1 present results for pairs of countries in the lower and upper halves of the bilateral trade distribution (the median trade intensity in our data is 0.85%). This split of the sample indicates a strong nonlinearity occurring at low levels of trade. In particular, we find that the trade-comovement relationship is much stronger in the higher trade sample. The coefficients for all specifications increase and remain highly significant in the ‘top 50%’ column, while the lower bilateral trade intensity sub-sample exhibits essentially no trade-comovement relationship. For the OLS specification, the coefficient on  $trade_{ij}$  is small and not significantly different from zero. For the IV specification, this sub-sample has a weak instrument problem<sup>31</sup>, and the coefficient is insignificant as well.

Summarizing, our results indicate a statistically and economically significant effect of bilateral trade intensity on GDP comovement across countries, which is also consistent with results in the literature.<sup>32</sup>

Table 1: Regression results: trade-comovement in the data.

Dependent Variable: GDP correlation			
	OLS	OLS bottom 50%	OLS top 50%
$trade_{ij}$	0.034** (0.016)	-0.017 (0.031)	0.055** (0.025)
$E_{ij}$	0.060 (0.093)	0.311 (0.221)	-0.076 (0.099)
R-squared	0.694	0.758	0.651
	IV	IV bottom 50%	IV top 50%
$trade_{ij}$	0.065*** (0.024)	0.325 (0.286)	0.070** (0.033)
$E_{ij}$	-0.028 (0.106)	-0.373 (0.661)	-0.103 (0.107)

\*\* , \*\*\* denote significance at 5% and 1% level. Numbers in parentheses are standard errors.

<sup>31</sup>In the lower trade subsample, the first stage regression F-statistic is 1.36, significantly below the cutoff of 10.

<sup>32</sup>This result has also been confirmed by other studies for a variety of specifications – see, for example Kose & Yi (2006), Baxter & Kouparitsas (2005) or Clark & van Wincoop (2001), among others.

### 3.3 Parameterization

This section describes how we choose functional forms and parameter values. First, we motivate our choice of targets for the elasticity of substitution  $\gamma$  and the market expansion friction  $\phi$ , which are crucial for our quantitative exercise, as they determine the long-run and short-run elasticities. Then, we describe the choice of the remaining parameters. The baseline period length in the model and in the data is one quarter. We parameterize the utility function in a standard constant relative risk aversion specification:

$$u(c, l) = \frac{(c^\eta(L - l)^{1-\eta})^{1-\sigma}}{1 - \sigma}.$$

Unless otherwise stated, we choose the same parameters for all three countries.<sup>33</sup>

**Short-run - long-run elasticity and the marketing frictions.** In the model, the long-run response of the product mix ratio<sup>34</sup>  $\frac{f}{d}$  to the relative price of the domestic good versus the foreign good is equal to the Armington elasticity  $\gamma$ . After a permanent tariff reduction of  $\Delta T$  percent,<sup>35</sup> the product mix ratio in the model changes by  $\Delta \log \frac{f}{d} \approx -\gamma \Delta T$ . Intuitively, in the long-run the marketing friction is slack, and the response of trade depends only on the elasticity of substitution between domestic and foreign good. Since an analogous equation has been estimated in the trade literature to measure the elasticity parameter, we can directly adopt these estimates. We set  $\gamma$  equal to 15, which is close to the upper limit of the values reported in the trade literature (see Ruhl (2008)).

Over the business cycle, the long-run relation between relative prices and trade flows is severed in our model—the marketing frictions limit the instantaneous response of quantities to price fluctuations, leading to a low estimated short-run elasticity of substitution, as in Drozd & Nosal (2012). We adopt the parameterization strategy and measurement of the short-run elasticity of that paper, and choose the market expansion friction parameter  $\phi$  to

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<sup>33</sup>Whenever possible, we make sure that the targets are obtained in all three economies, otherwise they are obtained exactly for the domestic economy.

<sup>34</sup>In the model, the same relationship holds for  $\frac{g}{d}$  and also for aggregate imports  $\frac{imports}{d}$ . We use only  $\frac{f}{d}$  for expositional simplicity.

<sup>35</sup>Which is equivalent to a permanent change of the the price of  $d$  relative to  $f$ .

match the measured value of *volatility ratio*<sup>36</sup> in our cross-section of countries of 1.17.

**Independently calibrated parameters.** Here we describe the choice of parameters which can be independently calibrated. They are: (i) the discount factor  $\beta$  chosen to reproduce the average annual risk free real interest rate of 4.1%, (ii) Cobb-Douglas production function  $k^\alpha l^{1-\alpha}$  with parameter  $\alpha$  chosen to reproduce the constant labor share of 64%, (iii) depreciation of physical capital of 2.5% (quarterly) as in David K. Backus, Patrick J. Kehoe & Finn E. Kydland (1995)<sup>37</sup>, and finally, (iv) the standard value for the intertemporal elasticity of substitution/risk aversion parameter  $\sigma$  of 2. We arbitrarily fix  $\delta_h = 0.1$ , implying that the matches in the economy last on average 2.5 years (10 quarters).<sup>38</sup> We also choose population sizes  $L_i$ , to be 20 times larger in World than Domestic or Foreign.

**Productivity shock process.** The country-specific productivity shock  $A(s^t)$  is assumed to follow an AR(1) process with no cross-country spillovers

$$\log(A_i(s^t)) = \psi_i \log(A_i(s^{t-1})) + \varepsilon_i(s_t),$$

where the residuals  $\varepsilon_i$  are assumed to be normally distributed with zero mean, standard deviation  $\sigma_i^2$ , and correlation coefficients  $\rho_{ij}$ .

We set the parameters of the productivity process to be symmetric for the bilateral pair in the model (domestic and foreign) and set it to match the median behavior of real GDP in our sample of countries. For the world country, we set its productivity process to match the median behavior of real GDP for the relative Rest of the World in our sample. Specifically, we set  $\psi_D = \psi_F$  and  $\sigma_D = \sigma_F$  to match the median autocorrelation and standard deviation

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<sup>36</sup>To construct the *volatility ratio* we use constant and current price values of imports and domestic absorption  $DA$ , given by  $DA = (C + G) + I - IM$ . The prices are taken to be their corresponding price deflators. Denoting the deflator price of domestic absorption by  $P_{DA}$  and the deflator price of imports by  $P_{IM}$ , the *volatility ratio* is then defined as  $\sigma(\frac{IM}{DA})/\sigma(\frac{P_{DA}}{P_{IM}})$ , where  $\sigma$  refers to the standard deviation of the logged and Hodrick-Prescott filtered quarterly time series. Notice that the *volatility ratio* places an upper bound on the regression coefficient between the two variables underlying its construction.

<sup>37</sup>It implies investment to GDP ratio of 25%. In the recent data we find 20% in US, 28% Japan, 22% Germany, 21% France. The OECD median is close to 20%. We adopt this number to make the model implications more comparable to the literature.

<sup>38</sup>For the details on national accounting in the model, see the Online Appendix.

of real GDP in our sample of 0.83 and 1.41%. For World, we set  $\psi_W$  and  $\sigma_W$  to match the autocorrelation of real GDP and standard deviations of 0.89 and 1.05%, respectively. To set the correlations of innovations, we target the median correlation of real GDPs within our bilateral pairs of 0.52 to set  $\rho_{DF}$  and target the median correlation of real GDPs of our sample countries with their relative Rest of the World of 0.66 to set  $\rho_{DW} = \rho_{FW}$ .

**Jointly calibrated parameters.** The group of parameters jointly calibrated include the marketing friction parameter  $\phi$ , depreciation of marketing capital  $\delta_m$ , the up-front search cost  $\chi$ , the bargaining power  $\theta$ , home-bias parameters  $\omega_i^j$  and the consumption share parameter  $\eta$ . Because each parameter influences more than one target at the same time, the calibration must be joint. We choose their values simultaneously using the following targets: (i) producer markups of 10% as estimated by S. Basu & J. G. Fernald (1995), (ii) volatility ratio of 1.17 equal to the median value in our sample (iii) relative volatility of the real export price  $p_x$  to the real exchange rate  $x$  of 37% consistent with the data for the US, (iv) marketing to GDP ratio of 4.5%, which is half way between estimates of marketing/sales of 7% reported by G. L. Lilien & J. D. C. Little (1976) and advertising/GDP of around 2% reported in R. Coen (June 2007), (v) the standard value for the share of market activities in the total time endowment of households equal to 30% (T. Cooley (1995)) and finally (vi) our measure of bilateral trade between two symmetric countries of 0.85% and between a small country and the rest of the world of 19.03% which are the median values in our sample. The parameter values in the benchmark calibration are presented in Table 2.

**Targeted trade patterns.** We vary bilateral tariffs in order to reproduce exactly three values for each of our 190 country pairs: (i) the bilateral trade intensity as defined in equation (24) and imports/GDP from the relative rest of the world of country 1 (ii) and country 2 (iii) in the pair.

**Frictionless model.** For comparison purposes, we report results from a 3-country version of the frictionless international business cycle model (Backus, Kehoe & Kydland (1995)). The frictionless model is parameterized in the same way, whenever applicable. We report

the values of the parameters in Table 3.

Table 2: Parameter values in the benchmark calibration

Parameter		Value
Preference Parameters		
$\gamma$	elasticity of substitution	15
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.3929, 0.2917, 0.3154
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	0.2917, 0.3929, 0.3154
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.3154, 0.3154, 0.3691
$\eta$	leisure weight in utility	0.346
$\sigma$	risk aversion	2
$\beta$	time discount factor	0.99
Technology Parameters		
$\alpha$	capital share	0.36
$\delta$	depreciation of physical capital	0.025
$\delta_H$	match destruction rate	0.1
$\chi$	search cost	1.13
$\delta_m$	depreciation of marketing capital	0.0149
$\phi$	adjustment cost of marketing capital	3.06
$\theta$	bargaining power of producers	0.449
Other Parameters		
$\psi_1, \psi_2, \psi_3$	persistence of the productivity shock	0.54, 0.54, 0.71
$\rho_{12}$	Cross-correlation of productivity shocks	0.43
$\rho_{13}, \rho_{23}$	Cross-correlation of productivity shocks	0.61, 0.61
$L_D, L_F, L_W$	population sizes	1, 1, 20

### 3.4 Results

This section presents quantitative results from the benchmark model, and compares them to our empirical findings in Section 3.2, as well as the predictions of a frictionless three-country business cycle model. We report regression coefficients based on model-generated data to quantitatively assess the trade-comovement relationship. We also report median typical business cycle moments from the model, in order to confirm that our friction implies good business cycle performance of the theory.

Table 3: Parameter values in the frictionless model

Parameter		Value
Preference Parameters		
$\gamma$	elasticity of substitution	1.17
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.8044, 0.0165, 0.1790
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	0.0165, 0.8044, 0.1790
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.1790, 0.1790, 0.6419
$\eta$	leisure weight in utility	0.332
$\sigma$	risk aversion	2
$\beta$	time discount factor	0.99
Technology Parameters		
$\alpha$	capital share	0.36
$\delta$	depreciation of physical capital	0.025
Other Parameters		
$\psi_1, \psi_2, \psi_3$	persistence of the productivity shock	0.83, 0.83, 0.90
$\rho_{12}$	Cross-correlation of productivity shocks	0.52
$\rho_{13}, \rho_{23}$	Cross-correlation of productivity shocks	0.67, 0.67
$L_D, L_F, L_W$	population sizes	1, 1, 20

### 3.4.1 Trade-Comovement Relationship

As described in the previous section, we choose bilateral tariffs to mimic exactly the trade patterns within the bilateral pair and of the pair countries with the rest of the world. The exercise produces 190 data points of real GDP correlations and trade intensity within the bilateral pair, on which we then run the same regression as in the data.

Table 4 presents results from regressions on model-generated data. The model implies a regression coefficient that is close to the data estimates. It accounts for 40 – 60% of the empirical slope in the overall sample. However, when we perform the same split into high- and low-trade sub-samples, the model turns out to feature the same kind of nonlinearity as the one we documented in the data. In particular, both the model and data show no trade-comovement relation in the low trade sub-sample, and a much stronger relation in the higher-trade sub-sample. In the top 50% of bilateral trade intensity sub-sample, the



Table 4: Regression results: Data versus Model.

Dependent Variable: GDP correlation			
Coefficient $\beta_{GDP}$	OLS	OLS bottom 50%	OLS top 50%
Data	0.034**	-0.017	0.055**
Model	0.022	0.003	0.044
Model/Data	64%	-	81%
Frictionless Model/Data	-0.3%	-	-1.0%
	IV	IV bottom 50%	IV top 50%
Data	0.065***	0.325	0.070**
Model	0.028	0.003	0.049
Model/Data	44%	-	70%
Frictionless Model/Data	0.2%	-	-0.4%

\*\* ,\*\*\* denote significance at 5% and 1% level for the data regression.

model accounts for 70 – 80% of the empirical relation.<sup>39</sup> By way of comparison and relation to previous quantitative studies, Table 4 also includes regression coefficients implied by a 3-country frictionless model parameterized in the same way as the benchmark model. As implied by our analytical results in Section 2, the frictionless model exhibits virtually no trade-comovement relationship.

**Shutting Down Elasticity Disconnect in the Model** In this section, we show that the solution identified in Section 2: disconnecting short-run and long-run elasticities and making the trade cost variation smaller, is what is responsible for the resolution of the trade-comovement puzzle in the benchmark model. To this end, we shut down the search frictions, and consider two parameterizations: one in which the elasticity ( $\gamma$ ) is set to the long-run target (15) and one in which we set it to the short-run target (1.17).<sup>40</sup> These two parameterizations correspond to two ways of parameterizing a model with a single elasticity— by either targeting the data estimate of the long-run elasticity or the short-run elasticity.

<sup>39</sup>The results from the model differ between OLS and IV because for the IV parameterization, we used theoretical trade levels implied by the first stage regression for parameterizing trade.

<sup>40</sup>To keep the exercise simple, we keep the other parameters at benchmark values. The numbers and conclusions do not change if we reparameterize the models.

We consider two parameterizations of trade intensity. First, we consider a case in which trade with the rest of the world and bilateral trade are set to their medians (19% and 0.85%, respectively). Then, we change the trade costs to match the 90th percentile of the bilateral trade intensity (3.83%). We then report the implied regression coefficients for output and trade cost changes needed to implement the increased trade. Table 5 presents the results in the benchmark model, the two frictionless versions of the benchmark, and the frictionless model. As suggested in Section 2, the tariff variation needed to induce the change in trade in the frictionless model (73%) is an order of magnitude higher than the one needed in the benchmark model (11%). Reducing long-run elasticity to the short-run target brings our model’s predictions in line with the frictionless model in terms of the required change in trade cost, and it also implies that the trade-comovement relationship implied by the model is counterfactually negative. This confirms our analytic results that point to the high trade cost change required in the model as the culprit for the counterfactual performance. Increasing short run elasticity to the long run target also implies negative comovement consistently with our analytic result that low short run elasticity is necessary for positive trade-comovement relationship.

Table 5: Shutting Down Elasticity Disconnect.

	Implied regression coefficient	Ratio to the data	Required change in trade cost
Benchmark Model	0.025	75%	11%
No elasticity disconnect, $\gamma = 15$	-0.012	-36%	11%
No elasticity disconnect, $\gamma = 1.17$	-0.031	-92%	82%
Frictionless Model	0.0005	1.6%	73%

**Quantifying the Intuition** Below, we decompose the response of the domestic country’s output to a foreign productivity shock into: a term driven by the complementarity channel, by which foreign demand for exports drives output correlation; and the residual. First, we show that the complementarity effect identified earlier is the main driving factor behind GDP comovement in our model—by shutting it down, we shut down the trade-comovement

in the model. Second, we show that the response of the residual is what un-does the complementarity effect in the frictionless model, but not in our model.

Consider the expression for real GDP in the benchmark model, given by

$$\begin{aligned}
 RGDP = & L_D(P_d d + P_f f + P_g g) + \sum_{i=F,W} x_i p_i^d L_i d_i - p_D^f L_D f - p_D^g L_D g \\
 & + v_D(a_D^f + a_D^g) - x_F v_F a_F^d - x_W v_W a_W^d
 \end{aligned} \tag{25}$$

where all the prices are evaluated at their steady state levels.<sup>41</sup> To isolate the complementarity effect, we will split the above equation into exports to country F, given by  $CE = x_F p_F^d d_F L_F$ , and the rest of the expression.

We consider output comovement for low and high bilateral trade intensity levels (50th and 90th percentiles of bilateral trade intensity, as described in the previous subsection) and compute implied regression coefficients. The first row of Table 6 shows that just like in the full quantitative exercise, the benchmark model exhibits a strong trade-comovement relationship, whereas the frictionless model shows essentially no effect. Row two of Table 6 reports counterfactual implied regression slopes, in which we fix the complementarity effect  $CE$  to its steady state level. We can see that the residual of the decomposition has a much stronger negative impact on GDP correlation in the frictionless model than in the benchmark model. This embodies the intuition that the risk-sharing channel has a much stronger negative effect on comovement for low long-run elasticities, and explains why the positive effect of the complementarity channel prevails in the benchmark model.

### 3.4.2 Business Cycle Statistics

In the final set of results, we verify that our model accounts for the trade-comovement relationship without sacrificing the performance in other respects, focusing on a comprehensive set of business cycle statistics. The results, presented in Table 7, report median business cycle statistics from our simulated model, as well as medians in our dataset. As the inspection of the table shows, the model matches the statistics fairly well, at least as

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<sup>41</sup>In the frictionless model, the last three terms are zero, since there are no marketing expenditures.

Table 6: Quantifying the Intuition

	Benchmark		Frictionless Model	
	Implied regression coefficient	Ratio to the data	Implied regression coefficient	Ratio to the data
With Complementarity Channel	0.025	75%	0.0005	1.6%
No Complementarity Channel	-0.014	-42%	-0.042	-124%

well as the frictionless model and often better. One notable improvement is the prediction that output is more correlated internationally than consumption, addressing the so called ‘quantity anomaly’.<sup>42</sup>

## 4 Conclusions

In this paper, we have demonstrated that exercises which take business cycle models to cross-country data will critically depend on the properties of dynamic elasticity within the model, as trade elasticity is a major determinant of how business cycle shocks spill over across borders. We have illustrated the quantitative relevance of this point by exploring the well documented link between trade and comovement in the cross-section of countries. We found that dynamic elasticity does indeed affect the findings in a quantitatively significant way. Hence, our paper advocates for using dynamic elasticity models in contexts that evaluate international business cycle theory vis-a-vis data on cross-country variation of business cycle moments.

## A Appendix

For additional details, see the Mathematica notebook available from the authors’ websites.

<sup>42</sup>Identified in D.K. Backus, P.J. Kehoe & F.E. Kydland (1992).

Table 7: Business Cycle Statistics: Data and Models<sup>a</sup>

Statistic	Data Median <sup>b</sup>	Benchmark Median	Frictionless Median
<i>A. Correlation</i>			
<i>domestic with foreign</i>			
TFP (measured)	0.44	0.54	0.52
GDP	0.52	0.53	0.52
Consumption	0.41	0.45	0.57
Employment	0.42	0.46	0.54
Investment	0.50	0.38	0.45
<i>GDP with</i>			
Consumption	0.71	0.92	0.93
Employment	0.60	0.81	0.99
Investment	0.71	0.98	0.98
Net exports	-0.20	-0.63	-0.69
<i>Terms of trade with</i>			
Net exports	-0.31	-0.89	-0.54
<i>B. Volatility relative to GDP</i>			
Consumption	0.79	0.28	0.26
Investment	3.04	3.90	3.66
Employment	0.71	0.83	0.52
Net exports	0.59	0.20	0.14

<sup>a</sup>Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter  $\lambda = 1600$ .

<sup>b</sup>Unless otherwise noted, data column refers to the median in our sample of countries for the period 1980Q1-2011Q4.

## A.1 Proof of Proposition 2

We use the fact that welfare theorems hold and the allocation solves an appropriate planning problem. Since the objective function of such a planning problem is additive, and the constraints are imposed on a state-by-state basis, it can be recast as a static “state-by-state” maximization. In the first period, the allocation can thus be obtained by solving:

$$\max_{d, f, d^*, f^*, l, l^*} \text{Log}(G_{\phi, \rho}(d, f)) - l + \text{Log}(G_{\phi, \rho}(f^*, d^*)) - l^*$$

subject to ( $\tau(x)$  is as defined in the text)

$$\begin{aligned} d(s) + d(s)^* + \tau(x)d(s)^* &= Al \\ f(s) + f(s)^* + \tau(x)f(s) &= A^*l^*. \end{aligned}$$

The second period planning problem is similar but with  $A = A^* = 1$  and  $G_{\omega,\gamma}$  in place of  $G_{\phi,\rho}$ . We guess and verify that the symmetric (deterministic) solution ( $A = 1, A^* = 1$ ) is given by:

$$\begin{aligned} l &= 1 \\ f &= \frac{1}{-\frac{\phi(\tau+1)^\rho}{\phi-1} + \tau + 1} \\ d &= \frac{1}{1 - \frac{(\phi-1)(\tau+1)^{1-\rho}}{\phi}}. \end{aligned}$$

The analogous solution of the second period can be obtained after appropriate modifications (e.g.  $\rho$  replaced by  $\gamma$ ). By definition of trade intensity stated in text, the symmetric solution applied to the second period gives:

$$x := \tau^{-1}(x) = \frac{1}{1 + \tau - \frac{\omega(\tau+1)^\gamma}{\omega-1}}.$$

Implicit differentiation gives

$$\frac{d\tau}{dx} = -\frac{1}{x \left( (1-\gamma)x + \frac{\gamma}{\tau+1} \right)}.$$

Next, define  $\phi$  to assure that the trade intensity (as defined in text) is equal to  $x$  for any fixed value of  $\tau$ . This way we obtain

$$\phi = \left( 1 + \frac{x(1+\tau)^\rho}{1-x-\tau x} \right)^{-1}.$$

For this value of  $\phi$ , the symmetric (deterministic) solution of the first order conditions stated above is given by  $d = 1 - (1+\tau)x$ ,  $f = x$  and  $l = 1$ . Log-linearizing the first order conditions at the symmetric solution and inverting the system, we derive the solution for domestic output  $y$ :

$$y = A [1 - 2x(1-\rho)(1+\tau)(1-x-\tau x)] + A^* [2x(1-\rho)(1+\tau)(1-x-\tau x)].$$

For detailed derivations refer to the Online Appendix and/or the Mathematica notebook posted online.

## A.2 Proof of Proposition 5

First, we establish a set of necessary conditions for the equilibrium allocation.

**Lemma 1** *Define the risk sharing function as*

$$R(s) := B(s) + (1-p)f. \tag{26}$$

*Then, the allocation in the first period (for any  $s = (A, A^*)$ ) must satisfy i) labor/leisure choice conditions*

$$\frac{\partial G_{\phi,\rho}/\partial d}{G_{\phi,\rho}} = 1/A, \tag{27}$$

$$\frac{\partial G_{\phi,\rho}^*/\partial f^*}{G_{\phi,\rho}^*} = 1/A^*, \tag{28}$$

ii) commodity market clearing conditions

$$\begin{aligned}\partial G_{\phi,\rho}/\partial f &= p(1+\tau)\partial G_{\phi,\rho}/\partial d \\ \partial G_{\phi,\rho}^*/\partial d^* &= \frac{1}{p}(1+\tau)\partial G_{\phi,\rho}^*/\partial f^*\end{aligned}\tag{29}$$

iii) budget constraint of the domestic country

$$d + f(1 + p\tau) = Al(s) + R,\tag{30}$$

iv) feasibility condition for foreign goods

$$f(s) + f^*(s) = A^*l^*(s) - \tau f,\tag{31}$$

v) feasibility condition for domestic goods

$$d(s) + d^*(s) = Al(s) - \tau d^*,\tag{32}$$

and, finally, vi) the risk sharing condition

$$\partial G_{\phi,\rho}/\partial d(1 + \tau) = \partial G_{\phi,\rho}^*/\partial d^*,\tag{33}$$

which effectively determines the risk-sharing function  $R(s)$ . The underlying bond portfolio  $B(s), B^*(s)$  can be recovered from the definition of  $R$  and market clearing  $B^*(s) = B(s)$ . (By Walras law the foreign country budget constraint is redundant.)

**Proof.** In Online Appendix. ■

We log-linearize the equilibrium system with respect to all variables except  $R$ . Since  $R$  is zero at the symmetric (deterministic) solution, we linearize the system with respect to  $R$ .

To derive the decomposition discussed in text, we proceed as follows. In the first step, we linearize / log-linearize the above equilibrium system by treating  $p$  and  $R$  as exogenous and dropping condition (iv) and the foreign part of (ii), which yields:

$$y = A - R - xp.$$

We then include equation for the foreign part of (ii) to obtain the equation characterizing the endogenous dynamics of the terms of trade  $p$  as function of state variables  $A, A^*$  and the still exogenous stochastic process for  $R$ :

$$p = \frac{R - x(1 + \tau)(A - A^*)}{x(-2\rho(\tau + 1) + \tau + 2(\rho - 1)(\tau + 1)^2x)}.$$

Finally, we include all equilibrium conditions to obtain the equation for  $R$ :

$$R = (A - A^*)x(1 + 2\tau - 2(1 + \tau)(\rho + x(1 - \rho)(1 + \tau)))$$

The last equation closes the system. The coefficients in text are derived from the above set of equations by evaluating appropriate derivatives with respect to  $A^*$ . For detailed derivations refer to the Online Appendix and/or the Mathematica notebook posted online.

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