Uncertainty as Commitment

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Abstract

When governments cannot commit to not providing bailouts, banks may take excessive risks and generate crises. At the outbreak of financial problems, however, governments are usually uncertain about their systemic nature, and may delay intervention to learn more from endogenous market outcomes. We show such delay introduces strategic restraint: banks restrict their portfolio riskiness relative to their peers to avoid being the worst performers and bearing the costs of delay. Hence, uncertainty has the potential to induce bank self-discipline, hence mitigating crises in the absence of commitment. We study the effects of policies and institutions on these novel forces.

1 Introduction

Few would disagree that bailouts are socially costly. Yet, they are ubiquitous during crises in most countries, dating as far back as the 1800s. In the recent global financial crisis, for example, the U.S. government used a variety of instruments to bail out, on an unprecedented scale, many financial entities that were exposed to systemic risk.

Equally ubiquitous is the uncertainty governments face at the onset of financial crises about their systemic nature. Anecdotal evidence suggests that policymakers are limited in their capacity to acquire and rapidly process information about the scope and severity of an
unraveling crisis – the most recent one being a prime example.¹ This government uncertainty is usually considered a shortcoming of policymaking while dealing with crises. We argue, however, that this uncertainty has a positive effect of inducing self-disciplining behavior of financial firms ex-ante, hence mitigating the likelihood and magnitude of financial crises.

One of the leading explanations of why bailouts are relatively common despite being costly relies on the time inconsistency of no-bailout policies (e.g. Holmström and Tirole (1998) and Farhi and Tirole (2012)). If bank failures are costly for an economy ex-post, a government may be tempted to bail out banks in distress. Without commitment, banks internalize this ex-post reaction and hence have no incentive to avoid exposing themselves to risks ex-ante, effectively leading to endogenous crises. Due to this moral hazard problem, the equilibrium outcome obtaining under no commitment is typically inferior to the one in which governments can commit to no bailouts during financial distress.

We show government uncertainty has the potential for sustaining commitment outcomes even when the government lacks commitment. Intuitively, at the onset of financial problems, governments are uncertain about their systemic nature, and hence about the actual need for a bailout – when the crisis is not systemic a distressed bank would be more efficiently handled by a take-over by other banks. In order to observe more signals and learn about the nature of the shock, the government may want to delay bailout and let the first bank(s) in distress fail, possibly avoiding a potentially costly and unnecessary intervention (when the shock is not systemic). Crucially, expected delays make the relative performance of banks’ portfolios critical since no bank wants to be amongst the first in line for government help. We call this effect strategic restraint, as banks endogenously restrict the riskiness of their portfolio relative to their peers in order to avoid being amongst the worst performers, inducing a sort of competition to reduce excessive risk-taking.

In the recent U.S. crisis, for example, Kelly, Lustig, and Van Nieuwerburgh (2011) show that U.S. policymakers avoided providing bailout (‘funds at subsidized rates’) until September 15, 2008, when it became evident that financial markets were experiencing a systemic event as Lehman Brothers filed for bankruptcy and no private party was willing to take over its operations. Before that moment, on March 14, 2008, the Federal Reserve Bank of New York refused to extend a collateralized loan to Bear Stearns, forcing the company to sign a

¹Kelly, Lustig, and Van Nieuwerburgh (2011) provide a thorough discussion of the timing of events and the evolution of government’s announcements of bailouts during the recent crisis. One of the striking features they document is the overlap of positive and negative announcements concerning the provision of bailouts. The bailout of Continental Illinois Bank and Trust Company in 1984 provides another example. The FDIC chairman at the time, William Isaac, stressed that the decision to bail out the bondholders was made given ‘the best estimates of our staff, with the sparse numbers we had at hand’, acknowledging the remaining uncertainty concerning the case at the time of intervention.
merger agreement with JP Morgan Chase two days later at $2 a share (less than 7% its market value just two days before). Similarly, on September 7, 2008, the Treasury announced plans to help Fannie Mae and Freddie Mac, but not with the provision of funds but rather by placing them into conservatorship. Cochrane and Zingales (2009) argue that the Lehman failure did not cause the subsequent unraveling of the financial market, but rather was the first convincing signal of a bigger problem.

We build a theoretical model to formally study the role of government uncertainty and strategic restraint in an economy without government commitment. In the model, bankers borrow short-term from households to finance projects that are illiquid. Projects may suffer shocks over time, in which case they require extra funds to bring them to fruition. The shock hitting a project may be idiosyncratic, hitting only certain banks, or aggregate, hitting all banks. High levels of short-term debt allow banks to invest in large projects, but at the same time hinder their ability to refinance if a shock hits. We study the problem of a central authority, which we call the government, which maximizes total welfare (bankers’ plus households’) by affecting the cost of borrowing to refinance in case of shocks. An intervention that reduces the cost of borrowing to bankers, which we call a bailout, is financed through taxes on households in a way that is socially costly (e.g. due to distortions). The benefits of bailouts, on the other hand, are naturally given by bringing banks’ projects to fruition, and thus increasing output.

When the government observes a bank in distress – which we define as the bank running out of cash to keep financing operations, and options for refinancing on the market disappear – it does not observe whether the shock is idiosyncratic or aggregate, information that is critical for taking appropriate action. If the shock is idiosyncratic, other banks have enough liquidity to take over the distressed bank, and no intervention is needed. If the shock is aggregate, intervention is the only way to avoid a project failure. Hence, the government’s decision to bail out the bank or not depends on its beliefs about the nature of the shock.

We show that, if the government is initially relatively optimistic about there being enough liquidity in the banking system (i.e. not facing an aggregate shock), then it chooses to learn more by delaying intervention, not bailing out the first distressed bank(s). By delaying the bailout, the government maintains the option of introducing the bailout at a later time – after observing subsequent signals (further bank distress) – under a more precise belief about the true state. For the banks, however, this delay makes their relative performance relevant, introducing incentives to avoid being the worst performers. In the model, this happens through banks leveraging less, downsizing their projects, and carrying more cash reserves.

The basic liquidity problem we set up builds on Farhi and Tirole (2012), with important extensions.
than their peers for the eventuality of being hit by the refinancing shock – a behavior we call *strategic restraints*.

In our benchmark, we consider a stark case in which banks can guarantee not being the worst performers by choosing slightly lower leverage than other banks – which gives rise to a Bertrand-type competition. In this stark setup, in the unique equilibrium of the economy, banks compete away all excessive leverage and the allocation coincides with the one which obtains under commitment. Here, however, it is driven by government uncertainty and strategic restraint instead. Our results are more general. In an extended environment, we consider shocks to cash holdings of individual banks, such that small deviations in leverage do not guarantee outperforming other banks. In this case, *government uncertainty* and *strategic restraint* forces still operate, moving the equilibrium allocation closer to the optimal (commitment) outcome – the unique equilibrium is characterized by leverage that is intermediate between the commitment (optimal) and non-commitment (inefficient) equilibrium, the benchmark model being a limiting case. In the more general setting, bailouts and crises still are observed on the equilibrium path, but they appear less frequently and with smaller magnitude than in the absence of uncertainty. Furthermore, crises are smaller and happen less frequently when shocks to cash holdings have small variance.\(^3\)

Since *government uncertainty* and *strategic restraint* forces robustly implement allocations that dominate the non-commitment equilibrium in terms of welfare (the benchmark achieving the optimum), natural policy questions arise regarding the effects of regulation and economic environment on the effectiveness of these forces. How does financial innovation affect uncertainty and the likelihood of endogenous crises? Is government uncertainty more effective when there are many banks? Is it more effective when banks are of similar size? Since uncertainty serves as commitment only when the probability that policymakers assign to systemic events are below a certain threshold, these extensions affect the applicability of the mechanism by either changing the beliefs of systemic events or by modifying the threshold under which uncertainty serves as commitment.

We model financial innovation as affecting the ability of banks to insure away part of their idiosyncratic risk, for example by trading securitized products or over-the-counter derivatives. We show that when banks hold large amounts of cross-insurance - like in the most recent crisis - the industry as a whole is more prone to moral hazard behavior and crises. Intuitively, cross-insurance links banks’ portfolio performance, and financial problems are more

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\(^3\)In the Appendix we provide a detailed discussion of extensions and modifications of our model, highlighting the robustness of our mechanism to different settings, with different timing assumptions, with non-targeted bailout policies, with gradual and temporary bailouts, and with strategic coordination motives.
likely to be observed earlier if the shock is aggregate since there is less liquidity in the pool of banks. Hence, the time at which a single bank shows distress reveals information about the nature of the shock, reducing government uncertainty and relaxing strategic restraint. There are, of course, benefits of cross insurance in financial markets, and in the paper, we derive a cap on the fraction of idiosyncratic risk that can be insured away, while still preserving the ability of government uncertainty to implement welfare-superior allocations. We find that this cap increases with the variance of the shocks to cash holdings, pointing to an unexplored effect of financial innovation, and suggesting a new rationale for regulating it.

We also explore the role of industry concentration, measured by the number of banks. We show that a larger number of banks strengthens the two forces that induce governments to delay intervention. First, more banks increase the chance that there is enough liquidity in the system for a takeover of a distressed bank. Second, more banks increase the option value of learning the true nature of the shock, by making the potential loss of one bank’s failure relatively smaller and the benefits of making better informed decisions later bigger. Hence, decreasing industry concentration makes allocations closer to commitment outcomes easier to sustain.

Finally, we study the effect of bank size heterogeneity – in particular, a case when a single bank is asymmetrically large in the industry. We show that our novel mechanism introduces a new rationale for capping bank size, over and above the one driven by concerns of contagion. Having banks whose balance sheets are overwhelmingly large relative to the industry makes them less likely to be acquired by its peers and hence more likely that the government must bail them out in case of distress, regardless of the nature of the shock or whether they are the first to show problems. Hence, in the absence of commitment, sufficiently large banks do not have any concern for their relative performance and choose excessive leverage. Given this behavior, smaller banks have a strategic incentive to expose themselves only slightly less than the large bank. In our setting, the ‘too big to fail’ problem shows up very differently than in the rest of the literature because large banks become ‘shields’ for smaller banks to take excessive risk, exerting a negative externality on the economy by inducing endogenous systemic crises of larger magnitude.

On a general level, our work suggests that it may be optimal for governments to design political structures that delay bailout decisions, or regulatory standards that maintain an optimal level of uncertainty, imposing lower burdens on preventive regulation and giving more room to facilitate optimal self-regulation. Clearly, an alternative would be macro-prudential regulation and direct oversight of banking activity. However, historically regulators have been incapable to design macro-prudential regulation that prevents crises without choking.
off growth. Our results suggest that, from an ex-ante perspective, governments that lack commitment and good enforcement ability would rather not have the possibility to learn rapidly, making more mistakes when crises happen, but also reducing the probability of those crises. Even though it may be difficult for a government to commit not to modify bailout decisions ex-post, they could easily avoid ex-ante the implementation of technologies to learn rapidly or to take fast decisions when distress happens.

**Related Literature** There is a large literature on the time-consistency of no bailout policies and moral hazard behavior of banks, to which this paper contributes. A number of papers, extensively reviewed in Stern and Feldman (2004), argue that the existence of ‘too big to fail’ banks is the source of the time inconsistency of policies, and at the root of crises. Another strand of the literature, most recently represented by Acharya and Yorulmazer (2007), Pasten (2011) and especially Farhi and Tirole (2012) argue that ‘too big to fail’ banks are not a pre-requisite since coordinated actions by smaller banks can also give rise to endogenous crises. Our paper, which introduces uncertainty and shows how it helps to achieve welfare-superior allocations, applies to both environments, with and without large banks.

Our setting builds on Holmström and Tirole (1998) and Farhi and Tirole (2012). Relative to their work, we introduce idiosyncratic shocks and the possibility of government uncertainty about the nature of the shock. We additionally allow for efficient takeovers of distressed banks by healthy banks, making the true nature of the shock crucial for the government. In contrast to their work, the timing of bank distress at the onset of a crisis is critical for us. We also choose to model our economy so that the collective moral hazard issues do not arise in our setup. We discuss in detail the role of our timing assumptions and a version of the model with collective moral hazard in the Appendix.

Our work also relates to Acharya and Yorulmazer (2007), who develop a model of ‘too-many to fail’ in an environment where bank takeovers are also possible and technologically superior to bailouts, as in our paper.\(^4\) In our model, the ‘wait and see’ strategy of the government has the additional gain of providing information to the government about the nature of the shocks, which creates *strategic restraint* and hinders the possibility of herding that they highlight. We discuss a mapping between their too-many-to-fail and our too-big-to-fail model in the online Supplementary Appendix.

Freixas (1999) shows in a banking setup that the optimal policy for a lender of last resort is to randomize ex-post between bailing out banks in distress and not. This “constructive

\(^4\)Perotti and Suarez (2002) model additional reasons why takeovers may be the superior outcome, based on takeover’s effect on the market power of the banks.
ambiguity" strategy, however, requires commitment. In our setting we show that a similar policy emerges in equilibrium when there is no commitment but the government is uncertain about the nature of distress.

Recently, Green (2010) and Keister (2011) argue that bailouts may be optimal to avoid excessive hoarding of liquidity. In a similar vein, Cheng and Milbradt (2010) suggest bailouts can instill confidence on credit markets. In our setup, whatever the optimal level of liquidity is, it can be attained as long as government uncertainty and strategic restraint forces are at work, even in the absence of commitment.

Bianchi (2012) concludes that moral hazard effects of bailouts are significantly mitigated by making bailouts contingent on the occurrence of a systemic financial crisis. In contrast, in our framework shocks are unobservable and hence the government cannot make bailouts contingent upon them. This gives rise to a positive option of delay and learning, which is exactly what mitigates the moral hazard problem.

As in our extension to asymmetric bank sizes, Davila (2012) also argues that large banks allow small banks to take more risks, increasing economy-wide leverage and bailout probability when large banks are present. While his results are based on banks’ uncertainty about bailout policies, ours depend on governments’ uncertainty about the nature of the shocks.

Uhlig (2010) proposes a model of systemic bank runs with the implication that delaying political intervention can increase the likelihood that a financial distress become systemic. In our model whether a shock is idiosyncratic or aggregate is exogenous. Endogenizing it to depend on delay is an interesting extension, but beyond the scope of this paper.

A recent strand of the literature highlights the effects of policy uncertainty in inducing crises and delaying recoveries. Cukierman and Izhakian (2014), for example, show that uncertainty about policymakers’s actions can induce sudden financial collapses when investors follow a max-min behavior. Baker, Bloom, and Davis (2012) argue that uncertainty about future policies delays recoveries since individuals prefer to ‘wait and see.’ In our case, it is the government who is uncertain about the nature of refinancing shocks and may like to ‘wait and see’ before intervening, reducing the likelihood of endogenous crises.

Finally, there is a previous literature that explores the ability of imperfect information to improve equilibrium outcomes under time inconsistency. Cremer (1995) shows in a static decision problem that the inability of a principal to observe workers’ types can serve as a commitment device to punishing low output realizations. Carrillo and Mariotti (2000) show that an agent with time-inconsistent preferences might optimally choose not to learn in order to restrict future selves. The effect of imperfect information in our model has a similar flavor, but in our banking setting, the competition that arises from the banks’ concerns for their
relative position is critical for our results, and is absent in their settings. Furthermore, our setting explores how dynamics affect decisions and incentives, and the government not only delays because of imperfect information but also because there are gains from learning and resolving the uncertainty.

2 The Model

The model environment builds on Holmström and Tirole (1998) and Farhi and Tirole (2012), with several important modifications. First, we introduce two types of shocks, aggregate and idiosyncratic, and allow for imperfect information about the nature of the shock. Second, we allow for a non-degenerate timing of events, in which banks with higher leverage ratios endogenously show distress earlier. Third, we admit the possibility that healthy banks take over distressed banks.

2.1 Environment

Time is continuous and finite, \( t \in [0, 2] \), and there is no discounting. There are three types of agents in the economy: two banking entrepreneurs (banks hereafter), a continuum of households and a government. Banks borrow short-term to finance illiquid projects which either pay off at \( t = 1 \) or need refinancing and pay off at \( t = 2 \). A bank’s project needs refinancing because of an aggregate shock (both banks need refinancing) or an idiosyncratic shock (only one of the two banks needs refinancing). These shocks hit only at date \( t = 1 \), and for the rest of time the economy is deterministic. We partition the state space so that we can denote the probability that (i) both banks need refinancing as \( P_2 \), (ii) only one bank needs refinancing as \( 2P_1 \) and (iii) no bank needs refinancing as \( P_0 \). Households are risk neutral providers of loans to banks. The government maximizes total welfare, using transfers between households and bankers as a policy instrument.

2.1.1 Banks

The two banks in the economy have the objective of maximizing their individual net worth. At \( t = 0 \), they choose the size \( i \) of an investment project, which is financed using own initial assets \( A \) and funds borrowed from the households. The size \( i \) also determines the speed of expense outflows (to pay suppliers, workers, etc), which happens at a rate \( idt \) during the period, such that all projects run out of funds at \( t = 1 \) and larger projects have a larger outflow rate than smaller projects. We assume that if at any moment expense outflows are
interrupted (the firms stop paying workers, for example, or stop buying supplies), the project is discontinued generates output that is scaled-down.5

The payoff from each project consists of two parts. The first part is deterministic, $\pi_t$ at time $t = 1$. The second part is random. If the project does not suffer any shock, it returns $(\rho_0 + \rho_1)i$ at time $t = 1$. If the project suffers a shock, it does not pay anything more than the deterministic $\pi_t$ at time $t = 1$, and the bank has the choice to refinance the project to size $j \leq i$ which returns $(\rho_0 + \rho_1)j$ at time $t = 2$. Refinancing a project, however, does not change its intrinsic rate of expenses outflow – if only half of a large project is refinanced ($j = i/2$), for example, the bank would run out of cash to pay expenses at $t = 1.5$. We introduce financial frictions by assuming that from the total output of the project, $\rho_1$ is a benefit that can only be captured by bankers, and hence it is not pledgeble.6 Parametric assumptions that make the model economically interesting are summarized below.

**Assumption 1 Assumptions about projects’s payoffs**

1. **Binding pledgeability:** $\pi < 1$ and $\rho_0 < 1$.
2. **Efficient projects:** $\pi + \rho_0 + \rho_1 > 1 + P_1 + P_2$.
3. **Efficient refinancing:** $\rho_0 + \rho_1 > 1$.

The first part of Assumption 1 guarantees that investment in period $t = 0$ is finite, and that refinancing depends on retained earnings7. The second part guarantees that financing the project at $t = 0$ is socially efficient. It states that the payoff of always seeing the project to fruition is greater than the expected cost of doing so (initial investment plus refinancing in case of a shock). The third part guarantees that refinancing the project at $t = 1$ is also socially efficient.8

### 2.1.2 Households and Government

A continuum of risk neutral households are born at dates $t = 0$ and $t = 1$. They are endowed with assets $S_t$ when born, which they allocate between holding cash (or storing at a return 1) and lending to banks. Then, the return on their savings is consumed in period $t + 1$.

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5A setup in which the whole output is lost upon discontinuing the project would make our results stronger.

6This can be derived from first principles as in Holmström and Tirole (1998), for example by moral hazard within the bank, in which the banker exerts hidden efforts that affect the outcome of the project.

7If $\pi > 1$, then infinite size of investment could be financed by just the promise of deterministic repayment, since the equilibrium interest rate is 1. If $\rho_0 > 1$, then zero retained earnings is enough to refinance at the market interest rate.

8The special case with $\pi = 0$ and $\rho_0 = 0$ satisfies all restrictions and represent a problem of self-financing. Still, all results regarding bailouts and uncertainty as commitment go through.
Assuming perfect competition of households as lenders, which imply their return is always 1, and denoting government taxes as $T$ (which may serve to finance potential bailouts), utility for each generation is given simply by $U_t = S_t - T_{t+1}$.

The government is benevolent and maximizes welfare $W = \beta V + U_0 + U_1$, the weighted sum of the banks’ surplus, $V$, and each generation of households’ surpluses, $U_0$ and $U_1$. To maximize $W$ the government may need to transfer resources from households to banks to refinance projects in distress. This is what we call a bailout. We assume governments bail out banks by lending to them directly at below-market interest rates, which we denote by $R(t)$ at $t \in [0, 2]$, financed by taxes on households. The weight $\beta < 1$ captures the idea that these transfers between households and banks are costly from a welfare perspective.\(^9\) In what follows, we assume bailouts are targeted, which means that a government can reduce interest rates only for certain banks and not others. In the Appendix, however, we show that non-targeted bailouts, under which all banks can borrow at a lower rate, regardless of whether they are in distress or not, strengthen the results.

Even though banks know at $t = 1$ the state of the economy (which banks suffered a shock, if any), we introduce different assumptions on what the government knows at $t = 1$. We denote as full information the case in which the government also observes the state at $t = 1$ and we denote as imperfect information the case in which the government does not. In the latter case, we denote generically as $t \in [1, 2]$ the calendar time at which the government observes a bank in distress, inferring then that the observed bank must have suffered a shock.

### 2.1.3 Timing

At $t = 0$, the government announces a bailout policy as a function of the time the first and second banks show distress, by which we mean the time $t \geq 1$ at which they eventually run out of refinancing opportunities in the market. If the government has commitment, it then just executes the announcement. In contrast, if the government lacks commitment, it can deviate from the announced policy ex-post.

At $t = 1$, either both banks suffer a refinancing shock (aggregate shock) with probability $P_2$, only one bank suffers a refinancing shock (idiosyncratic shock) with probability $2P_1$, or no bank suffers a refinancing shock with probability $P_0$.

**Idiosyncratic Shock:** If only one bank suffers a shock and shows distress (it may never show distress if it retained enough cash to fully refinance the project), the government moves

\(^9\)We could additionally include distortionary effects of transfers, or fixed costs of intervening, as in Farhi and Tirole (2012). These extensions, as discussed in the Appendix, do not change our main results, so we omit it here for expositional purposes.
first and decides whether to bail out or not. If the government does not bail out, the healthy bank moves second and decides whether to take over the distressed project or not.

*Aggregate Shock:* If both banks suffer a shock, at the time that the first bank shows distress the government decides whether to bail that bank out or not. If the government does not provide a bailout and the second bank shows distress, the government decides again whether to bailout or not, payoffs are realized at $t = 2$ and the game ends.

The above timing of events applies to both the full information and the imperfect information cases. With full information, the government knows how many projects need refinancing when deciding whether to bail out or not the first bank in distress. With imperfect information the government does not know how many projects need refinancing when deciding whether to bail out or not the first bank in distress.

In the above, the government moves first, and hence is not able to readjust after observing the action of the second bank. We explore alternatives to that assumption in the Appendix and show that the results are robust to different specification of timing.

### 2.2 Preliminaries

Below, we introduce notation and derive basic results that are used in the rest of the paper. We first describe the banks’ borrowing decisions at $t = 0$ and $t = 1$. Then, we describe the government’s bailout decision when observing a bank in distress at time $t \in [1, 2]$.

**Bank borrowing** We assume that bank borrowing is non-contingent. At $t = 0$ banks promise to repay $b$ per unit of investment, subject to limited liability, which implies $b \leq \pi$.\(^{10}\) Since the alternative use of cash for lenders (households) is storage with return 1, competitiveness and risk-neutrality of lenders implies that the market interest rate is $R = 1$ (i.e. households do not require a premium for non-contingent payments).\(^{11}\) This gives

\[
i - A = bi, \quad \text{and hence} \quad i = A/(1 - b).
\]  

\(^{10}\)The non-contingent debt assumption is not critical for our results: if repayment is conditional on success, the optimal level of investment will increase, but the liquidity choice considerations will remain. The crucial assumption is limited liability, which means that the banks cannot pledge future profits.

\(^{11}\)In this paper we focus solely on fully collateralized, risk-free, loans. Hence, all bailouts in the model are bailouts of equity holders and not debt holders. In an extension that allows for default, interest rates would include a premium for the expected probability of default. If banks and lenders expect bailouts to lenders, the premium is low and there is effectively a subsidy to risk-taking. Hence, restricting the model to bailing out only equity holders is not critical for our results.
to \( c = (\pi - b) \) per unit of investment, banks face a tradeoff between increasing the initial size of the project and holding some cash for refinancing if needed.

Then, the reinvestment scale \( j \) depends on the cash carried at \( t = 1, ci = (\pi - b)i \), that can be levered by taking a new loan, with the restriction that the reinvestment cannot increase the size of the project (that is, \( j \leq i \)). The second period payoff in case of refinancing needs is \((\rho_0 + \rho_1)j\), of which, crucially, only \( \rho_0j \) is pledgeable by the lenders. If the required market rate of return on bank lending is \( R \), then the maximum the bank can raise at \( t = 1 \) is

\[
R(j - ci) = \rho_0j, \quad \text{which implies} \quad j = \min\left\{\frac{c}{1 - \rho_0/R}, 1\right\}i. \tag{2}
\]

This clearly implies that at the market rate, \( R = 1 \), banks need to save cash \( c = 1 - \rho_0 \) per unit of investment if they want to refinance the whole project at market interest rate in case of a shock.

Denote by \( \bar{t}(c) \) the calendar time of distress after refinancing the maximum possible at the market rate. Then, we have

\[
\bar{t}(c) = \min\left\{1 + \frac{c}{1 - \rho_0}, 2\right\}. \tag{3}
\]

If banks hold \( c = 0 \), then they can invest in the largest feasible project, \( i = A/(1 - \pi) \) at \( t = 0 \), but will not be able to refinance anything at the market rate in case of a shock at \( t = 1 \). In contrast, if banks hold \( c = 1 - \rho_0 \), then they can invest in a smaller project, \( i = A/(2 - \pi - \rho_0) \) at \( t = 0 \), but will be able to refinance the project fully at the market rate in case of a shock at \( t = 1 \).

**Government Policy** Governments can implement bailouts – transfers from households to bankers – by issuing bonds and providing money directly to the distressed bank against the pledgeable amount of the projects, later covering those bonds with taxes to households. Taxes have to be equal to \( T = (1 - \rho_0)(i - j) \), which is the difference between what the bank cannot refinance itself \((i - j)\) (the issuance of bonds) and the maximal return the government can recover from the pledgeable part of the project, \( \rho_0(i - j) \). We assume bailouts are targeted – only available to the bank(s) chosen by the government.

This policy is isomorphic to the government modifying the interest rate that banks face to refinance. For example, if the government sets \( R = \rho_0 \), the interest rate is exactly equal to the pledgeable amount \( \rho_0 \) and banks are able to refinance any amount of reinvestment, up to the feasible level \( i \), with zero cash holdings. Without loss of generality, we assume that
this policy interest rate can only take two values: (i) a no intervention market rate of $R = 1$ and (ii) a bailout rate of $R = \rho_0$.

We denote the government’s belief that both banks need refinancing by $p$. Under full information, $p$ is either 0 (idiosyncratic shock) or 1 (aggregate shock). Under imperfect information, $p$ is updated to $P'_2$ according to Bayes’ rule, given the information contained in the first bank showing distress, that is:¹²

$$P'_2 = \frac{P_2}{P_1 + P_2} > P_2.$$ (4)

The decision of the government is a binary one: whether or not to bailout, given its belief that both banks are in need of refinancing $p$ and after observing at least one bank running out of cash at time $t \geq 1$. We summarize this decision by a function $\Pi(t,p) \in \{0,1\}$, where 1 is a bailout, and 0 lack thereof.

For the purposes of banks’ optimization, it is crucial what is the earliest time that the government is willing to bailout the first bank in distress.

**Definition 1** The earliest bailout time $t^*_p$ is the minimum time at which the government is willing to provide a bailout when the probability that both banks need refinancing is $p$ and the government observes a bank in distress (without funds to continue operations):

$$t^*_p = \min\{t | \Pi(t,p) = 1\}. \quad (5)$$

We are restricting the set of government policies $\Pi$ to ones that guarantee that $t^*_p$ is well defined. When it does not generate confusion, we denote the policy when an aggregate shock is certain simply as $t^*$.  

**Takeovers** In case of the idiosyncratic shock, if a government does not bail out the first bank in distress, the healthy bank gains $\rho_0 + \rho_1 - 1 > 0$ per unit of investment from taking it over with a take-it-or-leave-it offer.¹³ The value of a takeover is $V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$, which only depends on the other bank’s choice of $i'$ and $c'$ if the proceeds from a successful project are enough to take over the remainder of the failed bank’s project. This is satisfied in a symmetric equilibrium and it is always satisfied as long as the smallest project’s proceeds

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¹²Equation (4) assumes that banks are not anonymous, i.e. the conditioning event is that a given bank needs liquidity, not just any bank. Under the alternative assumption of bank anonymity, the denominator would be $2P_1 + P_2$, and the rest of analysis would go unchanged.

¹³Any bargaining arrangement which pays the healthy bank more than 1 per unit of investment delivers the same result.
can take over the largest project, which holds if $\rho_1 > 1/(1-\pi)$. To keep the exposition simple, we maintain this parametric restriction throughout, but relaxing this assumption would just increase the incentives for banks to hold cash, reinforcing our results.

**Returns** It is convenient to define the social gain per unit of investment from refinancing with private funds as $y = \beta \rho_1 - \beta (1 - \rho_0)$. Both the gains and the costs per unit of refinancing is weighted by the bankers’ $\beta$, because for the households, this is a zero net value operation. In turn, define the social gain from refinancing with public funds as $x = \beta \rho_1 - (1 - \rho_0)$. Here, the gains are weighted by the banker’s $\beta$ but the costs are weighted by the households’ $1$ which means a bailout is effectively a transfer from households to bankers.

We assume that bailouts are socially costly (raising public funds introduce distortions) but beneficial (bailouts save socially useful projects), which together with Assumption 1, implies $y > x > 0$.

**Assumption 2** Bailouts are socially costly, but beneficial: $\beta \rho_1 > 1 - \rho_0$.

In essence, even when beneficial, the cost of bailing out a bank when the shock is idiosyncratic is that the bailout prevents socially efficient takeovers.

### 2.3 Full Information

Here, we assume, as a full information benchmark, that the government observes at $t = 1$ how many projects are in distress.

#### 2.3.1 Commitment

Assume the government is able to commit to a policy announced at $t = 0$. We first solve the optimal reaction of banks given a policy announcement and then we compute the optimal policy announcement.

At $t = 0$, the bank chooses the project size, and then how much cash $c$ to retain at $t = 1$, conditional on the government’s policy and the refinancing problem described in Section 2.2. The value function of the bank as a function of the cash choice $c$ depends on whether $\bar{t}(c)$ (given by equation (3)) is larger or smaller than the government’s policy $t^*$ (given by equation (5) at $p = 1$). Specifically,

\[
V(c) = \begin{cases} 
V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)i(c)] & \text{if } \bar{t}(c) < t^* \\
V_s(c) + P_2[c + \rho_0 + \rho_1 - (t^* - 1) - \rho_0(2 - t^*)]i(c) & \text{if } \bar{t}(c) \geq t^*
\end{cases}
\]  

(6)
where \( V_s(c) = (P_0 + P_1)(c + \rho_0 + \rho_1)i(c) + P_1\rho_1 j(c) + P_1 V_{TO} \), is the expected value when there are no shocks or when the shock is idiosyncratic and hence it is independent of government policy, \( t^* \), and \( V_{TO} \) is the value of takeover, defined earlier and independent of \( c \).

Equation (6) implies a jump in the value function, generated by government policy. Since there are no bailouts before \( t^* \), if the bank does not hold cash to refinance until \( t^* \) (that is, \( \bar{t}(c) < t^* \)), then it will need to scale down the project to \( j(c) = (\bar{t}(c) - 1)i(c) < i(c) \). In that case, the payoff for the bank is \( ci + (\rho_0 + \rho_1)j \) minus the cost of refinancing \( j \) at a market-rate \( R = 1 \) per unit of reinvestment. Since \( j - ci = \rho_0 j \), the value function in this case can be rewritten simply as \( V_s(c) + P_2\rho_1 j(c) \).

In contrast, when \( \bar{t}(c) \geq t^* \), there is a bailout, which implies banks can borrow at rate \( \rho_0 \) at time \( t^* \). This implies banks refinance as little as possible at interest rate \( R = 1 \), and then refinance up to full scale at rate \( R = \rho_0 \). Then, a fraction \( (t^* - 1)i \) of the project is refinanced at cost of 1 per unit of refinancing and the rest (a fraction \( (2 - t^*)i \) of the project) is refinanced at cost \( \rho_0 \) per unit of refinancing. Naturally in this case, the gains are given by \( (\rho_0 + \rho_1)i \), for the full project, and are independent of \( c \) as long as \( c \) is enough for \( \bar{t}(c) > t^* \).

We now impose two assumptions that make the model economically interesting. First, Assumption 3 guarantees that banks care about refinancing scale \( j - \) when faced with a tradeoff between increasing investment \( i \) or sacrificing reinvestment \( j \), they choose not to sacrifice reinvestment. In other words, this assumption guarantees that banks will always choose \( c \) such that \( \bar{t} \geq t^* \), or that in the absence of bailouts they prefer to invest in a smaller project but refinance it completely in case of a shock.

**Assumption 3** *Banks care about reinvestment scale (\( \bar{t} \geq t^* \)):*

\[
(P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - (P_1 + P_2)\frac{1 - \pi}{1 - \rho_0}\rho_1 < 0.
\]

This condition is given by the derivative of the value function when \( \bar{t}(c) < t^* \) with respect to \( c \). The first term on the left is the cost of holding \( c \) and financing a smaller project. This cost accrues when there is no shock \( (P_0) \) or when only the other bank needs refinancing \( (P_1) \). The second term is the benefit due to upscaling \( j \) (the bank can leverage extra cash \( (1 - \pi) \) by \( (1 - \rho_0) \) to obtain a gain \( \rho_1 \) per unit of refinancing), which happens when the bank needs refinancing \( (P_2 \text{ and } P_1) \). Under Assumption 3, the costs of holding cash are smaller than the benefits of holding cash, and banks prefer to reduce \( i \) to face potential needs of refinancing.

Next, Assumption 4 ensures that, if the government provides a bailout, i.e. \( t^* < 2 \), then it is not optimal for banks to carry a cash level such that the implied \( \bar{t} \) is greater than \( t^* \).
Assumption 4  The promise of a bailout increases leverage ($\bar{t} \leq t^*$):

$$(P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1 \frac{1 - \pi}{1 - \rho_0} \rho_1 - P_2 > 0$$

This condition is given by the derivative of the value function when $\bar{t}(c) \geq t^*$ with respect to $c$, and evaluating it at the most stringent condition, $t^* = 2$. The first term on the left is again the cost of larger $c$ and smaller $i$. Compared with Assumption 3, this cost also accrues in case both banks fail (there is no change in reinvestment scale $j$ if $\bar{t} > t^*$). The second term is the benefit of upsizing the project, which only happens when an idiosyncratic shock pushes the bank to fail. The third term captures the benefit of consuming $c$ in case of a bailout when an aggregate shock occurs. Under Assumption 4, the costs of holding cash are larger than the benefits of holding cash and banks prefer to maximize $i$ since the government takes care of refinancing through bailouts in the case of an aggregate shock.\(^{14}\)

Figure 1 shows the banks’ expected payoffs when the government commits to never bailout (dashed line under Assumption 3) and when the government commits to bailout if the shock is aggregate (solid line under Assumption 4). Under our stated assumptions, the expected payoff of the bank is decreasing in cash if bailouts always happen after the aggregate shock hits, and increasing if bailouts are never used by the government. This implies that effectively, under commitment, the government is able to select one of the equilibrium outcomes ($c = 0$ or $c = 1 - \rho_0$) by announcing the appropriate bailout policy.

Proposition 1 below establishes that, with commitment and under the stated assumptions,

\(^{14}\)For details of the derivation Assumptions 3 and 4, see the Appendix.
banks optimally choose to hold cash to refinance fully in case of an aggregate shock, given a
government policy $t^*$, but not high enough to refinance fully when the shock is idiosyncratic.

**Proposition 1** Under Assumptions 1-4, given government policy $t^*$, the optimal choice of
cash is characterized by $c^*(t^*) = (1 - \rho_0)(t^* - 1)$, where $t^* \in [1, 2]$ is the earliest bailout time,
given in Definition 1.

**Proof** In appendix.

Given Proposition 1, and the solution to the bank’s maximization problem, $c^*(t^*)$, the
only characteristic that matters for welfare in terms of choosing a policy rule $\Pi(t, p)$, is the
earliest bailout time $t^*$, which under commitment is like choosing $c^*$ directly from the set
$[0, 1 - \rho_0]$. We will therefore express ex-ante welfare in terms of the cash choice of banks.
Ignoring constants, welfare can be expressed as

$$W^{ca}(c) = \beta[\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2]2i(c) - (1 - \beta)P_2((1 - \rho_0) - c)2i(c).$$

(7)

where $i(c) = \frac{A}{1-\pi+c}$ and $ci(c) = (\pi - 1)i(c) + A$.

The first term represents the ex-ante payoff per unit of investment, when the project is
always refinanced in case of a shock, weighted by the bankers’ $\beta$. The second term represents
the size of the bailout weighted by the difference between households’ and bankers’ weights
$1 - \beta$, this is, the distortion cost in terms of welfare.

Clearly, the optimal policy depends on the welfare weight on bankers, $\beta$. For $\beta = 1$,
which implies equal weights in the welfare function, the fact that banks are subsidized does
not change welfare per se, because utility is transferrable one to one between households and
banks. In that case, the government only cares about output, and ex-ante wants to transfer
resources from households to bankers, implying an optimal government policy of $t^* = 1$ and
$c^* = 0$. In contrast, when $\beta$ is low, the weight governments put on producing output is low,
since households gain nothing from it. As mentioned above, $\beta$ is a shortcut for potential
distortions introduced by the transfers. One could interpret low $\beta$ as destroying output from
households when transferring resources to bankers.

Definition 2 below states the equilibrium under commitment and Proposition 2 charac-
terizes equilibrium under commitment, showing how equilibrium outcomes depend on $\beta$.

**Definition 2 (Commitment Equilibrium)** A symmetric equilibrium of the economy un-
der commitment is a cash level $c^*$ and policy of the government $\Pi(t, p = 1)$, such that $c^*$ is
the optimal response of the banks to policy. That is, $c^*$ maximizes (6) given $\Pi(t, p = 1)$, and
$\Pi(t, p = 1)$ is such that $c^*$ maximizes welfare (7).
Proposition 2 (Optimal Policy with Commitment) Define

\[ \beta^* = \frac{P_2(2 - \rho_0 - \pi)}{(\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2) + P_2(2 - \rho_0 - \pi)} < 1. \]

Then,

(i) If \( \beta < \beta^* \), \( \frac{dW^{ea}(c)}{dc} > 0 \) for all \( c \in [0, 1 - \rho_0] \). The equilibrium cash holding is \( c^* = 1 - \rho_0 \), which corresponds to welfare maximizing policy choice of no bailout, this is \( t^* = 2 \).

(ii) If \( \beta > \beta^* \), \( \frac{dW^{ea}(c)}{dc} < 0 \) for all \( c \in [0, 1 - \rho_0] \). The equilibrium cash holding is \( c^* = 0 \), which corresponds to a welfare maximizing policy of immediate bailout, this is \( t^* = 1 \).

(iii) For \( \beta = \beta^* \), the equilibrium government policy is indeterminate. \( t^* \in [1, 2] \) and \( c^*(t^*) \) is determined as in Proposition 1.

The proof is evident from inspection of equation (7). It is instructive to compare welfare \( W^{ea}(c) \) evaluated at \( c = 0 \) (equilibrium when governments announces a bailout when the shock is aggregate) versus \( c = 1 - \rho_0 \) (the government commits to no bailouts). The benefits of committing to no bailouts are given by the social gains from private refinancing (using own savings) relative to the social gains from public refinancing (using households resources) when shocks are aggregate, \( P_2(y - x) = P_2(1 - \beta)(1 - \rho_0) \). The costs of committing to no bailouts are given by the reduction in the scale of the project times the benefits per unit of investment, \( \beta \left[ \frac{1 - \rho_0}{2 - \rho_0 - \pi} \right] [\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2] \). The cost is adjusted by \( \beta \), which is the weight governments assign to bankers. The equation for \( \beta^* \) in the proposition comes from equalizing these costs and benefits.

2.3.2 No Commitment

In this section we assume that the government is unable to commit to its policy announcements. Banks internalize the government’s optimal ex-post actions in their optimization problem, effectively making them first-movers and giving them the ability to choose the time of the bailout \( t^* \) to maximize (6).

Definition 3 (Non-Commitment Equilibrium) A symmetric equilibrium without commitment is a cash choice of banks \( c^* \) and a policy of the government \( \Pi(t, p = 1) \) such that given the banks’ choice of cash, the policy \( \Pi(t, p = 1) \) is the ex-post best response, and hence banks maximize (6) given the government’s reaction to both banks’ cash choices.

The government never intervenes in the case of an idiosyncratic shock. This is because \( y > x \), and, in a symmetric equilibrium, takeover is feasible and socially preferable. In
contrast, the government always intervenes when the shock is aggregate. This is because, by assumption 2, \( x > 0 \), and it is preferred to refinance the remainder of the project at a social cost than let it fail.

How much do banks save at \( t = 0 \) knowing this reaction of the government? Under Assumption 4, the unique non-commitment equilibrium is when all banks hold zero cash.

**Proposition 3 (Optimal Policy without Commitment)** Under Assumptions 2 and 4, the unique equilibrium without commitment is characterized by banks choosing \( c^* = 0 \), and the government immediately intervening when the shock is aggregate.

In what follows, we focus on the parameter space subset for which it is ex-ante optimal for governments to commit not bailing out banks when the shock is aggregate, but it is ex-post optimal for them to bail out banks in such a state, which implies:

**Assumption 5** Inefficient excessive leverage: \( \frac{1-\rho_0}{\rho_1} < \beta < \beta^* \).

This assumption, which combines Assumption 2 and Proposition 2, introduces into our model the tradeoff we set out to study: the time-inconsistency of government policies. Ex-ante, the government would like to commit to no bailouts, but without commitment, there is excessive inefficient leverage in the economy, with large projects but no liquidity to refinance in case both banks fail, with inefficient bailouts on the equilibrium path. This assumption is more likely to hold when \( P_2 \) is low, \( \rho_1 \) is low and \( \pi \) is relatively high with respect to \( \rho_0 \).\(^{15}\)

### 2.4 Imperfect Information

In contrast to the previous section, here we assume the government does not observe the realization of the shocks at \( t = 1 \). Specifically, at some time \( 1 \leq t < 2 \), the government may observe a bank in distress: not having any liquidity to continue the project. In such case, the government has to decide whether to bail out or do nothing, in which case the remainder of the project is lost if not taken over. The government that decides not to bail out the first bank in distress, however, always faces the concern that both banks suffered a shock and there is not enough liquidity in the system. The posterior probability of both banks in distress, conditional on one bank in distress, is given by \( P_2' \) in equation (4).

\(^{15}\)There are several ways this assumption may not hold. On the one hand, if \( \beta \) is either lower or higher than both cutoffs \( \beta^* \) and \( \frac{1-\rho_0}{\rho_1} \), then lack of commitment does not introduce any inefficiency, rendering the problem irrelevant. On the other hand, if \( \frac{1-\rho_0}{\rho_1} > \beta > \beta^* \), it is ex-ante optimal for governments to commit to bailout banks when the shock is aggregate, but it is ex-post optimal not bailing out banks in such a state. In this last case there is inefficient insufficient liquidity and the strategic restraint mechanism we highlight in the next sections are not effective to eliminate such an inefficiency.
In case of bailing out the first bank in distress, interim welfare is 
\[ [(1 - P')_2 x + P'_2 (2x)] (2 - \bar{t}) i, \]
where \( x = \beta \rho_1 - (1 - \rho_0) \) is the social gain from refinancing with public funds. With probability \( P'_2 \) the shock is aggregate and the government bails out the two banks, but with probability \( 1 - P'_2 \) the shock is idiosyncratic and the government bails out only one. In case of not bailing out the first bank in distress, interim welfare is 
\[ [(1 - P')_2 y + P'_2 x] (2 - \bar{t}) i, \]
where \( y = \beta \rho_1 - \beta (1 - \rho_0) \) is the social gain from refinancing with private funds. With probability \( P'_2 \) the shock is aggregate, the project of the first bank in distress fails and the government bails out just the second bank in distress. With probability \( 1 - P'_2 \) the shock is idiosyncratic and the bank in distress is efficiently taken over. In both cases, the payoffs are proportional to the size of the cash infusion needed to refinance the projects, \( 2 - \bar{t} \). Given these payoffs, ex-post, the government decides to delay the bailout of the first bank in distress if
\[ P'_2 < \bar{P} \equiv 1 - \frac{x}{y}. \] (8)

Condition (8) is more likely to hold the bigger is the difference between the social benefits of private takeover versus public bailout. If this condition does not hold, the first bank in distress is bailed out regardless of the nature of the shock. In such case, banks have even less incentives to reduce the scale of the project than under full information and no commitment, since the bank expects to be bailed out regardless whether the shock is aggregate or idiosyncratic.

In contrast, if condition (8) is satisfied, the first bank in distress is not bailed out, but the second bank in distress is. This implies that the banks value functions become:
\[
V(c) = \begin{cases} 
V_s(c) + P'_2 [c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)] i & \text{if } \bar{t}(c) < \bar{t}(c') \\
V_s(c) + \frac{1}{2} P'_2 [c + (\rho_0 + \rho_1 - 1)] i + \frac{1}{2} P_2 \rho_1 (2 - \bar{t}(c)) i & \text{if } \bar{t}(c) = \bar{t}(c') \\
V_s(c) + P'_2 [c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)] i + P_2 \rho_1 (2 - \bar{t}(c)) i & \text{if } \bar{t}(c) > \bar{t}(c') 
\end{cases} \] (9)

where \( V_s(c) \) is the same as before in equation (6).

Now, as in the full information case, there is a jump in the value function – in fact there are two. The additional one is the midpoint between being bailed out or not when the banks hold the same level of cash (this is, \( \bar{t}(c) = \bar{t}(c') \)). The difference between equations (9) and (6) is that now what matters is whether the bank runs out of cash before or after its competitor (summarized by the relation between \( \bar{t}(c) \) and \( \bar{t}(c') \)).

**Definition 4 (Non-Commitment Equilibrium with Delay)** A symmetric equilibrium without commitment in case of delay (this is, \( P'_2 < \bar{P} \)) is a policy \( \Pi(t, p) \) and the cash choice
of banks $c^*$, such that $\Pi(t, P_2) = 0 \ \forall t$ after observing the first bank in distress, and $\Pi(t, 1) = 1 \ \forall t$ after observing the second bank in distress and the cash choice of banks $c^*$ is such that, given the other bank’s choice of cash, each bank maximizes (9).

Next, we solve the ex-ante optimal cash choice of a bank, $c$, taking as given the cash choice of the other bank, $c'$. In particular, we ask whether it is optimal for a bank to deviate from a symmetric strategy $c = c'$ (which implies $\bar{t}(c) = \bar{t}(c')$). The crucial part of the argument is how any deviation $c \neq c'$ affects the probability that the bank is the first one showing distress, and hence the one failing when the condition (8) holds.

Note that a marginal deviation upwards from $c = c'$ (i.e. carrying slightly more liquidity that the other bank), has the benefit of increasing discontinuously the probability of a bailout (we discuss a version of the model in which the bailout probability changes continuously later), at the cost of downsizing the project slightly, from $i(c')$ to $i(c) < i(c')$. For any marginal change, the first effect dominates, and there are always incentives to deviate as long as

$$\frac{1}{2} P_2 \rho_1 [2 - \bar{t}(c')] i(c') > 0,$$

which holds for all $\bar{t}(c') < 2$. The fraction $1/2$ is the change in the probability of being bailed out, which is multiplied by the probability of an aggregate shock and the benefit of financing the project until completion with public funds. We will refer to equation (10) as the strategic restraint condition. How this condition affects incentives is presented graphically in Figure 2, which plots the value function of a bank with cash $c$ under imperfect information, when the other bank’s cash holdings are $c'$. For any $c \leq c' < 1 - \rho_0$, the bank would like to deviate
upwards, and in particular, this is true for any tie-break \((c = c')\). The only point where incentives for deviation vanish is at \(c = 1 - \rho_0\), which is the point of full refinancing. This is summarized in Proposition 4 below.

**Proposition 4 (Non-Commitment Equilibrium and Government Uncertainty)**

If \(P_2' < \bar{P}\), there is a unique symmetric equilibrium where \(c^* = 1 - \rho_0\) (that is, \(\bar{t}(c^*) = 2\)), which coincides with the optimal solution under commitment. The equilibrium policy of the government is \(\Pi(t, P_2') = 0 \forall t\) after observing the first bank in distress, and \(\Pi(t, 1) = 1 \forall t\) after observing the second bank in distress.

The statement of the proposition follows from applying the strategic restraint condition to all cases in which the delayed bailout condition holds. In all such cases, the value of being the second bank in distress is discontinuously higher that the value of being the first bank in distress. Following a Bertrand-style undercutting argument, banks want to deviate from a symmetric strategy in order to avoid being the first in distress. At \(\bar{t} = 2\), there is a corner solution and no more incentives to deviate, since banks can self-finance completely.

This result is purposefully made very stark to highlight the main forces in the model – it relies on a discrete change in probabilities of default driven by a continuous change in cash holdings, \(c\), and then on the Bertrand competition logic. In the next section, we consider a setup in which both change continuously, and show that the main forces of the model remain intact. In particular, we show in a more general setup with ex-post shocks to cash holdings, that an analogous result to Proposition 4 holds: strategic restraint limit banks’ moral hazard and bring leverage closer to the social optimum, just not completely.

**Remark on the timing of moves** In the benchmark model, we assume that the government decides whether to bail out a distressed bank or not before knowing if a potential non-distressed bank is willing to take over. In the Appendix, we explore the alternative assumption of giving the first move opportunity to a potentially existing healthy bank. The equilibrium in this case is determined by how the outside investment opportunities of healthy banks depend on government interventions and the stability of the financial sector. When a bank gets liquidated and taken over, there may be negative external effects on the investment opportunities of the healthy banks, for example due to contagion, runs, loss of confidence, higher collateral requirements, tightening on regulation or government oversight, etc. In this case, even though a healthy bank would choose to take over a distressed bank in the absence of a bailout, the presence of a bailout may increase their other investment opportunities. In
this case the bank would rather not to announce a take over to force the bailout, and the analysis goes through unchanged.

**Remark on non-targeted bailouts**  In the benchmark model, bailouts are targeted, i.e. only the distressed bank has access to the low interest rate offered by the government. In the Appendix, we consider the effects of an increased scope of the bailout so that the transfer is available to healthy banks as well. This extension makes our results stronger in two respects. First, it makes the delayed bailout condition easier to satisfy, because it increases the social benefit of a private takeover. Second, if healthy banks are able to make take over bids before bailout decisions, as discussed in the previous paragraph, non-targeted policies strengthen the incentives of healthy banks to *not* make takeover bids, then taking advantage of subsidized interest rates.

**Remark on gradual bailouts**  Our results are derived under the assumption that bailouts are discrete events – if the government does not provide the total amount of funds needed for reinvestment, the project is prematurely terminated. In the Appendix, we allow the government to introduce *gradual bailouts*, i.e. bailouts that keep the project running until some endogenously chosen time $t'$, where $t' = 2$ corresponds to the benchmark model. First, we show that if there are no incentives for healthy banks to make takeover bids (as explained previously), our benchmark analysis goes through without change – if the shock is idiosyncratic, the government would not learn about it by doing a gradual bailout and, foreseeing this time inconsistency, it would rather bail out the whole project at once. We also characterize government delay in a general case of choosing a gradual bailout duration optimally. In the general setup developed in the next section, we characterize how such possibility affects the cutoff for delayed bailout and show it is always bounded away from zero when cash holdings are risky.

**Remark on time of distress and running out of cash**  In the benchmark, time of distress is defined as the moment banks run out of cash, and then of refinancing options at market interest rates. This assumption captures the optimal strategy of banks under our set of assumptions. On the one hand, since an interruption of the project’s flow of expenses implies a premature termination, no bank would want to ask for a bailout strictly *after* it runs out of cash. On the other hand, under government uncertainty and the delayed condition holding, no bank would want to ask for a bailout *before* it runs out of cash – as there is a chance that the other bank will run out of funds first.
Remark on strategic coordination  Our benchmark model abstracts from forces that make banks’ leverage choices strategic complements (which are present, for example, in Farhi and Tirole (2012)). In the Appendix, however, we derive the delayed bailout condition when there is an additional fixed cost of intervention, independent of the total volume of bailouts, which introduces strategic complementarity in banks’ actions. Intuitively, banks would rather show distress together as this situation minimizes the average cost of the intervention relative to the benefits of saving more projects. In such extended setup, under full information banks have incentives to coordinate. However, under imperfect information, the delayed bailout condition is more easily satisfied, and strategic restraint forces make banks’ actions strategic substitutes, just like in the benchmark model.

Remark on informational assumptions  In the benchmark model, even though banks and investors know the refinancing actions of banks, the government does not. Otherwise government would perfectly infer the state of the economy just observing how many banks raised new funds at $t = 1$. Our results do not depend on whether the market participants know the state of each bank or not, as all refinancing is fully collateralized, but governments would like ex-post (at $t = 1$) to conduct a potentially costly (and noisy) monitoring activity in order to track cash levels. Our results, however, imply that ex-ante (at $t = 0$) the government would actually not want to spend resources on such monitoring, as it would improve outcomes under no commitment.

Remark on the possibility of contagion  An extension we do not formally consider is the possibility of contagion, where a failure of one bank may trigger the need for refinancing other banks, then making the probability of an aggregate shock endogenous to the decision of delay. Still, our benchmark model can be easily extended to include this possibility, interpreted as an increase in the belief about the probability of an aggregate shock, say from $P_2$ to $P_2 + \chi$, conditional on letting a project fail. If the belief of the government about the contagiousness of shocks is independent of the refinancing needs, then the delayed bailout condition becomes $\hat{P}_2' = \frac{P_2 + \chi}{P_1 + P_2 + \chi} > P_2'$, implying that the delayed bailout condition is more difficult to satisfy in the presence of contagion. Even though this is clearly an ad-hoc introduction of contagion, any micro-founded model of contagion would map into a distribution over $\chi$ and hence could be analyzed from the perspective of our forces. Giving a full evaluation of the effects of contagion would require modeling such micro-foundations explicitly as the details of the mechanism could potentially affect the welfare costs and benefits of delay. These considerations are outside of the scope of this paper.
Remark on crises and bailouts on the equilibrium path  The equilibrium of the benchmark model, when the delayed bailout condition holds, features banks holding enough cash reserves so that on the equilibrium path we do not observe bank failures and bailouts. This would also be the result under the assumption of commitment. This prediction, however, is just the result of the purposefully stark assumption about Bertrand type of competition we made in the benchmark model. In Section 2.5, we present an extension of our model that includes ex-post shocks to cash holdings. In that model, both crises and bailouts can happen on the equilibrium path. Banks still choose to hold more liquidity than in a full-information, no commitment equilibrium, but they choose levels that do not fully exclude bank failures. Hence, in the extended setup with government uncertainty, crises happen with lower probability and they are smaller in magnitude.

2.5 Ex-post Shocks to Cash Holdings

So far we have assumed that, once bankers choose leverage at $t = 0$, and hence cash holdings $c$ for potential refinancing at $t = 1$, they do not face any uncertainty about those cash holdings. In contrast, in this section we assume that bankers suffer idiosyncratic and independent shocks to their cash position at $t = 1$, after the refinancing shock has been realized. A positive shock to cash holdings implies that a bank holds more cash than planned (the return from savings is higher than expected, for example) while a negative shock implies the bank holds less cash than planned (there is an unexpected expense to cover, for example).

Formally, the cash available for refinancing at $t = 1$ is

$$\hat{c}(h)i = (c + h)i, \quad \text{where } h \sim N(0, \sigma_h^2) \text{ and i.i.d. across banks.}$$

(11)

For analytical tractability and simplicity of exposition, we assume that shocks to cash holdings (the potential extra cash $h$) can be used to refinance own projects in distress, but not other banks’ projects in distress. Were this assumption relaxed, we would need to consider the possibility that a distressed bank with good enough cash holdings can refinance both projects when the shock is aggregate, not adding to the conclusions and just making the equations cumbersome.\footnote{An alternative assumption is that banks can only save cash for potential refinancing in a risky technology that is subject to limited liability (at $t = 1$ cash is non-negative) and is subject to a maximum capacity (at $t = 1$ cash is never more than $1 - \rho_0$ per unit of investment). The analysis of this section would be substantially more complicated without providing additional insights to our analysis. This alternative analysis is available upon request.}

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Given the shocks, cash maps into time of distress analogously to (3):

\[
t(h|\bar{c}) = \begin{cases} 
1 & \text{if } \frac{h}{1-\rho_0} \leq -(\bar{t} - 1) \\
\bar{t} + \frac{h}{1-\rho_0} & \text{if } -(\bar{t} - 1) < \frac{h}{1-\rho_0} < (2 - \bar{t}) \\
2 & \text{if } (2 - \bar{t}) \leq \frac{h}{1-\rho_0}
\end{cases}
\]  

(12)

where \( \bar{t} = 1 + \frac{\bar{c}}{1-\rho_0} \) is the expected time of distress in case of a shock when holding \( \bar{c} \). Since \( h \) follows a normal distribution with zero mean, \( t(h) \) is distributed according to

\[
f(t|\bar{c}) = \begin{cases} 
\Phi\left(-\frac{1-\rho_0}{\sigma_h}(t - 1)\right) & \text{for } t = 1 \\
\phi\left(\frac{1-\rho_0}{\sigma_h}(t - \bar{t})\right) & \text{for } 1 < t < 2 \\
1 - \Phi\left(\frac{1-\rho_0}{\sigma_h}(2 - \bar{t})\right) & \text{for } t = 2
\end{cases}
\]  

(13)

where \( \Phi \) denotes the standard cumulative normal distribution and \( \phi \) denotes the density of the standard normal distribution.

We now derive the strategic restraint condition, similarly to the benchmark model. The value of a bank (net of the expected value of takeovers, \( V_{TO} \), which is constant as before, as \( h \) nets out in expectation) of deviating from a symmetric strategy \( \bar{c} \) by choosing \( c \) is

\[
V(c|\bar{c}) = \left[ (P_0 + P_1)(c + \rho_0 + \rho_1) + P_1\rho_1 \frac{c}{1-\rho_0} + P_2\rho_1 \left( \eta(c|\bar{c}) + (1 - \eta(c|\bar{c})) \frac{c}{1-\rho_0} \right) \right] i(c)
\]

where \( \eta(c|\bar{c}) \) is the probability of being bailed out conditional on saving \( c \) while the other bank saves \( \bar{c} \), given by

\[
\eta(c|\bar{c}) \equiv Pr(t > \bar{t}) = Pr(h' - h < c - \bar{c}) = \Phi\left(\frac{c - \bar{c}}{\sqrt{2}\sigma_h}\right).
\]

and \( h' \) denotes the shock to cash holdings of the other bank, such that \( h' - h \sim \mathcal{N}(0, 2\sigma_h^2) \).

Taking the derivative of the value function with respect to \( c \),

\[
\frac{dV(c)}{dc} = -C \frac{i^2(c)}{A} - \eta(c|\bar{c}) \frac{P_2\rho_1}{1-\rho_0} \left[ 2 - \rho_0 - \pi \right] \frac{i^2(c)}{A} + \eta'(c|\bar{c}) \frac{P_2\rho_1}{1-\rho_0} \left[ 1 - \rho_0 - \pi \right] i(c),
\]

(14)

where \( C \) is given by the left hand side of Assumption 3 (and then negative by assumption), and \( \eta'(c|\bar{c}) \) is the marginal change in the probability of bailout when a banker saves more
cash, conditional on the other bank still saving $\bar{c}$, given by

$$\eta'(c, \bar{c}) = \frac{\partial \eta(c|\bar{c})}{\partial c} = \left(\frac{1}{\sqrt{2\sigma_h}}\right) \phi \left(\frac{c - \bar{c}}{\sqrt{2\sigma_h}}\right).$$

In a symmetric equilibrium, in which both banks hold in expectation the same amount of cash (this is, $c = \bar{c}$), the probability of showing distress second is 50% when the shock is aggregate (this is, $\eta(\bar{c}|\bar{c}) = 0.5$ since $t = \bar{t}$) and the change in such probability from holding more cash is $\eta'(\bar{c}|\bar{c}) = \frac{\phi(0)}{\sqrt{2\sigma_h}}$, where $\phi(0)$ is the density of the standard normal distribution evaluated at 0.

To put the derivative (14) in perspective, assume full information and no bailouts, such that $\eta(c|\bar{c}) = 0$ and $\eta'(c|\bar{c}) = 0$. Then

$$\frac{dV(c)}{dc} = -C \hat{i}^2(c)A > 0,$$

since, by Assumption 3, $C < 0$. This implies that banks always want to increase cash reserves to refinance full scale. In contrast, if we assume full information and guaranteed bailouts, such that $\eta(c|\bar{c}) = 1$ and $\eta'(c|\bar{c}) = 0$, we have

$$\frac{dV(c)}{dc} = -[D + P_2 (2 - \rho_0 - \pi)] \frac{i^2(c)}{A} < 0$$

where $D$ is given by the left hand side of Assumption 4 (and hence positive). In this case, banks always want to reduce cash reserves to maximize the size of the project, given the certainty of a bailout.

Hence, if $\eta' = 0$, there is a cutoff $\bar{\eta}$, such that, for all probabilities of bailout low enough (this is, $\eta < \bar{\eta}$), banks would like to increase cash reserves to have the possibility of refinancing the project fully. We focus on symmetric strategies, and consider deviations evaluated at $c = \bar{c}$, which implies replacing $\eta(\bar{c}|\bar{c})$ and $\eta'(\bar{c}|\bar{c})$ in expression (14).

Define $Z$ by replacing $\eta(\bar{c}|\bar{c}) = \frac{1}{2}$ in equation (14), such that

$$Z \equiv C + \frac{P_2 \rho_1}{1 - \rho_0} \left[1 - \frac{\rho_0 + \pi}{2}\right].$$

Then

$$\frac{dV(c)}{dc} = -Z \hat{i}^2(c)A + \eta'(\bar{c}|\bar{c}) \frac{P_2 \rho_1}{1 - \rho_0} [1 - \rho_0 - c] \hat{i}(c)$$

Using on this expression, we can now characterize the equilibrium with shocks to cash.
holdings and relate it to the variance of those shocks, $\sigma_h$.

**Proposition 5 (Equilibrium with Shocks to Cash Holdings)**

(i) If $Z \leq 0$, the equilibrium is unique and coincides with the socially optimal level of cash holdings, this is $c^* = 1 - \rho_0$, independent of the shock volatility;

(ii) If $Z > 0$, the equilibrium $c^*(\sigma_h)$ is unique, decreasing in $\sigma_h$ and solves

\[
\frac{dV(c^*)}{dc} = -\frac{Z}{1 - \pi + c^*} + \frac{\phi(0)}{\sqrt{2}\sigma_h (1 - \rho_0)} [1 - \rho_0 - c^*] = 0, \tag{15}
\]

on the domain $c \in [0, 1 - \rho_0]$, for all $\sigma_h < \sigma_h$, where

$$
\sigma_h = \frac{P_2\rho_1\phi(0)}{\sqrt{2}(1 - \rho_0)Z}(1 - \rho_0)(1 - \pi).
$$

Furthermore, $\lim_{\sigma_h \to 0} c^*(\sigma_h) = 1 - \rho_0$.

**Proof.** Part (i) follows directly from the analysis in the text. For part (ii), notice that (15) can be expressed as

\[-Z + \frac{\phi(0)}{\sqrt{2}\sigma_h (1 - \rho_0)}(1 - \rho_0 - c)(1 - \pi + c) = 0\]

and is quadratic in $c$. Moreover, with $\sigma < \sigma_h$, one root is strictly positive and in $[0, 1 - \rho_0]$ (and a local max), and one is strictly negative (and a local min). What this means is that for any $\sigma_h < \sigma_h$, there is exactly one root $c^*(\sigma_h) \in [0, 1 - \rho_0]$ which solves (15) and the derivative is positive for all $0 \leq c < c^*(\sigma_h)$.

For the last part of the proof, notice that $c^*(\sigma_h)$ is decreasing in $\sigma_h$ and as $\sigma_h \to 0$, the variable term in (15) goes to $+\infty$ and is always positive, hence giving a corner solution in the limit, such that $c^*(\sigma_h = 0) = 1 - \rho_0$.

Intuitively, from expression (14), the benefits from deviating and holding more cash than the other bank is to increase the chances of being second in distress and obtain a bailout when the shock is aggregate. In a symmetric equilibrium the benefits are decreasing in $\sigma_h$ and $c$. In contrast, the costs from deviating come from downsizing the project. In a symmetric equilibrium, the costs are independent of $\sigma_h$. Since the bank is indifferent between deviating or not when expression (14) is zero, the larger is $\sigma_h$ the lower the benefits from deviating.

\footnote{To complete the characterization, as $\sigma_h \to \sigma_h$, the equilibrium cash holdings $c^*(\sigma_h)$ converges to zero if $\pi < \rho_0$ and to $\pi - \rho_0$ if $\pi > \rho_0$.}
Hence the indifference is recovered when $c$ is lower, or equivalently, when banks hold cash to refinance a smaller fraction of the distressed project in case of an aggregate shock.

Consider the possible values for $\sigma_h$. If $\sigma_h = \infty$, $\eta'(\bar{c}|\bar{c}) = 0$, the randomness of cash holdings is so large that banks cannot change their relative position by reducing their leverage. In this case, strategic restraint depends purely on the sign of (14) evaluated at $\eta = \frac{1}{2}$ and $\eta' = 0$ (that is, the sign of $Z$). For $\sigma_h < \infty$, a reduction in leverage increases the probability of not being the first bank in distress when the shock is aggregate, which is captured by $\eta' > 0$. The smaller the $\sigma_h$, the larger the marginal increase in such probability and the more likely that expression (14) is positive, inducing a reduction in leverage. In the limit, when $\sigma_h = 0$, $\eta' = \infty$, and we obtain the same conclusion as in the benchmark model. The gains from deviating are so large that we recover the commitment outcome with full refinancing.

3 Policy Implications

In the previous section, we showed that government uncertainty together with strategic restraint has the potential to implement allocations closer to efficient commitment outcomes, even in the absence of commitment. Below, we ask which market structures and features of the industry support the discipline on leverage implied by our informational friction. The discussion in this section illustrates the applicability of the insights from the theoretical model to regulation and design of the banking industry, as well as provides an overview of the limitations of the mechanism.

More specifically, we study the impact of financial innovation, industry concentration, and asymmetric bank sizes on the potential of government uncertainty to induce more efficient outcomes in the absence of commitment.

3.1 Financial Innovation

How government uncertainty and bank behavior change with financial innovation that allows banks to insure away part of their idiosyncratic risk? In this section we show that when banks are allowed to share risk with other banks, for example by holding a large amount of correlated assets, using swaps, over-the-counter derivatives, and other instruments to cross-insure each other’s cash flows, the timing of any individual bank distress becomes a more precise signal about the nature of the refinancing shock.

The message of this section is that in situations in which banks hold large amounts of cross-insurance - like in the most recent period before the great recession - the industry as
a whole is more susceptible to moral hazard behavior and crises. Intuitively, in such cases banks’ balance sheets are connected and therefore a single bank’s distress is a precise signal that the whole system is in distress. Hence, the government is more likely to bail out and banks are more likely to take excessive leverage ex-ante.

We call $s$ the fraction of a project’s risk a bank can diversify away by using financial instruments. We derive a cap on $s$, which can be more generally interpreted as a cap on risk sharing among financial firms, which allows government uncertainty to remain operational in implementing superior outcomes.\(^{18}\)

We consider the general setting of Section 2.5, with idiosyncratic shocks to the banks’ cash holdings, $h \sim \mathcal{N}(0, \sigma^2_h)$, which hit at $t = 1$, after the refinancing shock has been realized (either aggregate or idiosyncratic). The cash available to refinance when the shock is aggregate is again given by equation (11), while the cash available to refinance when the shock is idiosyncratic is

$$
\hat{c}(h, s)i = (c + s(\rho_0 + \rho_1) + h(1 - s))i
$$

Note the shock to cash, $h$, is proportional to the size of the project that needs refinancing.\(^{19}\)

Define the time of distress given an aggregate shock as $t^a(h)$ such that $(t^a(h) - 1)i$ is the amount of the investment that can be refinanced at market rate $R = 1$. When the shock is aggregate, this is equivalent to Section 2.5, and hence $t^a(h|\bar{c}) = t(h|\bar{c})$ is given by equation (12), and its density is given by $f^a(t|\bar{c}) = f(t|\bar{c})$ in equation (13).

Define the time of distress given an idiosyncratic refinancing shock as $t^i(h|\bar{c})$, where $(t^i(h|\bar{c}) - 1)i$ is the amount of the investment that can be refinanced at the market rate $R = 1$, equal to $c(h, s)/[(1 - \rho_0)(1 - s)]$. We can write $t^i(h|\bar{c})$ as function of the expected time of distress $\bar{t} = 1 + \frac{\bar{c}}{1 - \rho_0}$ as

$$
t^i(h|\bar{c}) = \begin{cases} 
\bar{t} - s & \frac{1}{1 - s} + \frac{s(\rho_0 + \rho_1)}{(1 - \rho_0)(1 - s)} + \frac{h}{1 - \rho_0} \\
2 & \text{if } \frac{h(1 - s) + s(\rho_0 + \rho_1)}{1 - \rho_0} < -\bar{t} - 1 \\
\frac{h(1 - s) + s(\rho_0 + \rho_1)}{1 - \rho_0} & \text{if } -\bar{t} - 1 < \frac{h(1 - s) + s(\rho_0 + \rho_1)}{1 - \rho_0} < (2 - \bar{t}) - s \quad (16) \\
(2 - \bar{t}) - s & \text{if } (2 - \bar{t}) - s < \frac{h(1 - s) + s(\rho_0 + \rho_1)}{1 - \rho_0}
\end{cases}
$$

\(^{18}\)We do not model the well-known benefits of diversification but rather focus on an unexplored cost of diversification, which makes the timing of distress of the first bank informative about the nature of the refinancing shock, then hindering the use of uncertainty as commitment.

\(^{19}\)This assumption is not crucial for our result, but allows for a clear evaluation of the monotonicity for the posterior belief of the government in (18), as in this case the likelihood ratio is monotone.
Given the distribution of $h$, $t_i(h)$ is distributed according to following density $f^i(t|\bar{c})$

$$f^i(t|\bar{c}) = \begin{cases} 
\Phi \left( \frac{-1 - \rho_0(1 - s)}{\sigma_h} - \frac{s(\rho_0 + \rho_1)}{\sigma_h(1 - s)} \right) & \text{for } t = 1 \\
\phi \left( \frac{1 - \rho_0(1 - s)}{\sigma_h} \right) - \frac{s(\rho_0 + \rho_1)}{\sigma_h(1 - s)} \right) & \text{for } 1 < t < 2 \\
1 - \Phi \left( \frac{1 - \rho_0(1 - s)}{\sigma_h} \right) - \frac{s(\rho_0 + \rho_1)}{\sigma_h(1 - s)} \right) & \text{for } t = 2 
\end{cases} \tag{17}$$

where $\phi$ and $\Phi$ are the pdf and cdf of the standard normal distribution, and the mean of $t^i(h|\bar{c})$ (i.e. setting $h = 0$) is equal to $\frac{\bar{c}}{(1 - s)} + \frac{s(\rho_0 + \rho_1)}{(1 - \rho_0)(1 - s)}$.

When $s = 0$, $f^a(h|\bar{c}) = f^i(h|\bar{c})$. When $s > 0$, $f^i(1|\bar{c}) < f^a(1|\bar{c})$ and $f^i(2|\bar{c}) > f^a(2|\bar{c})$, which implies it is more likely to see aggregate shocks earlier than idiosyncratic shocks. The updated probability of an aggregate shock, after observing a bank in distress, is then

$$P'_2 \equiv P(Agg|t) = \frac{P(t^a(h) = t|Agg)P_2}{P(t^a(h) = t|Agg)P_2 + P(t^i(h) = t|Id)P_1}.$$

The government does not bailout the first bank in distress as long as the probability of an aggregate shock is smaller than the cutoff $\bar{P}$ from the delayed bailout condition (8)

$$P'_2 = \frac{f^a(t)P_2}{f^i(t)P_1} \leq \bar{P}, \tag{18}$$

Note that, when $s = 0$, $f^a(t) = f^i(t)$, and $P'_2$ is the one obtained in the benchmark, $P'_2 = \frac{P_2}{P_1 + P_2}$. Additionally, for $s > 0$, the likelihood ratio under normality is declining in $t$. A sufficient condition for (18) to hold at any moment $t$ is that it holds at $t = 1$, which gives

$$\left( \frac{\bar{c}}{(1 - s)} + \frac{s(\rho_0 + \rho_1)}{(1 - \rho_0)(1 - s)} - 1 \right)^2 - (\bar{c} - 1)^2 \leq 2\frac{s^2}{\sigma_h^2} \ln \left( \frac{\bar{P}}{(1 - \bar{P})} \frac{P_1}{P_2} \right). \tag{19}$$

Since the left hand side of (19) is strictly increasing in $s$ and the right hand side is a constant, there is a strictly positive $\bar{s}$ that is the minimum between $1/2$ (the maximum possible level of cross-insurance) and the value of $s$ that satisfies (19) with equality.

Any level of cross-insurance lower than or equal to $\bar{s}$ guarantees an outcome more efficient than the non-commitment one, as described in Proposition 5. Finally, it is straightforward to see that $\bar{s}$ is weakly increasing in $\sigma_h^2$. There is a $\sigma_h^2$ large enough such that $\bar{s} = 1/2$ and full insurance does not prevent uncertainty to implement the commitment outcome. In contrast, if $\sigma_h^2 = 0$, we are back in the benchmark case, in which $\bar{s} = 0$ and any level of cross-insurance eliminates government uncertainty.
Remark on systemic risk  Cross-insurance here does not operate through affecting the systemic risk in the market, but instead by allowing governments to learn better about a systemic event. Alternatively, financial innovations could also allow banks to coordinate on a particular risk factor, making systemic events more likely. This is a straightforward extension of our model in which banks can directly affect the value of $P_2$. If banks could coordinate to induce a relatively high probability of an aggregate shock (i.e., $P'_2 > \bar{P}$) then the government would not delay bailouts and, in the absence of strategic restraint, the outcome would be the full information no-commitment solution of our model. Naturally, the policy prescription in such situation is that governments should also limit the use of these types of financial innovation to guarantee there is enough pessimism about the likelihood of a systemic event to deliver government delay ex-post.

3.2 Number of banks

In what follows we study the government incentives to bail out the first bank in distress when $N > 2$. We show that having more banks in the economy reduces the incentives to bail out the first bank in distress due to two forces. First, more banks implies a higher chance that there is enough liquidity in the system for a takeover of a distressed bank. Second, it also means that the size of a distressed bank relative to the industry is smaller, which increases the option value of delaying and learning. Both having a smaller ‘test case’ and larger ‘rest of industry’ at stake makes it more valuable for governments to wait and see in order to take a better informed action later on.

The many-bank case analysis is simplified by the observation that for any number of banks, not bailing out the first bank in distress is enough to trigger strategic restraint and to obtain outcomes closer to the commitment allocation. Hence, we focus on the government’s incentives to bail out the first bank in distress. Denote by $p_{(N,d)|n}$ the probability of having $d$ banks in distress conditional on having $N$ banks in total and observing $\hat{d}$ banks already in distress. For example, $p_{(2,2)|1} \equiv P'_2$ is the updated probability of an aggregate shock after observing one bank in distress from the previous discussion when $N = 2$.

We initially impose two restrictions, which will be relaxed later. First, the second bank in distress is always bailed out. Second, the probability of an aggregate shock is independent on the number of banks. The next Lemma, proved in the Appendix, is an important benchmark to understand the effects of $N$ on delay decisions. Under these two restrictions the number of banks $N$ does not affect the conditions for delaying intervention on the first bank.
Lemma 1 Let $p_{(N,N)|1} = p$ for all $N$. If the government bails out for sure the second bank in distress, then the condition for delayed bailout is $p < 1 - \frac{x}{y}$ for all $N$.

In the next two lemmas, also proved in the Appendix, we separately relax the two restrictions and isolate the two forces for which more banks induce governments to delay intervention more likely.

First, Lemma 2 relaxes the assumption that governments have to intervene if a second bank is in distress. This introduces the option value of not bailing out the first bank and having the chance to make the optimal decision of not bailing out if other banks show distress. This Lemma shows that having more banks makes more valuable the option value of learning and making better decisions ex-post.

Lemma 2 For all $N$, let $p_{(N,N)|1} = p$, $p_{(N,N-1)|2,to} = p'$ conditional on the first bank being taken over (to) and $p_{(N,N-1)|2, nto} = 1$ conditional on the first bank being not taken over (nto). The delayed bailout condition of the first bank is more easily satisfied for larger $N$.

Lemma 3 relaxes the assumption that the probability of an aggregate shock is independent of $N$. Effectively, if the idiosyncratic shocks are i.i.d., and in particular, independent of the aggregate shock, then it is less likely that no bank is healthy when there are many banks, and then less likely that the first bank in distress is not taken over and fails in case of not being bailed out by the government.

Lemma 3 Assume governments always bailout second banks in distress. Then the probability to delay the bailout of the first bank in distress weakly increases with $N$.

Together, the previous two lemmas give the following Proposition, which says that the incentives to provide a bailout are decreasing with the number of banks.

Proposition 6 The larger the number of banks in the economy, the more likely governments delay interventions when banks start showing distress.

3.3 Too Big To Fail

This section studies the effects of asymmetric bank sizes on governments’ incentives to delay bailouts. Our goal is to analyze the impact of too big to fail banks – banks whose balance sheets are very large relative to the whole industry. We show that our novel mechanism introduces a new rationale for capping bank size, over and above the one driven by concerns
of contagion highlighted in the literature. Having banks whose balance sheets are over-
whelmingly big relative to the size of the industry makes them impossible to take over, and
effectively immune to concerns about not being bailed out. In equilibrium, the existence of
‘too big’ banks allow smaller banks to restrain leverage just slightly below that of big banks.

Formally, we modify the benchmark by assuming that Bank 1 has higher initial assets
than Bank 2, i.e. $A_1 > A_2$. Such ex-ante asymmetry will imply ex-post asymmetry in
investment size and consequently a healthy Bank 2 may not have enough funds to take over
a distressed Bank 1. Specifically, Bank 2’s available cash, which potentially can be used to
refinance Bank 1’s project, is equal to $(\rho_0 + \rho_1 + c_2)i_2$. Hence, the reinvestment scale in case
Bank 2 needs to take over Bank 1’s project is

$$I = \min \left\{ \frac{(\rho_0 + \rho_1 + c_2)i_2}{1 - \rho_0}, (2 - \bar{t}_1)i_1 \right\}.$$ 

The reinvestment scale is either equal to the part of the project that needs refinancing,
$(2 - \bar{t}_1)i_1$, or to the maximal amount of money that Bank 2 can raise by levering up its own
cash. Clearly, for large enough asymmetry, the latter is going to be strictly smaller than the
former and the project will be scaled down even under takeover.

The delayed bailout condition after seeing the large bank in distress becomes $(1 - P'_2)yI \geq
x(2 - \bar{t}_1)i_1$, or

$$P'_2 \leq \bar{P} \equiv 1 - \frac{x}{y \left( \frac{I}{(2 - \bar{t}_1)i_1} \right)}.$$  \hfill (20)

When $A_2/A_1$ goes to zero, $i_2/i_1$ also goes to zero and hence both terms in the denominator approach zero. This implies there is a level of asymmetry such that $\bar{P} = 0$ and the
government always bails out the large bank in distress, regardless of the updated belief about
the probability of an aggregate shock. In contrast, as $A_2/A_1$ goes to one, $\bar{P}$ converges to the
original cutoff shown in equation (8). Any level of asymmetry implying $\frac{I}{(2 - \bar{t}_1)i_1} < 1$ makes
the cutoff smaller, such that delay is more difficult to occur.

If condition (20) does not hold, Bank 1 has no incentive to restrain leverage, since it
would be bailed out anyways, then choosing $c_1 = 0$. This implies that it is optimal for
Bank 2, conditional on Bank 1 holding no cash, to hold a slightly positive amount of cash
to guarantee showing distress in second place when the shock is aggregate. The large bank
becomes a ‘shield’ for the small bank to engage in inefficient levels of leverage. This points
to a new and unique negative externality of ‘too big to fail’ banks for households, who may
not only need to bailout large banks but also excessive risk exposure of small banks.
4 Conclusions

At the onset of financial crises, banks usually show distress sequentially. At least initially, governments are uncertain about the nature of the problem at hand, and may decide to delay intervention to learn further about the underlying situation from market outcomes. Crucially, such delays introduce incentives for banks to ‘compete’ for not being among the first ones in trouble, giving rise to endogenous strategic restraint of their risk taking and their exposure to liquidity shocks.

We show that these novel forces have dramatic effects on equilibrium outcomes. In seminal models of banking and liquidity, Holmström and Tirole (1998) and Farhi and Tirole (2012) show that non-commitment can lead to endogenous crises and inefficient bailouts. Introducing government uncertainty radically changes these results, moving the economy from inefficiently high levels of leverage to more efficient levels, even in the absence of commitment.

Based on these insights we provide a novel discussion of how financial innovations, banking concentration and asymmetric bank sizes can induce endogenous systemic events. In our case this works through the effects of these aspects of the banking industry on the inference problem of the government and the ensuing strategic behavior of banks.

The literature has identified the time-inconsistency of governments’ policies as an important justification for macro-prudential regulation and direct oversight of banking activity. However, historically regulators have been incapable to design macro-prudential regulation that prevents crises without choking off growth. Our work suggests that it may be optimal for governments to design political structures that delay bailouts decisions, or regulatory standards that maintain an optimal level of uncertainty, imposing lower burdens on preventive regulation and giving more room to facilitate optimal self-regulation. Contrary to the common view that information and speed of action are desirable characteristics of policymaking, we make the case that banks’ perception about governments reacting fast to systemic events give them incentives to coordinate on those events, endogenously generating crises.

We have assumed that governments cannot learn the nature of a bank’s distress, but clearly in the model, ex-post they would prefer to have such knowledge and react optimally. From an ex-ante perspective, however, governments that lack commitment would rather not have the possibility to learn rapidly, making more mistakes when crises happen, but also reducing the probability of those crises. Even though it may be difficult for a government to commit not to modify bailout decisions ex-post, they could easily avoid ex-ante the implementation of technologies to learn rapidly or to take fast decisions when distress happens. Examples of these ex-ante institutional designs include cumbersome bureaucratic steps for
decision-making, parliamentary systems with lengthy discussions and supermajority rules to pass certain law changes, the fragmentation of supervisory bodies with separate functions and access to incomplete pieces of information, insufficient information sharing across government bodies, etc.

References


A Appendix Proofs and Derivations

A.1 Derivation of Assumptions 3 and 4
If \( \bar{t} < t^* \), value functions in equation (6) can be rewritten (net of the value of taking over another bank, \( V_{TO} \)) as

\[
V(c|\bar{t} < t^*) = (P_0 + P_1)[c + \rho_0 + \rho_1]i(c) + P_1\rho_1 \frac{c}{1 - \rho_0} i(c) + P_2 \left[ c + (\rho_0 + \rho_1 - 1) \frac{c}{1 - \rho_0} \right] i(c)
\]

when replacing \( j(c) = (\bar{t} - 1)i(c) \) by \( j(c) = \frac{c}{1 - \rho_0} i(c) \). Hence

\[
V(c|\bar{t} < t^*) = (P_0 + P_1)[c + \rho_0 + \rho_1]i(c) + (P_1 + P_2)\rho_1 \frac{c}{1 - \rho_0} i(c)
\]

Since \( ci = (\pi - 1)i(c) + A \),

\[
V(c|\bar{t} < t^*) = \left[ P_0 + P_1 + (P_1 + P_2) \frac{\rho_1}{1 - \rho_0} \right] [(\pi - 1)i(c) + A] + (P_0 + P_1)(\rho_0 + \rho_1)i(c).
\]

Taking derivatives with respect to \( c \),

\[
\frac{dV(c)}{dc} = \frac{\partial V(c)}{i(c)} \frac{\partial i(c)}{dc}
\]

where

\[
\frac{\partial V(c)}{\partial i} = (P_0 + P_1)[\pi - 1 + \rho_0 + \rho_1] + (P_1 + P_2) \frac{\rho_1}{1 - \rho_0} (\pi - 1)
\]

\[
\frac{\partial i(c)}{\partial c} = -\frac{i^2(c)}{A} < 0
\]

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Then
\[
\frac{dV(c)}{dc} = -C\frac{i^2(c)}{A}
\]
where
\[
C \equiv (P_0 + P_1) [\rho_0 + \rho_1 + \pi - 1] - (P_1 + P_2) \frac{1 - \pi}{1 - \rho_0} \rho_1
\]
Then, we guarantee that \( \frac{dV(c)}{dc} > 0 \) for \( t < t^\ast \) if \( C < 0 \), as in Assumption 3.

If \( \bar{t} > t^\ast \), value functions can be rewritten as
\[
V(c|\bar{t} > t^\ast) = (P_0 + P_1)[c + \rho_0 + \rho_1]i(c) + P_1 \rho_1 \frac{c}{1 - \rho_0} i(c) + P_2 [c + \rho_0 + \rho_1 - (t^\ast - 1) - \rho_0(2 - t^\ast)] i(c)
\]
A sufficient condition for this value function to decrease in \( c \) can be determined at \( t^\ast = 2 \), when the incentives to hold cash are maximized. Then
\[
V(c|t^\ast = 2) = (P_0 + P_1)[c + \rho_0 + \rho_1]i(c) + P_1 \rho_1 \frac{c}{1 - \rho_0} i(c) + P_2 [c + \rho_0 + \rho_1 - 1] i(c)
\]
Again
\[
\frac{dV(c)}{dc} = -D\frac{i^2(c)}{A}
\]
where
\[
D \equiv (P_0 + P_1 + P_2) [\rho_0 + \rho_1 + \pi - 1] - P_1 \frac{1 - \pi}{1 - \rho_0} \rho_1 - P_2
\]
Then, we guarantee that \( \frac{dV(c)}{dc} < 0 \) for \( t < t^\ast \) if \( D > 0 \), as in Assumption 4.

### A.2 Proof of Proposition 1

Fix \( t^\ast \) and consider \( c \) such that \( \frac{c}{1 - \rho_0} < t^\ast - 1 \). The bank’s value function on this domain is
\[
V(c) = [P_2(c + (\rho_0 + \rho_1 - 1)\bar{t}(c) - 1)] + (P_0 + P_1)(c + \rho_0 + \rho_1) + P_1 \rho_1 (\bar{t}(c) - 1) i + P_1 V_{TO}
\]
where \( V_{TO} = (\rho_0 + \rho_1 - 1)(i^\prime - j^\prime) \) is independent of \( c \). Since
\[
ci = (\pi - 1)i + A
\]
\( V(c) \) can be written just in terms of \( i(c) \), replacing \( ci \) into \( j = (\bar{t}(c) - 1)i = \frac{ci}{1 - \rho_0} \).

Since \( \frac{dV(c)}{dc} = V'(i)i'(c) \) and \( i'(c) = -\frac{A}{(1 - \pi + \delta)^2} < 0 \), then \( \frac{dV(c)}{dc} > 0 \) if and only if \( V'(i) < 0 \), which is the case if
\[
V'(i) = (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - (P_1 + P_2) \frac{\rho_1}{1 - \rho_0} (1 - \pi) < 0.
\]
Assumption 3

Whenever \( c \) is too low to fully refinance at \( R = 1 \), it is optimal for the bank to increase cash holdings. For \( 1 > \frac{c}{1 - \rho_0} > t^\ast - 1 \), the bank always refinances fully on the market and the value function is
\[
V(c) = [P_2(c + \rho_0 + \rho_1 - (t^\ast - 1) - \rho_0(2 - t^\ast))] + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1 \rho_1 (\bar{t}(c) - 1) i + P_1 V_{TO}.
\]
Again, $\frac{dV(c)}{dc} = V'(i)i'(c)$ and $i'(c) < 0$, which implies that $\frac{dV(c)}{dc} < 0$ if and only if $V'(i) > 0$.

$$V'(i) = (P_0 + P_1 + P_2)\left(\pi - 1 + \rho_0 + \rho_1\right) - P_1\rho_1\left(\frac{1 - \pi}{1 - \rho_0}\right) - P_2\left((1 - \rho_0)(t^* - 1) + \rho_0\right)$$

Then $V'(i) > 0$ if and only if

$$(P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1\rho_1\frac{1}{1 - \rho_0}(1 - \pi) - P_2(\rho_0 + (1 - \rho_0)(t^* - 1)) > (P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1\rho_1\frac{1}{1 - \rho_0}(1 - \pi) - P_2 > 0.$$  

Hence, on this part of the domain, it is optimal to decrease cash holdings.

Finally, for $\frac{c}{1 - \rho_0} > 1$, the value of the bank is

$$V(c) = (P_2 + P_1)(c + \rho_1 + \rho_0 - 1)i + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1V_{TO},$$

taking the derivative with respect to $i$ and considering $\frac{dV(c)}{dc} < 0$ if $V'(i) > 0$

$$V'(i) = (\pi + \rho_1 + \rho_0) - (1 + P_1 + P_2) > 0.$$  

Therefore, here also it is optimal to decrease cash, which completes the proof. ■

### A.3 Proof of Lemma 1

For $N = 2$ the condition for not bailing out the first bank in distress is given by condition (8) with $P_2^\prime = p_{(2,2)|1} = p$. In the case of $N = 3$, the social gains from bailing out the first bank in distress are

$$p3x + (1 - p)\left[\frac{p_{(3,2)|1}}{1 - p}2x + \frac{p_{(3,1)|1}}{1 - p}x\right],$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p2x + (1 - p)\left[\frac{p_{(3,2)|1}}{1 - p}(y + x) + \frac{p_{(3,1)|1}}{1 - p}y\right].$$

Since $\frac{p_{(3,2)|1}}{(1 - p)} + \frac{p_{(3,1)|1}}{(1 - p)} = 1$, the condition to delayed bailout is also $p_{(3,3)|1} = p < 1 - \frac{x}{y}$. More generally, for any arbitrary $N$, the social gains from bailing out the first bank in distress are

$$pNx + (1 - p)\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p)}dx$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p(N - 1)x + (1 - p)\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p)}(y + (d - 1)x).$$

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Since $\sum_{d=1}^{N-1} \frac{p(N,d)1}{(1-p)} = 1$, the condition to delayed bailout is again

$$p_{(N,N)|1} = p < 1 - \frac{x}{y}.$$  

\[\square\]

### A.4 Proof of Lemma 2

For the $N = 3$ case, the delayed bailout condition is exactly as in the two bank case. For $N = 4$, the social gains from bailing out the first bank in distress are

$$p4x + (1 - p)[x + q(p'2x + (1 - p')x)]$$

while the social gains from not bailing out the first bank in distress are

$$p3x + (1 - p)[y + q \max\{p'2x + (1 - p')x, p'x + (1 - p')y\}]$$

where $q = \frac{p(4,3)|1 + p(4,2)|1}{(1-p)}$, $p' = p(4,3)|2, to = \frac{p(4,3)|1}{q(1-p)}$ and $(1 - p') = p(4,2)|2, to = \frac{p(4,2)|1}{q(1-p)}$. Then it is optimal to not bail out the first bank in distress if

$$(1 - p)(y - x) + (1 - p)q \max\{-p'x + (1 - p')(y - x), p'x - (1 - p')(y - x)\} > px$$

(22)

The last term in equation (21) is the value of keeping the option of introducing the bailout at a later time, under new posterior $p'$ which is necessarily more accurate. Since this number is at least weakly bigger than the value of immediate bailout, the actual cutoff value for delayed bailout condition $\bar{P}$ is weakly higher than for the two bank case. For values of $p'$ for which it is optimal to not bail out the second bank in distress, the increase is strictly positive. This shows that the cutoff probability for delayed bailout in the $N = 4$ case is equal or larger than the two or three banks case.

For the more general $N$ bank case the condition for delayed bailout is exactly (22) if we restrict $p'$ and $q$ to be the same across $N$, and we restrict government policies to always bail out the third bank in distress. More generally, incentives to delay depend on the sequence of Bayesian updates in response to observing takeovers. Let $p'_{(N,N-k)|k, to}$ be the posterior probability that $N - k$ banks are in distress conditional on observing $k$ takeovers (well defined for $2k < N$). Suppose that for two different numbers of banks in the economy, $M > N$, $p'_{(M,M-k)|k, to} = p'_{(N,N-k)|k, to}$, whenever both are well defined. In this case, giving the government more chances to introduce the bailout always has an effect of relaxing the delayed bailout condition. It is a sum of nonnegative numbers for both $M$ and $N$, and these numbers are the same in both cases up to the point where you cannot delay further with $N$. In the $M$ case, however, there are at least one more nonnegative term, as in the comparison between $N = 3$ and $N = 4$. Since the government will have at least weakly more chances for $M$ than in $N$, the left hand side of the analog of (22) will be weakly larger.\[\square\]
A.5 Proof of Lemma 3

The social gains from bailing out the first bank in distress is

\[ p(N)Nx + (1 - p(N)) \sum_{d=1}^{N-1} \frac{p(N,d|1)}{(1 - p(N))} dx \]

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

\[ p(N)(N-1)x + (1 - p(N)) \sum_{d=1}^{N-1} \frac{p(N,d|1)}{(1 - p(N))} (y + (d-1)x) \]

Since \( \sum_{d=1}^{N-1} \frac{p(N,d|1)}{(1 - p(N))} = 1 \), the condition to delay intervention is

\[ p(N) < 1 - \frac{x}{y} \]

While the cutoff for delay does not change, the updated probability \( P(N) \) is decreasing in \( N \) if the idiosyncratic shocks are independent, triggering strategic restraint more likely for larger number of firms. For example, if the probability of no aggregate shock is \( \alpha \) and the probability of no idiosyncratic shock is \( \lambda \), then under independence, the probability of no healthy banks after observing the first one is given by

\[ p(N) \equiv p(N,N|1) = \frac{(1 - \alpha) + \alpha(1 - \lambda)^N}{1 - \alpha \lambda} \]

clearly decreasing in \( N \). ■

B Appendix Extensions

This section provides a discussion of the robustness of our results to modifications of the main assumptions of the model.

Timing of Moves In the benchmark model, we assume that the government decides whether to bail out a distressed bank or not before knowing if the non-distressed bank is willing to take over. Furthermore, this decision was assumed irreversible – the government cannot ex-post reverse the bailout, even if the true shock turns out to be idiosyncratic (i.e. the healthy bank would have been interested in taking over the distressed project). In this section we give the first move opportunity to the healthy bank, before the bailout decision by the government.

Specifically, assume that in case of a bank’s distress, the healthy bank moves first with a takeover bid, then the government decides to implement the bailout, and then the healthy bank moves again and has another chance to make a takeover bid. The idea is that, if a
takeover bid is not made in the first move of the non-distressed bank, it can still be made ex-post, once the government’s action is known.

The outcome of the model under the new assumption depends crucially on the outside investment opportunities of the healthy bank. Below, we sketch a modification of the model under which each bank has access to alternative investment opportunities, and the payoffs of these opportunities depend on the stability of the financial sector. This assumption captures the idea that, when a bank gets liquidated, there may be negative external effects on the investment opportunities of other banks, for example due to contagion, fire sales, runs, higher collateral requirements, tightening on regulation or government oversight, etc.

Formally, we assume that i) banks have an additional investment opportunity to undertake at \( t = 2 \), and ii) such additional investment opportunity is more valuable when the projects of both banks come to fruition. More specifically, we assume that the additional investment opportunity pays \( \hat{\rho} > 1 \) per unit of investment if both banks’ projects continue until \( t = 2 \), and zero otherwise. The healthy bank’s payoffs from taking over are \( \rho_0 + \rho_1 > 1 \). In contrast, the healthy bank’s payoffs if not taking over depend on the government’s bailout choices. If there is a bailout, then the healthy bank can invest in the alternative project and gain \( \hat{\rho} > 1 \) per unit of investment. If there is no bailout, the healthy bank still has a second chance to take over and to gain \( \rho_0 + \rho_1 > 1 \) per unit of investment.

Clearly, for \( \rho_0 + \rho_1 \geq \hat{\rho} \), the healthy bank always makes an immediate takeover bid, and the government gets to make the bailout decision under full information, bailing out only if there was no take over. In contrast, if \( \rho_0 + \rho_1 < \hat{\rho} \), the healthy bank makes a takeover bid only when there is no bailout ex-post. In this case, the government is uncertain when making its decision and the analysis of the benchmark model with imperfect information applies.

Note that so far we have maintained the assumption of targeted bailouts. Hence, the type of the bailout policy is not crucial for our mechanism, but the ability and willingness of healthy banks to communicate truthfully their own realized shock is.

**Non-targeted bailouts** We can increase the scope of the bailout so that the transfer is also available to the healthy banks, a case which we call non-targeted bailout. In this case, the expected social cost of bailouts may increase if there is a social cost to providing below-market-rate lending to healthy banks. Denoting that social cost as \( \hat{x} \), the delayed bailout condition becomes

\[
P'_2 < P \equiv 1 - \frac{x}{y - \hat{x}}
\]

which is clearly more easily satisfied if \( \hat{x} < 0 \) and it is socially costly to transfer subsidized resources to otherwise safe banks.

With non-targeted policies it is easier that uncertainty acts as commitment for two reasons: it is less likely that healthy banks reveal the true state and it is more likely that governments delay interventions.

Non-targeted policies constitute a special case in which healthy banks prefer governments to introduce bailouts and obtain cheap funding than taking over distressed projects, revealing the true state to the government and giving up those subsidized funds. Using the previous
notation, when the gains from alternative investment opportunities are $\hat{\rho}$, the social costs of bailing out healthy banks are $\hat{x} = \beta(\hat{\rho} - \rho_0) - (1 - \rho_0)$, which is negative as long as the welfare gains from the transfer to healthy banks, $\beta(\hat{\rho} - \rho_0)$, are smaller than the social costs from subsidized funds, $1 - \rho_0$.

Additionally, with non-targeted bailouts, healthy banks take over immediately and eliminate uncertainty when $\rho_0 + \rho_1 > \hat{\rho} + (1 - \rho_0)$, which is more difficult to fulfill than with targeted policies.

This discussion implies that a government would like to implement ex-ante institutional arrangements that induce the use of non-targeted bailouts when they know they will face a bailout decision in the future. This is especially true if monitoring or ex-ante regulation is deemed ineffective or inefficient.

**Gradual bailouts** In the benchmark model, we assume that bailouts are discrete events – if the government does not provide the total amount needed to bring the project to fruition, the project is prematurely terminated.

Here we allow for an alternative technology that allows governments to inject funds gradually rather than completely, all the while keeping the project running. In principle a government would prefer this possibility because the nature of the shock may be eventually revealed, in which case the government would rather induce a take over from the healthy bank at least of a part of the distressed project.

Below, we discuss two cases. First, if the government knows that will always provide a gradual bailout to keep the option value of learning in the future, then the analysis in the paper goes through without change. Intuitively, at the moment one bank shows distress, the government internalizes its own dynamic inconsistency and rationally chooses to not bail out at the moment of observing distress, inducing the distressed project to be terminated if the shock were aggregate. The delayed bailout condition in this case is identical to the one in the paper. Second, we allow the government to provide a bailout between the first moment a bank shows distress, $t$, and $t' > t$, where $t'$ is chosen optimally. We derive the delayed bailout condition for this case. We discuss the details of these two cases below.

Suppose first that the government is always tempted to provide a bailout for an extra ‘second’ while keeping the option of reversing the bailout later. In the meantime, a healthy bank does not have incentives to take over the distressed project (that is, if $\hat{\rho} > \rho_0 + \rho_1$). At moment $t$, when a bank shows distress, a forward looking government internalizes these decisions, and hence the expected value of prolonging the bailout is actually bailing out until the true state is revealed to be aggregate – in which case the government bails out the second bank as well. The value of such strategy is

$$(2 - t)ix + P_2(t)(2 - \hat{t}(t))ix,$$

where $\hat{t}(t)$ is the expected time of distress of the second bank, conditional on an aggregate shock and conditional on the second bank not having showed distress until $t$ when the shock is aggregate. Alternatively, if the government does not bail out the current distressed bank, which implies termination if the shock is aggregate, the value is

$$(1 - P_2(t))(2 - t)y_i + P_2(t)(2 - \hat{t}(t))xi.$$
The government, anticipating the behavior of future selves, chooses to discontinue the bailout immediately if

\[(1 - P_2(t))(2 - t)y > (2 - t)x \iff P_2(t) < 1 - \frac{x}{y}\]

which is exactly the delayed bailout condition (8). Hence, if the government knows it will be always tempted to provide gradual bailouts, and in case the idiosyncratic shock the healthy bank does not have incentives to take over, then it would rationally delay bailout under the same conditions as in the full model, and the same equilibrium analysis goes through.

Consider now a more general case, where there is an optimal time \(t'\) such that if until \(t'\) there is no failure of the second bank, then it is optimal to stop the bailout, but not before. Hence, \(t'\) is defined by indifference between continuing the bailout for another moment,

\[V_b(t) = ixdt + \frac{d}{dt}\max[V_b(t), V_{nb}(t)]\]

and, as before, stopping has a value

\[V_{nb}(t) = (1 - P_2(t))(2 - t)y + P_2(t)(2 - \hat{t}(t))xi.

\(t'\) is then implicitly defined by

\[V_b(t') = V_{nb}(t').\]

Consider now this alternative strategy of choosing \(t'\) optimally. The value of keeping a bailout until \(t'\) is

\[(1 - P_2(t))[(2 - t')y + (t' - t)x]i + P_2(t)[(2 - \hat{t})x + p(t_2 > t')(t' - t)x + p(t_2 \leq t')(2 - t)x]i\]

where \(p(t_2 < t')\) is the probability that in case of an aggregate shock the second bank will show distress before the stopping time \(t'\),

\[p(t_2 \leq t') = \frac{F(t') - F(t)}{1 - F(t)}\]

where \(F\) is the cdf of the time of distress, conditional on surviving until \(t\). The delayed bailout condition is now

\[(1 - P_2(t))(2 - t)y + P_2(t)(2 - \hat{t})xi -
\[(1 - P_2(t))[2 - t']y + (t' - t)x]i - P_2(t)[(2 - \hat{t})x + p(t_2 > t')(t' - t)x + p(t_2 \leq t')(2 - t)x]i > 0\]

which simplifies to

\[(1 - P_2(t))(2 - t)y - [(1 - P_2(t))[2 - t']y + (t' - t)x]i - P_2(t)[p(t_2 > t')(t' - t)x + p(t_2 \leq t')(2 - t)x]i > 0\]

and hence

\[(1 - P_2(t))(t' - t)(y - x) - P_2(t)[(t' - t)x + p(t_2 \leq t')(2 - t')x] > 0\]
Which gives the new condition on the posterior $P_2(t)$:

$$P_2(t) < \frac{y - x}{y + x(2 - t') \frac{p(t_2 \leq t')}{t' - t}}$$

Clearly, when bailouts are once and for all, then $t' = 2$ and the equation reduces to (8). In the general case, there is a new expression $x(2 - t') \frac{p(t_2 \leq t')}{t' - t}$ in the denominator, potentially making the cutoff smaller, i.e. less likely to be satisfied. This additional term depends on $(2 - t')$ and

$$\frac{p(t_2 \leq t')}{t' - t} = \frac{1}{1 - F(t)} \frac{F(t') - F(t)}{t' - t}.$$

The constraint becomes tighter when the product of these expressions increase. While $(2 - t')$ is clearly positive and decreasing in $t'$, $\frac{p(t_2 \leq t')}{t' - t}$ is bounded below by the value of the pdf evaluated at the unconditional mean. In fact, for small bailouts, i.e. $t'$ close to $t$, the first term is proportional to the value of the pdf evaluated at $t$, then bounded for a fixed $\sigma_h$.

It is instructive to look at the limiting cases. For $\sigma_h$ approaching zero, the upper bound of $\frac{p(t_2 \leq t')}{t' - t}$ becomes arbitrarily large – the pdf at the mean goes to infinity, and we know that delay cannot be sustained if it is only momentary. The reason for that is that the government knows that it will learn right away if the shock was idiosyncratic or not, so gradual bailouts are very attractive. For $\sigma_h$ going to infinity, the pdf at any $t$ approaches zero and the delayed bailout condition becomes (8). Effectively, in that case the shock $h$ has a two point distribution – if the distress hasn’t happened yet (i.e. it is not at the lower end) then it will probably not happen (i.e. the shock is at the high end of the distribution).

To summarize, the possibility of gradual bailouts affects the cutoff belief about an aggregate shock under which a government delays bailing out the first bank in distress, even gradually, but not the main mechanism of uncertainty as commitment. In particular, the cutoff is bounded away from zero for all positive values of the variance $\sigma_h$.

Even though our mechanism is robust to the introduction of gradual bailouts, these types of bailouts are usually not feasible for financial intermediaries with very high levels of leverage and short term debt. A bank facing a bank run, for example, requires a large immediate volume of funds to keep their assets without being liquidated at fire sale prices. In thinking about non-financial corporations, however, gradual bailouts seems more plausible, which is consistent with their ubiquitous implementation in non-financial activity, such as the reorganization of car companies or airlines, for example.

**Equilibrium Crises and Bailouts** As we mention in the text, the benchmark model features banks holding enough cash reserves such that on the equilibrium path, there are never bank failures. As a result, effectively the government never has to decide about bailouts. However, as we argue below, this is just a specific feature of the benchmark, which we purposefully keep stark and simple to highlight the main mechanism. In particular, in our extension to the continuous case in Section 2.5, bank failures and bailouts do happen on the equilibrium path. With shocks to cash holdings, banks still hold more liquidity than in the non-commitment (or full information) equilibrium, but not enough to survive fully a refinancing shock.
Even in the limiting case when they hold full liquidity, shocks may cause them to run out of cash prematurely, and then the government makes a bailout decision in consistency with the delayed bailout condition (8). Hence, when uncertainty is large enough, with positive probability, a path is realized in which both banks are in distress, the first one fails and the second one is bailed out. If uncertainty is not large enough, then our mechanism is not at work, and both the first and second banks are bailed out.

There certainly exist a variety of extensions of our model which is also consistent with bank distress and bailouts on the equilibrium path. One example may be a case of imperfect observability of the government’s type, parameterized by the weight on bankers in the welfare function $\beta$. Banks may have prior beliefs that $\beta$ is high enough such that the delayed bailout condition is not satisfied, and hence take on excessive leverage (belief about $\beta$ affects the right hand side of (8)). Ex-post, it may turn out otherwise and we may see the government not bailing out the first bank(s). Another example would be an environment with imperfect observability of the government’s prior about the state of the economy (say $P^g_2$). This modification would affect the left hand side of (8), and again could lead to suboptimal behavior. Both of these modifications imply a dynamic response in bank beliefs to observed bailouts or lack thereof. For example, observing a bank failure would imply an update and potential increased discipline in bank risk-taking that develops with learning about government’s types or beliefs over time.

Strategic Coordination and Too Many To Fail

Both Farhi and Tirole (2012) and Acharya and Yorulmazer (2007) stress the idea that governments are more prone to bail out when many banks are in distress. In Farhi and Tirole (2012) these considerations come into play because the cost of a bailout is independent of the size of the bailout and depends only on the bailout interest rate. In Acharya and Yorulmazer (2007), many banks in distress means that efficient takeovers are less likely, and government takeover is more efficient.

Strategic Complementarities. In our setup, the costs of a bailout are proportional to the required transfers, and hence the tradeoffs of the government are the same, independently of the number of banks in distress or the amount of money at stake. As a straightforward extension, we could introduce an additional cost of a bailout, independent of the size of the transfer, for example a constant $L$. This constant captures the idea of fixed costs, either bureaucratic or political, that governments face when implementing bailouts, regardless of their magnitude. In this case the delayed bailout condition is

$$P'_2 \leq \frac{y - x + \frac{L}{2-t}}{y + \frac{L}{2-t}}.$$  

First, notice that for $L = 0$, this simplifies to (8). Second, a higher $L$ makes governments more prone to delay bailouts. In particular, the government is less likely to bail out if the distress comes late (high $t$). Intuitively, the size of the saved part of the project becomes relative small compared to the cost of bailout, which now includes the fixed constant $L$.

Finally, introducing $L$ may give rise to multiplicity in the full information economy, just like in Farhi and Tirole (2012). In particular, there is a range of parameters for which
it is optimal for governments to bailout two banks when the shock is aggregate (this is, 
\(2xi - L > 0\), but not only one bank (this is \(xi - L < 0\). This effect creates strategic 
complementarity in bank actions – they want to go down together to maximize the likelihood 
of bailouts.

Naturally, in case \(xi - L < 0\) even a single bank would be bailed out if not taken over, and 
so unique equilibrium is guaranteed. Whichever the parameter case, imperfect information 
and strategic restraint give rise to bank actions being strategic substitutes (a very extreme 
one indeed in the benchmark case, where banks compete a la Bertrand), potentially flipping 
the strategic properties of the equilibrium actions by the banks.

Too Many To Fail. The too-many-to-fail case analyzed in Acharya and Yorulmazer (2007) 
can be mapped into our Too Big To Fail example in Section 3.3. What matters is the total 
amount of assets (projects) under distress, and the single big bank can be thought of as a 
subset of banks that fall in distress as the same time. In such instance, if \(too many\) banks 
show distress initially, i.e. the size of the distressed asset group is too large to be taken over, 
then the government may be more compelled to bail out.

Two sets of conclusions may be drawn from this analogy. First, too big too fail and too 
many to fail are the same problems in our economy. What really matters is whether there 
is enough liquidity in the system to refinance projects in distress. Second, this may give 
banks an additional incentive to correlate their risks to become too big too fail as a group, 
specifically in order to coordinate their failures. What we show in Section 3.3 is that if such 
activity passes a certain threshold level of coordinated failures, then the economy switches 
from one extreme (best) equilibrium to the opposite (worst, high leverage) equilibrium.