Corridor Invoicing: Real Hedging in International Trade

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Abstract

Using the universe of Italian customs from 2000 to 2021, matched to firm balance-sheets, we study how invoicing currency shapes exchange-rate exposure and profitability for two-sided trading firms. We document four facts. First, when a firm begins invoicing imports in dollars, it becomes far more likely to invoice exports in dollars the same year, with import-side adoption leading. Second, this matching is bilateral: a firm importing from a country in dollars disproportionately invoices exports to that country in dollars, a corridor structure that aggregate hedging cannot rationalize. Third, corridor alignment reduces profit variance beyond what aggregate net dollar exposure explains. Fourth, exchange-rate movements transmit to profits mainly through transactions rather than balance-sheet revaluation, and dollar-invoiced import quantities rise after a euro depreciation, concentrated in inputs linked to exports. We interpret these facts through a model of invoicing currency choice with a rich sourcing and export destination structure, driven by price-stability motives and the incentive to hedge country-specific risk. In the richest framework, invoicing currency choice is a real hedge against country risk, not only currency risk.

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1 Introduction

In the recent literature, the dominant currency paradigm has been extensively documented as a prevailing way firms set prices in international transactions (Gopinath et al., 2020; Boz et al., 2022).[1] Under price stickiness, invoicing currency choice determines how exchange-rate movements pass through to prices and quantities (Engel, 2006; Gopinath et al., 2010), and through them to firm profits (Barbiero, 2022). Consistent with a motive of hedging this exposure, firms align the invoicing of the two sides of their trade: importers of dollar-invoiced inputs disproportionately invoice their exports in dollars (Amiti et al., 2022). Much less is known about how this alignment comes about: at what level of the firm's trade it is organized, how it is built and unwound over time, and what it delivers for the level and the volatility of realized profits. In this paper, we aim to fill this gap by studying the invoicing choices and profits of the universe of Italian exports and imports between 2000 and 2021, combining transaction-level customs declarations that record the invoicing currency with firm-level balance-sheet and income-statement data.

Aggregated at the firm level, our data confirm the cross-sectional pattern of Amiti et al. (2022), with dollar invoicing on the two sides of the trade account strongly correlated. However, digging deeper uncovers a subtler organization of invoicing choices, and the matched balance-sheet data allow a precise evaluation of what it delivers. First, matching is a dynamic decision *within* firms: adopting dollar invoicing on one side of the trade account predicts adoption on the other within the same year, and adoption typically starts on the import side. Second, the matching is arranged at the *bilateral corridor* level: a firm that begins invoicing its imports from a country in dollars disproportionately begins invoicing its exports to that same country in dollars. Third, this geographic structure pays, as corridor matching lowers profit volatility beyond what the firm's aggregate net dollar position explains. Fourth, exchange-rate shocks move profits mainly through the operational margin, i.e., the part of profits generated by current transactions,[2] rather than through balance-sheet revaluation, with sign and magnitude of the effects governed by the firm's net dollar position. On the quantity side, a euro depreciation raises rather than lowers dollar-invoiced import quantities, with the in-

[1] See Gopinath and Itskhoki (2022) for a survey.

[2] Throughout, the operational margin is pre-tax profit net of the gains and losses that exchange-rate movements generate on the firm's outstanding foreign-currency assets and liabilities, the balance-sheet revaluation recorded in the income statement. Section 4.1 defines it. It is not the operating margin of financial analysis, operating income divided by sales.

crease concentrated among imported inputs linked to the firm's export basket. Finally, we analyze these empirical patterns through the lens of a model of invoicing currency choice based on Mukhin (2022), extended to allow for a rich sourcing and destination structure. This allows us to pin down the key elements and extensions of the theory required to account for the data. In the richest version of our framework, invoicing currency choice is a real hedge against country risk, not only against currency risk.

Our empirical analysis is based on the universe of Italian customs declarations from the COE register of the Italian National Institute of Statistics (ISTAT). The register covers all exporting and importing firms between 2000 and 2021 and reports trade values, physical quantities (in the relevant units), the currency of invoicing at the level of the firm, the 8-digit product, and the partner country, allowing price and quantity responses to be separated at a fine level of disaggregation. The resulting data set comprises roughly 664,000 firms, of which about 272,000 are active on both sides of the trade account, and more than 90 million firm-product-country-currency-year observations. We match these records to FrameSBS, ISTAT's register of structural business statistics, and to AIDA, the Bureau van Dijk database of Italian limited-liability companies, which provide employment, turnover, and sector of activity for nearly all firms and standardized financial statements for 71 percent of them. In particular, we observe the gains and losses generated by exchange-rate movements on the firm's outstanding foreign-currency assets and debts, which we use to separate the balance-sheet revaluation channel from the operational margin.

This setting is particularly well suited to the question we ask. Italian firms are, on average, small and have limited access to the formal hedging instruments and capabilities of large multinationals, such as derivatives markets and dedicated treasury functions, so a real hedge built into the invoicing decision itself is plausibly a first-order margin of adjustment rather than a refinement layered on top of financial hedging. At the same time, the Italian economy is highly open, with a large share of firms trading internationally. As the vast majority of transactions are invoiced in either euros or dollars, and euro invoicing generates no direct exchange-rate exposure for Italian firms, currency risk from trade arises almost entirely from dollar-invoiced transactions. This allows us to focus on euro- and dollar-invoiced transactions, sharpening the link between invoicing choice and realized exposure. Finally, the mandatory disclosure of foreign-exchange gains and losses described above is uncommon outside Italy and, combined with the transaction-level granularity of the customs data, lets us trace exchange-rate

transmission from the invoicing decision through to its balance-sheet and operational consequences with a precision rarely available in administrative data.

We document four main empirical findings. First, matching the invoicing currency of imports and exports is a dynamic choice. When a firm begins invoicing its imports in dollars, the probability that it also begins invoicing its exports in dollars in the same year more than triples relative to the baseline adoption rate.[3] The reverse relationship is also positive and precisely estimated, but consistently weaker, so dollar-invoicing regimes emerge asymmetrically, with adoption on the import side leading. Our estimates are identified by within-firm variation rather than cross-sectional differences across firms. In the cross section, the same co-movement reproduces the pattern of Amiti et al. (2022), but our panel estimates reveal it to be the outcome of an ongoing choice rather than a fixed characteristic of internationally active firms.

Our second finding is that currency matching is organized at the level of the bilateral corridor. When a firm begins invoicing its imports from a given country in dollars, the probability that it also begins invoicing its exports to that same country in dollars rises to almost five times the baseline, with the reverse direction again positive but weaker. The corridor-level estimates compare corridors within the same firm and year, with firm-by-time, country-by-time, and firm-by-country fixed effects absorbing the firm's overall dollarization in each year, conditions specific to each partner country, and persistent bilateral relationships. The pattern also survives at the firm-country-product level, where the comparison runs across countries within the same firm, product, and year, ruling out the product composition of trade with different partners as the driver. The response is confined to the corridor, with import adoption from one country not predicting export adoption to any other. This corridor matching of invoicing currency, moreover, appears to operate largely through existing trade: firms switch the invoicing of relationships they already have with the country rather than forming new ones.

For hedging exchange-rate risk, the corridor organization is a puzzle, as a firm's exchange-rate exposure depends only on its aggregate net dollar position, independent of the corridor decomposition. Yet our third finding is that corridor matching reduces profit volatility over and above what aggregate dollar exposure implies. To show that, we first compute residual profit fluctuations after controlling for aggregate

[3]Throughout the paper, we focus on the choice between euro and dollar invoicing. The dollar is the natural foreign-currency benchmark in our setting: it is the dominant vehicle currency in international trade, and, in our data, transactions invoiced in currencies other than the euro or the dollar account for only a small fraction of overall trade.

net exposure, and then estimate how the remaining volatility varies with the share of a firm's dollar-active corridors that are matched (*matching quality*). We find that corridor matching matters: firms with higher matching quality display lower residual profit volatility, and moving from the 25th to the 75th percentile of matching quality in our most conservative specification reduces the residual profit variance by 3.2 percent. By contrast, the balance-sheet revaluation component of profits behaves exactly as pure exchange-rate hedging predicts: conditional on aggregate exposure, matching quality has no effect on its volatility. The volatility that corridor matching removes is therefore in the operational margin.

Our fourth finding is that exchange-rate shocks affect profits primarily through the operational margin: a one-standard-deviation net-exposure shock moves profits by approximately 9.2 percent of their mean, and roughly three quarters of this response is operational rather than balance-sheet revaluation. Digging deeper, we find that pass-through of EUR/USD movements into euro prices is high and symmetric, about 60 percent on both sides of the trade account, so a depreciation raises the euro value of dollar revenues and dollar costs alike and the net effect on the operational margin follows the firm's net dollar position.[4] The surprise is in quantities: export quantities rise, as the depreciation improves the competitiveness of Italian exporters, but dollar-invoiced import quantities rise as well, even though the depreciation makes them more expensive in euros. The increase is concentrated among inputs linked to the firm's export basket, consistent with expanding exports raising production and, in turn, the demand for the inputs used to produce them. We refer to this mechanism as the scale effect on linked imports. A placebo supports this interpretation, with the estimated import response no stronger when the input comes from a country the firm also exports to, so the scale effect is a property of the imported input's link to the export basket, not of the corridor through which it is sourced.

We interpret these findings through the lens of a multi-country model of invoicing-currency choice building on Mukhin (2022). In the model, a firm serving many destinations chooses, for each one, the invoicing currency that minimizes the variance of its

[4]We also consider the effects of movements in the dollar-destination-currency exchange rate for dollar-invoiced transactions. The results are consistent with the dollar serving as a vehicle currency: prices respond strongly to EUR/USD movements but remain largely unaffected by fluctuations in the dollar relative to the destination currency, implying that they are relatively stable when expressed in dollars.

desired price.[5] Each destination's output is produced with inputs drawn from many source countries through an input-output structure. The model points to two elements of the environment that matter most for replicating our empirical patterns: the own-corridor weight in each destination's input mix, and the dollar pricing of competitors in each destination relative to the firm's own dollar costs. The first element delivers the invoicing patterns. When a destination draws disproportionately on inputs sourced from itself, dollar invoicing of imports from that country raises the dollar content of the input bundle serving it more than of any other bundle. The incentive to invoice exports to that destination in dollars rises with that dollar content. This is the corridor matching of our second finding. Aggregated across corridors, the same mechanism produces the firm-level co-movement of our first finding. The second element delivers the quantity side. A euro depreciation raises the euro prices of dollar-pricing competitors by more than it raises the firm's costs. Where such competitors are a sufficiently large fraction, sales therefore expand by enough to outweigh the substitution away from the more expensive dollar-invoiced input. This is the scale effect of our fourth finding. The two elements also interact to pin down whether the scale effect is a linked-inputs phenomenon or a corridor phenomenon. Corridor-sourced inputs respond more strongly to exchange rate shocks than other linked imports only if the destinations facing the most dollar-priced competition also draw their inputs disproportionately from countries the firm exports to. The equal responses in the data point to no such systematic alignment, bringing the model in line with the placebo of our fourth finding.

By construction, the model so far is silent on our third finding, as it features complete markets and no motive to reduce idiosyncratic profit volatility. To capture such incentives, we extend the model to a mean-variance objective and introduce country-specific shocks that jointly move the demand for the firm's exports to a country and the price of its imports from it. Such a shock impacts the firm's dollar position as well by scaling the corridor's flows, which scales the currency exposure they carry. Aggregate netting and currency derivatives cannot absorb this risk, because they are set before the shock realizes. Matching the invoicing currencies of the two legs of a corridor is the instrument that helps track it: when the shock enlarges the corridor's flows, the dollar exposures of the revenue leg and the cost leg grow together and offset. Corridor matching therefore lowers profit volatility beyond what aggregate exposure explains, which is our third finding. The extension leaves the pricing structure of the model unchanged,

[5] As shown in Section 6, the variance-minimization criterion is the second-order representation of the firm's expected-profit-maximization problem.

so the other three findings survive; if anything, the country shock reinforces corridor matching, since the hedging motive pushes the export denomination toward the import denomination on the same corridor. In this extended framework, invoicing currency choice is a real hedge against country risk, not only against currency risk.

Related literature This paper contributes to the literature on operational hedging through the joint currency denomination of imports and exports (Bartram et al., 2010; Hoberg and Moon, 2017). The two papers closest to ours are Amiti et al. (2022), which shows that firms importing dollar-priced inputs are more likely to invoice their exports in dollars, thereby aligning the exchange-rate sensitivity of revenues and marginal costs, and Barbiero (2022), which documents the corresponding implications for firm cash flows, showing that invoice-currency mismatches generate substantial exposure to exchange-rate movements and that exporters partly offset dollar-invoiced exports with dollar-invoiced imports.[6] Our identifying variation comes not from the cross section but from the firm-level dynamics of currency choice, and we provide new evidence of bilateral corridor matching of invoicing currency.[7] Additionally, we show that corridor matching lowers profit volatility beyond what firm-wide net dollar positions can explain.

Our findings also speak to theories of endogenous invoicing-currency choice (Bacchetta and van Wincoop, 2005; Goldberg and Tille, 2008; Burstein and Gopinath, 2014). Engel (2006) shows that, when prices are sticky, firms prefer to invoice in the currency in which their flexible-price target is most stable. Gopinath et al. (2010) extends this insight to a dynamic setting and Mukhin (2022) embeds the same firm-level decision in a multicountry general-equilibrium framework.[8] Our evidence points to a finer geographical dimension: the matching of import and export invoicing currencies is concentrated within the same bilateral corridor. To rationalize this pattern, we extend this class of models by introducing a richer source-destination structure that gives rise to

[6]Related work on natural and financial hedging includes Fauceglia et al. (2014), Allayannis et al. (2001), Lyonnet et al. (2022), Berthou et al. (2022), and Alfaro et al. (2024); see also Chung (2016) on imported inputs and invoicing-currency choice.

[7]Related evidence on the dynamics of invoicing choice comes from UK exporters in Corsetti et al. (2022), Crowley et al. (2022), Crowley et al. (2024), and Garofalo et al. (2024), and from Chilean importers in Cristoforoni and Errico (2025). These studies focus primarily on adjustments on one side of the trade account, often around a single large depreciation; we trace the joint evolution of invoicing choices on both sides over two decades, so that coordination emerges as a recurrent feature of firm behavior rather than the response to one episode. The stability of aggregate invoicing shares documented by Boz et al. (2022) can coexist with this reallocation beneath the surface.

[8]See also Gopinath et al. (2020) on the implications of dominant-currency pricing and Gopinath and Itskhoki (2022) for a survey.

destination-specific marginal costs. We also explore the role of corridor-specific shocks and competitor invoicing choices.

Finally, the paper relates to the literature on the firm-level transmission of exchange-rate movements through prices and quantities (Gopinath and Rigobon, 2008; Burstein and Gopinath, 2014; Pennings, 2017). Amiti et al. (2014) shows that exporters relying more heavily on imported inputs adjust their export prices less in response to exchange-rate movements because these movements also affect their production costs. Berman et al. (2012) shows that more productive exporters respond to a depreciation by increasing their markups more and their export quantities less.[9] We contribute to this literature by showing that the transmission also runs in the opposite direction: after a euro depreciation, expanding dollar-invoiced exports raise the quantities of the linked dollar-invoiced inputs used to produce them, the scale effect on linked imports.

The paper is organized as follows. Section 2 describes the data. Section 3 documents firm-level invoicing-currency matching and establishes its corridor-specific structure. Section 4 studies the implications for firm profits and shows that corridor matching reduces profit volatility beyond aggregate dollar exposure. Section 5 examines price and quantity responses to exchange-rate movements and documents the scale effect on linked imports. Section 6 develops the theoretical framework. Section 7 concludes.

2 Data

Our analysis combines several sources of firm-level data. Our primary data source is the COE register, compiled by the Italian National Institute of Statistics (ISTAT). The COE business register (Integrated International Trade Database) is based on administrative customs data and provides comprehensive information on foreign trade for the universe of Italian exporting and importing firms. The dataset is available at annual frequency over the period 2000–2021 and includes detailed information on trade flows by firm, product, partner country (destination country for exports and country of origin for imports), and currency of invoicing. The data are highly granular: products are defined at the 8-digit level of the Combined Nomenclature (CN8), and each firm–product–country–year observation reports both trade values and physical quantities. This feature is central to our analysis of exchange-rate pass-through in Section 5, as it allows us to separately identify price and quantity adjustments at a very fine level of disaggregation. The final sample comprises approximately 664,000 firms engaged in

[9] See also Amiti et al. (2019), Auer and Schoenle (2016), and Devereux et al. (2017) on how strategic complementarities, market structure, and invoicing choice shape exchange-rate pass-through.

international trade between 2000 and 2021. Of these, about 500,000 are exporters and 436,000 are importers, with roughly 272,000 firms active on both sides of the trade account. The resulting dataset contains approximately 3.6 million firm–year observations and more than 90 million firm–product–country–invoicing-currency–year observations.

We complement the customs records with two sources of firm-level information: FrameSBS, ISTAT's register of structural business statistics, which provides employment, turnover, and sector of activity for nearly all firms in the customs sample, and AIDA, the Bureau van Dijk database of Italian limited-liability companies. AIDA provides standardized financial statements, together with information on firms' legal status, year of incorporation, industry classification, registered location, ownership structure, and corporate-group affiliation. The financial statements include net foreign exchange income (NFXI), the statutory item that records, under Italian accounting standards (OIC 26), realized and unrealized exchange-rate gains and losses on firms' foreign-currency positions, which we use in Section 4 to separate the balance-sheet revaluation channel from the operational margin. These balance-sheet data can be matched to 71% of the internationally active firms in our customs sample, providing broad coverage of these additional dimensions.

We complement the dataset with exchange-rate data from the Bank of Italy. As shown in Figure B1 in Appendix B, the EUR/USD rate, the series of interest in our analysis, displays substantial variation over the sample period and several pronounced depreciation episodes, most notably in 2000, 2009–2010, 2012, and 2015. These episodes coincide with the major macroeconomic crises of the sample period.

Table B1 reports summary statistics for the main variables used in the analysis. Relative to the broader Italian business population, internationally active firms tend to be relatively large, employing nearly 30 workers on average, and mature, with a mean age of approximately 18 years, although both characteristics display substantial variation. These firms are also active across multiple trade corridors, exporting to more than five destination countries and importing from more than two origin countries on average. They trade a broad range of products as well, with the average firm exporting more than five products and importing more than six. The substantial cross-firm variation along these dimensions provides ample scope for our corridor-level analysis.

Moreover, pre-tax profits scaled by lagged total assets average 0.9%, with a median of 1.2% and a standard deviation of 15.5 percentage points, indicating substantial dispersion in profitability. Lagged dollar-invoiced exports amount to approximately 3.5% of lagged total assets on average, compared with 7.9% for dollar-invoiced imports, in-

dicating negative net dollar trade exposure on average. Turning to the variables used in the pass-through analysis, annual export and import price growth averages approximately 2.7% and 2.6%, respectively, while export and import quantities decline by approximately 1.4% and 1.9%. All four variables display substantial dispersion, providing considerable variation for the analysis of price and quantity adjustments.

Tables B2 and B3 report the geography of trade and of dollar invoicing. Euro-area partners account for roughly half of Italian trade but essentially none of the dollar-invoiced flows; trade invoiced in currencies other than the euro or the dollar is comparatively negligible, at only 3.0% of export value and 2.1% of import value, which is why we restrict attention to the euro–dollar margin throughout the paper. Dollar-invoiced trade instead concentrates in North America and Asia on the export side and in Asia and commodity-exporting countries on the import side, with dollar invoicing about as prevalent in trade with China as with the United States, consistent with the dollar functioning as a vehicle currency rather than tracking the trading partner's own currency.

Turning to invoicing choices, dollar invoicing is more prevalent on the import side than on the export side. At both the country and country–product levels, approximately 12% of import relationships are invoiced in dollars, compared with about 6–7% of export relationships. Invoicing choices also change meaningfully over time. In any given year, firms display a substantial probability of beginning to invoice either imports or exports in dollars, at both levels of aggregation (2–7%). The probability of discontinuing dollar invoicing is much lower, indicating that entry into dollar invoicing is relatively frequent and that, once adopted, the practice tends to persist.

3 Invoicing currency matching

Below, we establish the invoicing patterns in our data using within-firm variation across time, destinations, and products. We first document that matching the dollar invoicing of imports and exports is a dynamic decision: within a firm, adopting dollar invoicing on one side of the trade account predicts adoption on the other within the same year, with the import side leading (Section 3.1). We then exploit the destination and product dimensions of the data to show that this matching is organized at the level of the bilateral corridor: using only variation across corridors within the same firm and year, dollar adoption on one side of a corridor predicts adoption on the other side of that same corridor and not elsewhere (Section 3.2).

3.1 Firm-level evidence

If firms face exchange-rate risk on both sides of their trade account, simultaneously holding dollar positions on both sides offers a natural hedge: a euro depreciation against the dollar raises both the euro value of dollar-denominated export revenue and the euro cost of dollar-denominated imports. This hedging motive predicts positive co-movement between dollar export and import invoicing at the firm level. Amiti et al. (2022) and Barbiero (2022) provide cross-sectional evidence consistent with this prediction. Our 22-year Italian panel complements this evidence by tracing how currency matching develops and unwinds within firms: whether adopting dollar invoicing on one side of the trade account predicts adoption on the other, and whether firms similarly coordinate their exit from dollar invoicing.

To provide answers to these questions, we estimate linear probability models of the form:

$$y_{i,t} = \beta x_{i,t} + Z'_{i,t}\gamma + \alpha_i + \alpha_t + \varepsilon_{i,t}, \tag{1}$$

where $y_{i,t}$ can be either $\mathbb{I}\{X^{\$}_{i,t}\}$ or $\mathbb{I}\{M^{\$}_{i,t}\}$, two binary variables indicating whether firm i invoices exports or imports, respectively, in dollars in year t. The regressor $x_{i,t}$ captures the firm's dollar-invoicing behavior on the opposite side of its trade account, with its precise definition varying across specifications. The vector $Z_{i,t}$ includes controls for strategic complementarities in currency choice, the firm's market position, size, and importer status, while α_i and α_t denote firm and year fixed effects.

We estimate three variants of equation (1), which differ in the definition of the regressor of interest, $x_{i,t}$. First, $x_{i,t}$ is a dollar-invoicing indicator, $\mathbb{I}\{M^{\$}_{i,t-1}\}$ or $\mathbb{I}\{X^{\$}_{i,t-1}\}$, being equal to one if the firm used dollar invoicing on the opposite side of its trade account in the previous year. Second, it is an adoption indicator, Start $M^{\$}_{i,t}$ or Start $X^{\$}_{i,t}$, equal to one when the firm begins invoicing transactions on the opposite side in dollars in year t. Third, it is an exit indicator, Stop $M^{\$}_{i,t}$ or Stop $X^{\$}_{i,t}$, equal to one when the firm ceases to do so. The estimation samples are restricted accordingly to isolate adoption and exit on the dependent-variable side. The dependent variable is divided by its unconditional mean, so that β measures the change in the outcome probability as a proportion of its unconditional probability.

Table 1 reports the results. The co-movement is precisely estimated and economically meaningful across all three measures. Column 1 shows that dollar import invoicing in the previous year is associated with an increase in the probability of dollar export invoicing equal to 17.2 percent of its unconditional probability. This association is

roughly four times as large when we restrict the sample to firms that did not invoice exports in dollars in the previous two years (column 2). Beginning to invoice imports in dollars is associated with an increase in the probability of simultaneously adopting dollar export invoicing equal to 202.2 percent (column 3).

The relationship in the opposite direction is precisely estimated, but consistently weaker. The corresponding coefficients are 10.7 percent for prior dollar export invoicing in column 5, 12.7 percent in the restricted sample of column 6, and 167.9 percent at adoption in column 7. This asymmetry indicates that dollar-invoicing regimes tend to emerge first on the import side. At the exit margin, the coefficients in columns 4 and 8 are negative and precisely estimated, although economically modest: ceasing dollar invoicing on one side is associated with declines of 7.6 and 3.6 percent of the baseline probability that dollar invoicing continues on the other. Thus, the two sides also co-move as firms abandon dollar invoicing, but substantially less strongly than when these regimes are formed.

These patterns are consistent with firms deliberately aligning their foreign-currency exposures to reduce the net position exposed to exchange-rate movements. As we show next, however, this matching does not occur indiscriminately across countries.

3.2 The invoicing currency corridor

Next, we leverage the granularity of our data to show that dollar invoicing across exports and imports is, to a large extent, organized along *bilateral corridors*: firms that invoice imports from a given country in dollars are substantially more likely to invoice exports to that same country in dollars.

To document this pattern, we estimate the country-specific extension of the linear probability model in equation (1):

$$y_{i,c,t} = \beta x_{i,c,t} + Z'_{i,c,t}\gamma + \alpha_{i,t} + \alpha_{c,t} + \alpha_{i,c} + \varepsilon_{i,c,t}, \tag{2}$$

where $y_{i,c,t}$ and $x_{i,c,t}$ are the country-specific counterparts of the dependent variable and main regressor used in the firm-level analysis of Section 3.1. The vector $Z_{i,c,t}$ contains the same set of controls as in the firm-level specification, defined at the firm–country–year level where appropriate. Finally, $\alpha_{i,t}$, $\alpha_{c,t}$, and $\alpha_{i,c}$ denote firm–year, country–year, and firm–country fixed effects, respectively. These absorb each firm's overall tendency to dollarize its transactions over time regardless of the country, country-specific shocks, and persistent firm-specific invoicing preferences toward particular countries. Identification comes from within-firm, within-year variation across corridors. As before,

Table 1: USD invoicing in exports and imports: Firm-level evidence

	$\mathbb{I}\{X^{\$}_t\}$ (1)	$\mathbb{I}\{X^{\$}_t\}$ (2)	$\mathbb{I}\{X^{\$}_t\}$ (3)	$\mathbb{I}\{X^{\$}_t\}$ (4)	$\mathbb{I}\{M^{\$}_t\}$ (5)	$\mathbb{I}\{M^{\$}_t\}$ (6)	$\mathbb{I}\{M^{\$}_t\}$ (7)	$\mathbb{I}\{M^{\$}_t\}$ (8)
$\mathbb{I}\{M^{\$}_{i,t-1}\}$	0.172***	0.716***						
	(0.00591)	(0.0190)						
Start $M^{\$}_{i,t}$			2.022***					
			(0.0346)					
Stop $M^{\$}_{i,t}$				−0.0762***				
				(0.00599)				
$\mathbb{I}\{X^{\$}_{i,t-1}\}$					0.107***	0.127***		
					(0.00490)	(0.0159)		
Start $X^{\$}_{i,t}$							1.679***	
							(0.0283)	
Stop $X^{\$}_{i,t}$								−0.0360***
								(0.00368)
Sample	Full	$X^{\$}_{t-}$=0	$X^{\$}_{t-}$=0 & $M^{\$}_{t-}$=0	$X^{\$}_{t-}$=1	Full	$M^{\$}_{t-}$=0	$M^{\$}_{t-}$=0 & $X^{\$}_{t-}$=0	$M^{\$}_{t-}$=1
Firm FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.585	0.261	0.292	0.534	0.628	0.363	0.379	0.552
Identifying obs	3,038,173	2,506,617	2,112,396	429,605	3,038,173	2,363,657	2,112,396	559,508
Total sample	3,255,299	2,734,369	2,358,790	442,426	3,255,299	2,600,748	2,358,790	576,047

Notes: The unit of observation is the firm-year. $t-$ refers to both $t-1$ and $t-2$. Dependent dummy variables are rescaled by their unconditional means; coefficients measure percentage changes relative to the baseline adoption probability. Columns (1)–(4) report the effect of dollar import invoicing on dollar export invoicing; columns (5)–(8) report the reverse. Additional controls include the share of exports invoiced in USD by other Italian exporters in the same HS4 industry–destination (strategic complementarities as in Amiti et al. 2022, collapsed to firm-year level), a proxy for firm market share in the destination, log employment, log sales, and a dummy for importers. Standard errors clustered at the firm level. *, **, *** significant at 10%, 5%, 1%.

dependent variables are rescaled by their unconditional means so that each coefficient is the proportional effect of the regressor relative to the baseline adoption probability.

Table 2 reports the results. As before, a positive co-movement emerges across all measures: dollar invoicing on one side of a firm's trade relationship with a given country strongly predicts dollar invoicing on the other side of the relationship with that same country. As in the firm-level analysis, the relationship is generally stronger from imports to exports, especially at the adoption margin. Beginning to invoice imports from a given country in dollars raises the probability of beginning to invoice exports to that same country in dollars by 385 percent (column 3), compared with 175 percent from exports to imports (column 7). By contrast, dollar-invoicing regimes dissolve symmetrically, as the exit coefficients in columns 4 and 8 are nearly identical.

A potential concern with the results in Table 2 is that they may be driven by differences in the product composition of firms' trade across countries. To address this concern, Table 3 re-estimates the analysis at the firm–country–product level, augmenting each set of fixed effects in the country-level specification with a product dimension. The comparison therefore holds the product dimension fixed, ruling out the possibility that the country-level pattern merely reflects the mix of products traded with different countries. The same qualitative pattern emerges, although the coefficients are generally smaller: the relationship remains stronger from imports to exports at the adoption margin, while exit remains broadly symmetric across the two sides.

Two further robustness checks are in Appendix B. Tables B7 and B8 exclude the countries whose national currency is the dollar and leave every pattern of Tables 2 and 3 in place. Table B9 moves to the intensive margin, where the dollar share of exports to a country rises with the lagged dollar share of imports from it and vice versa, in levels and in first differences, at both the country and the country-product level.

The firm–year fixed effects included in Tables 2 and 3 rule out the possibility that the same pattern of dollar-invoicing matching occurs uniformly across all of a firm's trading partners. However, they do not exclude a weaker but still positive response in other countries. For instance, firms may match invoicing currencies more strongly within the same country relationship while also adjusting their invoicing choices elsewhere in response to aggregate hedging considerations. In such a case, the corridor would be the strongest expression of a firm-wide adjustment rather than an exclusively bilateral phenomenon. To test this possibility, Table 4 extends the product-level adoption specifications of Table 3 with an indicator for dollar adoption on the opposite side

of the firm's trade account in any country other than the one under consideration. The same-country coefficients remain consistent with the previous results. On the export side, column 1 shows that dollar-import adoption in other countries has no statistically significant effect on the adoption of dollar invoicing for exports to the country under consideration. The cross-corridor coefficient remains statistically insignificant in column 2, which restricts the sample to firms with multiple active corridors, thereby ensuring that alternative countries in which matching could occur are actually available. Coordination from the import side therefore appears to involve an increase in dollar export invoicing within the corridor, without a corresponding adjustment elsewhere. Dollar-export adoption in other countries is instead associated with a lower probability of beginning to invoice imports from the country under consideration in dollars (columns 3–4). This pattern is consistent with substitution across import origins, involving a reallocation of dollar import invoicing toward the same country and away from other trading partners. The positive matching response is therefore specific to the same country relationship, supporting an invoicing-currency channel that operates exclusively within the corridor.

A natural question about this corridor-level matching is whether it primarily reflects the creation of new dollar-denominated relationships or the switching of relationships that already existed in another currency. We cannot settle this with certainty, since our data record firm–product–country–year cells rather than individual buyer-level transactions, so we cannot distinguish a currency switch within the same relationship from a shift toward a different counterparty within the same cell, but Tables B5 and B6 in Appendix B suggest that matching most likely operates through the switching of pre-existing non-dollar relationships into dollar invoicing, rather than through the formation of new ones. Table B5 shows that the co-movement documented above is substantially stronger when the firm already had a non-dollar relationship with the same country and product in the preceding years, consistent with an existing relationship being re-denominated rather than a new one forming alongside it. Table B6 points in the same direction: when a firm begins invoicing one side of a corridor in dollars, non-dollar activity on the opposite side of the same country-product relationship falls. This evidence indicates that the corridor-level matching we document reflects a firm-level decision to reallocate the currency of existing trade relationships in order to achieve bilateral dollar alignment, rather than a mechanical byproduct of new relationships happening to form in dollars.

From the perspective of exchange-rate exposure, however, this corridor-level matching is somewhat counterintuitive: what should ultimately matter for hedging currency risk is the firm's aggregate net dollar position, rather than the invoicing currency of trade with any individual country. A firm with dollar exports to the United States and dollar imports from China is just as well hedged against aggregate exchange-rate movements as a firm with dollar exports and imports involving the same country. The corridor structure therefore introduces a geographic alignment that is redundant for hedging aggregate exchange-rate risk. Section 4 sheds light on this puzzle by examining whether corridor coordination affects firm profits beyond what can be explained by the aggregate net dollar position.

4 Real hedging and firm profitability

The corridor coordination documented in Section 3.2 is redundant for hedging aggregate exchange-rate exposure. We now examine whether it nevertheless stabilizes firm profits beyond what can be explained by the firm's aggregate net dollar position.

To answer this question, we first establish the firm-level benchmark. Under aggregate exchange-rate hedging, the effect of exchange-rate movements on profits should depend on the firm's net dollar exposure, that is, the difference between dollar-invoiced exports and imports. The firm-level matching documented in Section 3.1 is consistent with this logic. Section 4.1 below establishes this channel in our data and decomposes the resulting response into balance-sheet revaluation and the operational margin. Section 4.2 then conditions on this aggregate-exposure channel and asks whether the corridor structure documented in Section 3.2 provides additional stabilization.

4.1 Aggregate dollar exposure and firm profits

We begin by quantifying how firms' dollar exposure shapes the response of profits to exchange-rate movements. To estimate this relationship, we consider the following firm-level specification:

$$\frac{Y_{i,t}}{A_{i,t-1}} = \beta \times \Delta e_t^{€,\$} \times \left(\frac{X_{i,t-1}^{\$}}{A_{i,t-1}} - \frac{M_{i,t-1}^{\$}}{A_{i,t-1}} \right) + \delta \times \left(\frac{X_{i,t-1}^{\$}}{A_{i,t-1}} - \frac{M_{i,t-1}^{\$}}{A_{i,t-1}} \right) + \alpha_i + \alpha_t + \varepsilon_{i,t}^{Y}, \quad (3)$$

where $Y_{i,t}$ denotes a profit-related outcome described below, $\Delta e_t^{€,\$}$ is the log change in the EUR/USD exchange rate, with positive values denoting a euro depreciation, $X_{i,t-1}^{\$}$

Table 2: USD invoicing in exports and imports: Firm-country level evidence

	$\mathbb{I}\{X^{\$}_{c,t}\}$ (1)	$\mathbb{I}\{X^{\$}_{c,t}\}$ (2)	$\mathbb{I}\{X^{\$}_{c,t}\}$ (3)	$\mathbb{I}\{X^{\$}_{c,t}\}$ (4)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (5)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (6)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (7)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (8)
$\mathbb{I}\{M^{\$}_{c,t-1}\}$	0.375***	0.847***						
	(0.00904)	(0.0219)						
Start $M^{\$}_{c,t}$			3.854***					
			(0.0507)					
Stop $M^{\$}_{c,t}$				−0.0473***				
				(0.00500)				
$\mathbb{I}\{X^{\$}_{c,t-1}\}$					0.182***	0.440***		
					(0.00486)	(0.0119)		
Start $X^{\$}_{c,t}$							1.750***	
							(0.0232)	
Stop $X^{\$}_{c,t}$								−0.0452***
								(0.00401)
Sample	Full	$X^{\$}_{c,t-}$=0	$X^{\$}_{c,t-}$=0 & $M^{\$}_{c,t-}$=0	$X^{\$}_{c,t-}$=1	Full	$M^{\$}_{c,t-}$=0	$M^{\$}_{c,t-}$=0 & $X^{\$}_{c,t-}$=0	$M^{\$}_{c,t-}$=1
Country × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Country FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.682	0.425	0.448	0.672	0.807	0.521	0.565	0.722
Identifying obs	13,438,449	12,000,804	10,813,099	781,562	13,438,449	11,551,756	10,813,099	942,822
Total sample	16,226,000	14,792,344	13,619,147	1,052,404	16,226,000	14,323,085	13,619,147	1,521,663

Notes: The unit of observation is the firm-country-year. $t-$ refers to both $t-1$ and $t-2$. Dependent dummy variables are rescaled by their unconditional means; coefficients measure percentage changes relative to the baseline adoption probability. Additional controls include the share of exports invoiced in USD by other Italian exporters in the same HS4 industry–destination and a proxy for firm market share in the destination. Standard errors clustered at the firm-country level. *, **, *** significant at 10%, 5%, 1%.

Table 3: USD invoicing in exports and imports: Firm-country-product level evidence

	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (1)	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (2)	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (3)	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (4)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (5)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (6)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (7)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (8)
$\mathbb{I}\{M^{\$}_{c,t-1}\}$	0.0745***	0.237***						
	(0.00997)	(0.0129)						
Start $M^{\$}_{c,t}$			1.484***					
			(0.0262)					
Stop $M^{\$}_{c,t}$				−0.184***				
				(0.0100)				
$\mathbb{I}\{X^{\$}_{c,t-1}\}$					0.0560***	0.182***		
					(0.00392)	(0.00525)		
Start $X^{\$}_{c,t}$							0.428***	
							(0.00799)	
Stop $X^{\$}_{c,t}$								−0.178***
								(0.00962)
Sample	Full	$X^{\$}_{c,p,t-}$=0	$X^{\$}_{c,p,t-}$=0 & $M^{\$}_{c,t-}$=0	$X^{\$}_{c,p,t-}$=1	Full	$M^{\$}_{c,p,t-}$=0	$M^{\$}_{c,p,t-}$=0 & $X^{\$}_{c,t-}$=0	$M^{\$}_{c,p,t-}$=1
Country × Product × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Product × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Country × Product FE	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.823	0.701	0.709	0.809	0.901	0.722	0.778	0.826
Identifying obs	21,890,522	20,076,424	16,441,857	459,922	21,890,522	19,735,993	16,468,272	496,463
Total sample	50,309,894	48,539,859	40,611,794	1,770,035	50,309,894	47,264,295	41,884,876	3,045,599

Notes: The unit of observation is the firm-country-product-year. $t-$ refers to both $t-1$ and $t-2$. Dependent dummy variables are rescaled by their unconditional means; coefficients measure percentage changes relative to the baseline adoption probability. Standard errors clustered at the firm-country-product level. *, **, *** significant at 10%, 5%, 1%.

Table 4: Corridor specificity of bilateral dollar invoicing coordination

	$\mathbb{I}\{X^{\$}_{c,p,t}\}$		$\mathbb{I}\{M^{\$}_{c,p,t}\}$	
	(1)	(2)	(3)	(4)
Start $M^{\$}_{c,t}$ (same corridor)	1.486***	1.449***		
	(0.0339)	(0.0331)		
Start $M^{\$}_{c',t}$ (other corridors)	0.00447	0.00436		
	(0.0507)	(0.0494)		
Start $X^{\$}_{c,t}$ (same corridor)			0.339***	0.378***
			(0.00928)	(0.0103)
Start $X^{\$}_{c',t}$ (other corridors)			−0.246***	−0.274***
			(0.0167)	(0.0186)
Sample	$X^{\$}_{c,p,t-}=0 \,\&\, M^{\$}_{c,t-}=0$	$X^{\$}_{c,p,t-}=0 \,\&\, M^{\$}_{c,t-}=0$	$M^{\$}_{c,p,t-}=0 \,\&\, X^{\$}_{c,t-}=0$	$M^{\$}_{c,p,t-}=0 \,\&\, X^{\$}_{c,t-}=0$
Multi-corridor only	No	Yes	No	Yes
Country × Product × Time FE	Y	Y	Y	Y
Firm × Product × Time FE	Y	Y	Y	Y
Firm × Country × Product FE	Y	Y	Y	Y
R-squared	0.709	0.709	0.778	0.778
Observations	16,441,857	16,441,857	16,468,272	16,468,272

Notes: The unit of observation is the firm–country–product–year. The table tests whether bilateral dollar invoicing coordination is corridor-specific by augmenting the baseline entry specification with a placebo regressor: whether the firm started dollar invoicing on the opposite side on *other* corridors $c' \neq c$ in the same firm–year. Start $M^{\$}_{c',t}$ equals one if the firm initiated dollar import invoicing on at least one country other than c in year t; Start $X^{\$}_{c',t}$ is defined analogously. Columns (2) and (4) restrict to firms with at least two active bilateral corridors (*multi-corridor*), so that the cross-corridor variable is non-trivially defined. Dependent dummy variables are rescaled by their unconditional means. $t-$ refers to both $t-1$ and $t-2$. Standard errors clustered at the firm–country–product level. *, **, *** significant at 10%, 5%, 1%.

and $M^{\$}_{i,t-1}$ denote lagged dollar-invoiced exports and imports, and α_i and α_t capture firm and year fixed effects, respectively.[10]

Three points are worth noting. We scale profits and trade exposures by lagged total assets to make them comparable across firms of different sizes. We use lagged rather than contemporaneous exposures so that the effect of an exchange-rate movement is evaluated against positions determined before firms can adjust their trading or invoicing choices. The level term in the net position absorbs any direct association between exposure and profitability that is unrelated to contemporaneous exchange-rate movements.

The interaction between the exchange rate and firm-level exposure is essential because the same exchange-rate movement has different implications for firms with different dollar positions. In particular, the effect should change sign depending on whether the firm is a net dollar exporter or a net dollar importer. A regression on the exchange-rate movement alone would average across these heterogeneous responses.

In Table 5 we estimate equation (3) for both overall pre-tax profits, $\pi_{i,t}$, and net foreign exchange income, $NFXI_{i,t}$. Under Italian accounting standards (OIC 26), $NFXI_{i,t}$ records both realized exchange-rate gains and losses on foreign-currency transactions and unrealized year-end revaluations of foreign-currency monetary items.[11] Comparing their responses serves two purposes. First, it separates the direct foreign-exchange accounting effects from the overall response of pre-tax profits, allowing us to isolate the response specific to the firm's operations, which we refer to throughout the paper as the *operational margin*.[12] Second, it provides a natural benchmark for the corridor-level analysis: once aggregate net dollar exposure is controlled for, the bilateral allocation

[10]Table B10 in Appendix B also reports estimates from three alternative specifications. The first includes only dollar export exposure, the second includes only dollar import exposure, and the third includes both exposures separately. In the main text, we take the baseline net specification as the natural benchmark for aggregate exchange-rate hedging. Economically, the firm's exposure to a common EUR/USD movement depends on the difference between its dollar-invoiced exports and imports rather than on either gross position in isolation. Statistically, the strong firm-level co-movement between dollar export and import invoicing documented in Section 3.1 implies that specifications considering only one side may also capture the omitted effect of the other, while the separate coefficients in the joint specification are identified from the more limited variation in which the two exposures move independently. The net specification instead summarizes the two positions in the aggregate exposure that is relevant for firm-level exchange-rate risk.

[11]The corresponding line item in Italian statutory income statements is *utili e perdite su cambi*.

[12]Because $NFXI_{i,t}$ enters pre-tax profits as a separate accounting item, profits can be decomposed as $\pi_{i,t} = NFXI_{i,t} + \text{Operational margin}_{i,t}$. When the two outcomes are estimated using the same specification and sample, linearity implies that, for each exchange-rate exposure coefficient, $\widehat{\beta}^{\text{Operational}} = \widehat{\beta}^{\text{Profits}} - \widehat{\beta}^{\text{NFXI}}$.

of dollar positions should not systematically affect $NFXI_{i,t}$, whereas it may affect the operational margin.

Two patterns emerge. First, the coefficient on the interaction is positive for both outcomes: following a euro depreciation, net foreign exchange income and pre-tax profits rise for net dollar exporters and fall for net dollar importers, with the magnitude of the effect increasing with the size of the net position. Second, the response of total profits $\pi_{i,t}$ is substantially larger than that of $NFXI_{i,t}$. In the net-exposure specification, the coefficient rises from 0.059 for $NFXI_{i,t}$ to 0.241 for pre-tax profits. A one-standard-deviation increase in the net-exposure shock raises profits by approximately 9.2% relative to mean pre-tax profits, of which 2.2% reflects balance-sheet revaluation and 6.9% the operational margin.[13] Taken together, these results show that exchange-rate movements affect profits predominantly through the operational margin, while direct foreign-exchange accounting effects account for only one quarter of the total response.

These results complete the first step of our analysis by establishing the aggregate-exposure benchmark. The next step asks a distinct question: once this aggregate net-exposure channel has been accounted for, does the bilateral allocation of dollar positions across countries provide any additional stabilization? Section 4.2 addresses this question by examining whether corridor matching reduces the component of profit fluctuations left unexplained by aggregate dollar exposure.

4.2 Corridor matching and profit volatility

The firm-year residuals from the net-exposure specification estimated in Section 4.1 capture the component of profits left unexplained after accounting for the firm's aggregate dollar exposure, persistent firm characteristics, and year-specific factors common to all firms. In this section, we use the square of these residuals and examine whether the magnitude of these unexplained profit fluctuations varies systematically with how completely the firm's dollar trade is matched within bilateral corridors.[14]

[13] Each figure equals $\hat{\beta} \times \text{SD(shock)}/\bar{\pi} \times 100$, where SD(shock) is the standard deviation of the net-exposure shock $\Delta e_t^{\text{€,\$}} \times (X_{t-1}^{\$} - M_{t-1}^{\$})/A_{t-1}$ and $\bar{\pi}$ is mean pre-tax profits scaled by lagged total assets, both computed in the net-shock sample (Table 5).

[14] Because $E[Y^2] = \text{Var}(Y) + (E[Y])^2$, using squared profit levels would conflate variation in expected profits with variation around those expected profits. Residualizing profits first removes the conditional-mean component captured by the aggregate-exposure specification, so that the squared residuals isolate the remaining profit variability.

Table 5: Currency shocks and profits

	$\frac{NFXI_t}{A_{t-1}}$	$\frac{\pi_t}{A_{t-1}}$
	(1)	(2)
$\Delta e_t^{€,\$} \times \frac{X^{\$}_{t-1}-M^{\$}_{t-1}}{A_{t-1}}$	0.0586***	0.241***
	(0.00268)	(0.0484)
$\frac{X^{\$}_{t-1}-M^{\$}_{t-1}}{A_{t-1}}$	0.000554***	−0.0261***
	(0.000213)	(0.00593)
Sample	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	
Firm FE	Y	Y
Time FE	Y	Y
R-squared	0.288	0.673
Observations	208,028	208,028

Notes: The dependent variable in column (1) is net foreign exchange income (*utili e perdite su cambi*) scaled by lagged total assets. The dependent variable in column (2) is pre-tax profits scaled by lagged total assets. $\Delta e_t^{€,\$}$ is the log change in the EUR/USD rate (positive = euro depreciation). $X^{\$}$ ($M^{\$}$) are lagged dollar-invoiced exports (imports); A is lagged total assets. Both columns restrict to two-sided USD traders ($X^{\$}_{t-1} > 0$ & $M^{\$}_{t-1} > 0$); column (2) additionally restricts to firms with non-missing NFXI to ensure comparability with column (1). All specifications include firm and year fixed effects. Standard errors clustered at the firm level. *, **, *** significant at 10%, 5%, 1%.

We measure this alignment with matching quality $MQ_{i,t}$, the share of the firm's dollar-active corridors in which both exports and imports are invoiced in dollars:

$$MQ_{i,t} = \frac{\sum_c \mathbb{I}\left\{X^{\$}_{i,c,t} > 0\right\} \mathbb{I}\left\{M^{\$}_{i,c,t} > 0\right\}}{\sum_c \mathbb{I}\left\{X^{\$}_{i,c,t} > 0 \text{ or } M^{\$}_{i,c,t} > 0\right\}}, \tag{4}$$

where $X^{\$}_{i,c,t}$ and $M^{\$}_{i,c,t}$ denote the values of dollar-invoiced exports to and imports from country c, respectively. The numerator counts corridors in which the firm invoices both exports and imports in dollars, while the denominator counts dollar-active corridors, defined as corridors in which the firm uses dollar invoicing on at least one side. The measure ranges from zero to one, with higher values indicating more complete corridor-level matching.[15]

[15]Throughout the analysis, we restrict the sample to firms with positive dollar-invoiced exports to and imports from at least one country. The export and import countries may be the same or different, ensuring that the denominator of $MQ_{i,t}$ is always greater than or equal to one.

We then estimate the following specification separately for pre-tax profits and net foreign exchange income:

$$\left(\hat{\varepsilon}_{i,t}^{Y}\right)^2 = \beta^{MQ} MQ_{i,t} + \alpha_i + \alpha_t + u_{i,t}, \tag{5}$$

where $\hat{\varepsilon}_{i,t}^{Y}$ denotes the residual from the specification in Section 4.1, estimated for either pre-tax profits, $\hat{\varepsilon}_{i,t}^{\pi}$, or net foreign exchange income, $\hat{\varepsilon}_{i,t}^{NFXI}$, and α_i and α_t denote firm and year fixed effects, respectively.[16] A negative estimate of β^{MQ} indicates that firms whose dollar export and import exposures are more closely aligned within the same corridors experience smaller profit fluctuations even after the aggregate net-dollar-exposure channel has been accounted for, thereby identifying a hedging margin that aggregate netting cannot deliver.

Table 6 reports the results for both profit-related outcomes. Columns 1–3 use the squared residuals from the net foreign exchange income regression, $\left(\hat{\varepsilon}_{i,t}^{NFXI}\right)^2$, while columns 4–6 use the squared residuals from the total-profit regression, $\left(\hat{\varepsilon}_{i,t}^{\pi}\right)^2$. Focusing first on total profits, the coefficient on matching quality is negative and statistically significant across all specifications. Column 5 restricts the sample to *switchers*, defined as firms that cross the pooled-sample median of $MQ_{i,t}$ at least once during the observation period. The result remains robust in this subsample, assuaging concerns that the baseline estimate may be partly driven by firms with little meaningful variation in $MQ_{i,t}$.[17] Column 6 shows that it also remains robust when matching quality is lagged, $MQ_{i,t-1}$, thereby mitigating concerns that contemporaneous profit fluctuations may themselves affect firms' matching choices. The estimates are also economically meaningful. The estimates in column 6, our most conservative specification, imply that moving from the 25th to the 75th percentile of lagged matching quality in the switcher sample is associated with a 3.2% reduction in mean squared profit residuals.

Columns 1–3 provide an important consistency check by repeating the analysis for $NFXI_{i,t}$. The coefficient on matching quality is economically negligible and statistically insignificant. This is precisely the pattern implied by the accounting nature of net foreign exchange income, as it records the gains and losses generated by exchange-

[16] Although the first-step regressions already include firm and year fixed effects, we include them again in the second stage because orthogonality of the residuals in levels does not imply orthogonality of their squares.

[17] With both firm and year fixed effects, firms whose own $MQ_{i,t}$ changes little may still contribute to identification because changes in the yearly cross-sectional average alter their relative position in the distribution. Restricting the sample to switchers ensures that the estimate is driven by firms exhibiting genuine within-firm changes in matching quality.

rate movements on the firm's foreign-currency positions. Conditional on aggregate net dollar exposure, reallocating an otherwise identical dollar position across bilateral corridors should therefore not affect this item through the corridor allocation itself. The absence of any relationship between matching quality and residual $NFXI_{i,t}$ variability thus provides a reassuring placebo-style check on the empirical strategy.[18]

Total profits $\pi_{i,t}$, instead, also include the operational margin, which is exposed to country-specific risks for which the bilateral allocation of trade relationships may matter. The negative total-profit coefficients therefore suggest that the same alignment of dollar exports and imports plays two hedging roles at once: it offsets aggregate exchange-rate risk and, when organized within bilateral corridors, also provides protection against country-specific risks that are unrelated to pure exchange-rate revaluation but remain relevant for overall profit volatility. In Section 6, we formalize this mechanism.

5 Exchange rate pass-through

Section 4 shows that EUR/USD movements have large effects on the operational margin through firms' dollar-invoiced trade exposure. The firm-level profit regressions, however, do not reveal why this response is so large. The component of the operational margin associated with this exposure is driven by the revenues and expenditures generated by dollar-invoiced export and import transactions. In this section, we unpack these revenues and expenditures by examining separate price and quantity responses to exchange-rate movements, on both the export and import sides.

Specifically, we estimate pass-through regressions at the firm–country–product–year level, separately for exports and imports.[19] The baseline specifications are

$$\begin{aligned}\Delta \ln p_{i,c,p,t} = {} & \beta^{€} I^{€}_{i,c,p,t} \Delta e^{€,j}_{t} + \beta^{\$,€} I^{\$}_{i,c,p,t} \Delta e^{€,\$}_{t} + \beta^{\$,j} I^{\$}_{i,c,p,t} \Delta e^{\$,j}_{t} \\ & + \alpha_{i,p,t} + \alpha_{i,c,p} + \alpha_{a,p,t} + \varepsilon_{i,c,p,t}, \qquad (6) \\ \Delta \ln q_{i,c,p,t} = {} & \beta^{€} I^{€}_{i,c,p,t} \Delta e^{€,j}_{t} + \beta^{\$,€} I^{\$}_{i,c,p,t} \Delta e^{€,\$}_{t} + \beta^{\$,j} I^{\$}_{i,c,p,t} \Delta e^{\$,j}_{t} \\ & + \alpha_{i,p,t} + \alpha_{i,c,p} + \alpha_{a,p,t} + \varepsilon_{i,c,p,t}, \qquad (7)\end{aligned}$$

[18] Both patterns survive dispensing with the first-step regression altogether: in Table B4 in Appendix B, which measures volatility directly as the squared deviation of each outcome from its firm-specific leave-one-out mean, matching quality lowers total-profit volatility by a similar magnitude (4.0 percent for the same interquartile move), while leaving $NFXI_{i,t}$ volatility unaffected.

[19] To keep the notation simple, we use the same coefficient and fixed-effect symbols across equations, although all coefficients and fixed effects are estimated separately for prices and quantities and for exports and imports.

Table 6: Bilateral invoicing matching and profit volatility

	$\hat{\varepsilon}^2_{i,t}(NFXI)$			$\hat{\varepsilon}^2_{i,t}$(total)		
	(1)	(2)	(3)	(4)	(5)	(6)
$MQ_{i,t}$	0.0109	0.0132		-0.103***	-0.106***	
	(0.00892)	(0.00908)		(0.00889)	(0.00912)	
$MQ_{i,t-1}$			-0.00394			-0.0216**
			(0.01000)			(0.0103)
Sample	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	Switchers$_t$	Switchers$_{t-1}$	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	Switchers$_t$	Switchers$_{t-1}$
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
R-squared	0.459	0.447	0.445	0.505	0.481	0.482
Observations	195,461	150,404	154,362	195,461	150,404	154,362

Notes: Unit of observation: firm-year. The dependent variable is the squared residual $\hat{\varepsilon}^2_{i,t}$ from the first-step regression in equation (3), which relates profits to exchange-rate shocks interacted with lagged net dollar exposure and includes firm and year fixed effects. Residuals therefore capture the component of profit fluctuations not explained by the aggregate net-exposure channel. The squared residual outcome is standardized to have mean zero and unit standard deviation, so all coefficients are expressed in units of standard deviations of residual profit volatility. Columns (1)–(3) use the standardized squared NFXI residual as the outcome; columns (4)–(6) use the standardized squared total-profit residual. $MQ_{i,t}$ is the share of dollar-active corridors (i.e. corridors in which the firm uses USD on at least one side) in which both exports and imports are invoiced in USD (extensive margin). Switchers are firms that cross the pooled-sample median of $MQ_{i,t}$ at least once over the observation window, so that identification comes from within-firm variation in matching quality. The switcher sample also requires the two-sided-trader condition $X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$ and a non-missing matching-quality regressor at the column's own date (t in columns (2) and (5), $t-1$ in columns (3) and (6)); switcher status itself is always determined from the firm's history of contemporaneous $MQ_{i,t}$. Because a firm's $MQ_{i,t}$ can be missing while its lag is not, or vice versa, Switchers$_t$ and Switchers$_{t-1}$ are not the same set of observations, which is why columns (2)/(5) and (3)/(6) differ in sample size. Column (6) provides the main quantitative specification. Moving from the 25th to the 75th percentile of lagged matching quality in the switcher sample (IQR = 0.444) is associated with a 3.2% reduction in mean squared profit residuals. Standard errors are clustered at the firm level. *, **, *** denote significance at the 10%, 5%, and 1% levels.

where the dependent variables, $\Delta \ln p_{i,c,p,t}$ and $\Delta \ln q_{i,c,p,t}$, are the log-changes of prices in euros and quantities for either exports or imports. The indicators $I^{€}_{i,c,p,t}$ and $I^{\$}_{i,c,p,t}$ identify whether a transaction is invoiced in euros or dollars, respectively. The variables $\Delta e^{€,j}_t$, $\Delta e^{€,\$}_t$, and $\Delta e^{\$,j}_t$ denote log-changes in the euro–j, euro–dollar, and dollar–j exchange rates, respectively, where j is the currency of partner country c.[20] The baseline specifications include firm–product–year fixed effects, $\alpha_{i,p,t}$, firm–country–product

[20]We maintain the same notation as in the previous section: a positive $\Delta e^{k,j}_t$ denotes a depreciation of currency k relative to currency j.

fixed effects, $\alpha_{i,c,p}$, and area–product–year fixed effects, $\alpha_{a,p,t}$, where a indexes the area associated with country c.

Table 7 reports the estimates. For dollar-invoiced transactions, pass-through of EUR/USD movements into euro prices is high and similar across the two sides of the trade account. The coefficients on $I^{\$}_{i,c,p,t} \times \Delta e^{€,\$}_{t}$ are approximately 0.6 for both exports in panel A and imports in panel B, implying pass-through rates of about 60 percent. The high and symmetric pass-through implies substantial changes in both the euro value of dollar-invoiced export revenues and the euro cost of dollar-invoiced imports. The price component of the operational margin is therefore particularly sensitive to the firm's net dollar trade position.

For dollar-invoiced transactions with non-dollar partner countries, by contrast, the response of euro prices to movements in the dollar–j exchange rate is close to zero. These results imply that transaction prices are relatively stable when expressed in dollars, consistent with the dollar playing a vehicle-currency role. Pass-through for euro-invoiced transactions is much smaller on both sides of the trade account.[21] More generally, these patterns are consistent with transaction prices being relatively sticky in the currency of invoicing.[22]

Turning to quantities, a euro depreciation is associated with an increase in dollar-invoiced export quantities, with a coefficient of 0.156, consistent with an improvement in the competitiveness of Italian exporters. Dollar-invoiced import quantities also increase, with a coefficient of 0.311. These responses have two consequences. First, because quantities rise on both sides of the trade account, they reinforce rather than offset the price responses: dollar-invoiced revenues and expenditures move by more than price pass-through alone would imply, which helps account for the large effect of dollar exposure on the operational margin documented in Section 4.1. Second, the increase in import quantities is counterintuitive, given the rise in import costs, and calls for a closer investigation, which we undertake in the next subsection.

5.1 The scale effect on linked imports

A euro depreciation makes dollar-invoiced imports more expensive in euros, yet it is accompanied by an increase in quantities. Below, we investigate whether this response

[21] Although the euro-invoiced price coefficients are statistically significant in columns 1 and 5 for exports and in all three specifications for imports, given the large number of observations, their magnitudes are much smaller than the approximately 60-percent pass-through estimated for dollar-invoiced transactions.

[22] These estimates are in line with the evidence on invoicing currency and exchange-rate pass-through in Gopinath et al. (2010), Devereux et al. (2017), Corsetti et al. (2022), and Chen et al. (2022).

reflects a production *scale effect*. Intuitively, by expanding dollar-invoiced exports, a euro depreciation may raise production and, in turn, firms' demand for the imported inputs used in that production. If this scale effect is strong enough to dominate the substitution effect arising from higher import prices, dollar-invoiced import quantities may rise despite the depreciation.

This interpretation yields a clear empirical test: the increase in import quantities should be concentrated among inputs linked to the firm's export activity. We evaluate this prediction by re-estimating equations (6) and (7) for imports, allowing the exchange-rate coefficients to differ according to three increasingly targeted measures of export–import complementarity. The first classifies an import observation as export-linked when the firm's total exports exceed the 25th percentile of the export distribution in both the current and previous year. This broad measure asks whether the scale effect is concentrated among firms with economically significant export activity. The second classifies an imported product as export-linked when the same firm exports at least one product belonging to the same two-digit HS category. The third is a data-driven measure of input linkages. For each exported HS4 product p and imported HS4 product m, we compute the share of m in the total imports of the firms that export p, divided by the share of m in aggregate imports; this ratio exceeds one when exporters of p import m disproportionately. We drop the most ubiquitous imported products, those in the top 5 percent of aggregate import shares, and rank the remaining pairs by the ratio multiplied by the log of the import value behind the link, so that a high rank requires the link to be both disproportionate and economically material. For each exported product, we retain the 20 top-ranked imported products as its linked inputs. An import observation is then classified as export-linked when its product is a linked input of at least one product in the firm's export basket. Appendix C provides the details.

Across all three specifications, Table 8 displays a consistent pattern. Price pass-through is similar across groups: dollar import prices respond to EUR/USD movements with comparable elasticities regardless of export linkage. Quantity responses, instead, depend strongly on whether imports are linked to the firm's export activity. Export-linked imports exhibit large and significant responses, with estimated coefficients of 0.339, 0.333, and 0.424 under the three classifications described above, respectively. Non-export-linked imports, instead, display much smaller and statistically insignificant responses. The contrast is clearest under the most targeted linkage measure. These

Table 7: Export and import pass-through

Panel A:	Export pass-through					
	$\Delta \ln p^X$ (1)	$\Delta \ln q^X$ (2)	$\Delta \ln p^X$ (3)	$\Delta \ln q^X$ (4)	$\Delta \ln p^X$ (5)	$\Delta \ln q^X$ (6)
$I^{€}_{c,t} \times \Delta e^{€,j}_t$	0.00632*	0.157***	-0.0338	-0.193***	0.00920**	0.141***
	(0.00330)	(0.00738)	(0.0319)	(0.0711)	(0.00466)	(0.0102)
$I^{\$}_{c,t} \times \Delta e^{€,\$}_t$	0.637***	0.156***	0.597***	0.164*	0.638***	0.141***
	(0.0176)	(0.0385)	(0.0387)	(0.0876)	(0.0204)	(0.0443)
$I^{\$}_{c,t} \times \Delta e^{\$,j}_t$	0.00913	0.121***	–	–	0.00489	0.120***
	(0.00649)	(0.0161)	–	–	(0.00674)	(0.0175)
Sample	Full		Full		$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	
Firm × Product × Country × Time FE	N	N	Y	Y	N	N
Firm × Product × Time FE	Y	Y	–	–	Y	Y
Firm × Country × Product FE	Y	Y	–	–	Y	Y
Area × Product × Time FE	Y	Y	–	–	Y	Y
R-squared	0.415	0.387	0.526	0.516	0.397	0.386
Identifying obs	25,080,611	25,080,612	396,580	396,584	12,064,233	12,064,233
Total sample	34,735,058	34,735,059	34,735,058	34,735,059	15,805,980	15,805,980
Panel B:	Import pass-through					
	$\Delta \ln p^M$ (1)	$\Delta \ln q^M$ (2)	$\Delta \ln p^M$ (3)	$\Delta \ln q^M$ (4)	$\Delta \ln p^M$ (5)	$\Delta \ln q^M$ (6)
$I^{€}_{c,t} \times \Delta e^{€,j}_t$	0.121***	-0.0620	-0.0651*	-0.0101	0.112***	0.000864
	(0.0193)	(0.0379)	(0.0356)	(0.0695)	(0.0329)	(0.0583)
$I^{\$}_{c,t} \times \Delta e^{€,\$}_t$	0.606***	0.311***	0.370***	0.979***	0.656***	0.252**
	(0.0399)	(0.0821)	(0.0500)	(0.112)	(0.0625)	(0.122)
$I^{\$}_{c,t} \times \Delta e^{\$,j}_t$	0.0389***	-0.0407	–	–	0.0245	-0.0322
	(0.0129)	(0.0296)	–	–	(0.0187)	(0.0397)
Sample	Full		Full		$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	
Firm × Product × Country × Time FE	N	N	Y	Y	N	N
Firm × Product × Time FE	Y	Y	–	–	Y	Y
Firm × Country × Product FE	Y	Y	–	–	Y	Y
Area × Product × Time FE	Y	Y	–	–	Y	Y
R-squared	0.532	0.532	0.526	0.497	0.502	0.519
Identifying obs	4,811,929	4,811,938	177,044	177,040	1,744,145	1,744,142
Total sample	17,069,801	17,069,804	17,069,801	17,069,804	4,961,125	4,961,121

Notes: The unit of observation is firm–country–product–year. Additional regressors: $I^{€}_{c,t}$. Standard errors clustered at the firm–product–country–invoice currency level. *, **, *** significant at the 10%, 5%, and 1% levels.

results show that the increase in import quantities is not a general response to exchange-rate movements, but is concentrated among inputs connected to export production.

The heterogeneity in Table 8 is, however, silent on where the linked inputs are sourced, since all import transactions are considered together regardless of their country of origin. In light of the corridor-specific invoicing patterns documented in Section 3.2, we next ask whether the scale effect on linked imports also operates within the same bilateral corridor. To test this, Table 9 reports estimates of equations (6) and (7) for imports, allowing the exchange-rate coefficients to vary jointly with export-linkage status and corridor status. Specifically, an import is classified as belonging to a corridor when the firm also exports to the same country in the same year. In addition to the fixed effects used in the baseline specifications, we include currency-by-export-linkage-status-by-corridor fixed effects, which absorb average differences in price and quantity growth across the four linkage–corridor groups for each invoicing currency. Identification of the corridor margin comes from firm–product–year observations in which the same input is sourced from different countries, some of which are also destinations for the firm's exports.

The coefficients for linked imports sourced within and outside the corridor are similar in magnitude, and their difference is not statistically significant. The scale effect therefore appears to be tied to the firm's export basket rather than to the bilateral corridor in which the input is sourced. Taken together, these findings suggest that corridor matching is a hedging strategy that operates through invoicing-currency choices to stabilize profits, rather than a mechanism through which firms accommodate higher production by increasing their demand for imported inputs. We return to this corridor invariance in the next section, which provides the economic mechanism behind it.

6 Theoretical framework

In this section, we confront our empirical findings with the predictions of a model in which invoicing currency is an outcome of firm optimization. In our framework, we derive a set of analytical results that establish which elements of the economic environment are needed to account for the empirical facts documented above.

We start by restating four findings coming out of our empirical analysis.

1. *Firm-level co-movement* (Section 3.1). When a firm begins to invoice its imports in dollars, the probability that it begins to invoice its exports in dollars in the same year more than triples relative to the baseline. Adoption is asymmetric and led by the import side.

Table 8: Import pass-through: export-related heterogeneity

	$\Delta \ln p^M$ (1)	$\Delta \ln q^M$ (2)	$\Delta \ln p^M$ (3)	$\Delta \ln q^M$ (4)	$\Delta \ln p^M$ (5)	$\Delta \ln q^M$ (6)
$I^{€}_{c,t} \times \Delta e^{€,j}_t \times \mathbb{I}\{\text{Group 1}\}$	0.127***	-0.0630	0.129***	-0.0654	0.126***	-0.0729
	(0.0211)	(0.0407)	(0.0215)	(0.0415)	(0.0236)	(0.0490)
$I^{€}_{c,t} \times \Delta e^{€,j}_t \times \mathbb{I}\{\text{Group 0}\}$	0.0939**	-0.0577	0.0908***	-0.0479	0.117***	-0.0446
	(0.0368)	(0.0731)	(0.0340)	(0.0660)	(0.0287)	(0.0514)
$I^{\$}_{c,t} \times \Delta e^{€,\$}_t \times \mathbb{I}\{\text{Group 1}\}$	0.611***	0.339***	0.611***	0.333***	0.582***	0.424***
	(0.0423)	(0.0865)	(0.0430)	(0.0877)	(0.0463)	(0.0975)
$I^{\$}_{c,t} \times \Delta e^{€,\$}_t \times \mathbb{I}\{\text{Group 0}\}$	0.573***	0.0948	0.585***	0.183	0.643***	0.150
	(0.0791)	(0.153)	(0.0701)	(0.137)	(0.0617)	(0.120)
$I^{\$}_{c,t} \times \Delta e^{\$,j}_t \times \mathbb{I}\{\text{Group 1}\}$	0.0365**	-0.0528	0.0334**	-0.0459	0.0370***	-0.0848**
	(0.0143)	(0.0323)	(0.0144)	(0.0324)	(0.0142)	(0.0352)
$I^{\$}_{c,t} \times \Delta e^{\$,j}_t \times \mathbb{I}\{\text{Group 0}\}$	0.0506**	0.0168	0.0613***	-0.0201	0.0423*	0.0466
	(0.0225)	(0.0609)	(0.0217)	(0.0604)	(0.0233)	(0.0481)
$\mathbb{I}\{\text{Group 1}\}$	$\mathbb{I}\{X_{t,t-1} > p25\}$		$\mathbb{I}\{X^{HS2(p)}_{t,t-1} > 0\}$		$\mathbb{I}\{X^{Linked(p)}_{t,t-1} > 0\}$	
Sample	Full		Full		Full	
Firm × Product × Time FE	Y	Y	Y	Y	Y	Y
Firm × Country × Product FE	Y	Y	Y	Y	Y	Y
Area × Product × Time FE	Y	Y	Y	Y	Y	Y
R-squared	0.532	0.532	0.532	0.532	0.532	0.532
Identifying obs	4,811,929	4,811,938	4,811,929	4,811,938	4,811,929	4,811,938
Total sample	17,069,801	17,069,804	17,069,801	17,069,804	17,069,801	17,069,804

Notes: The unit of observation is firm–country–product–year. Group splits: columns (1)–(2), $X_{t,t-1} > p25$ identifies firms above the 25th percentile of the export distribution; columns (3)–(4), $X^{HS2(p)}_{t,t-1} > 0$ identifies firm–product pairs where the firm also exports in the same HS2 class; columns (5)–(6), $X^{Linked(p)}_{t,t-1} > 0$ uses a data-driven linked-products mapping (see text). Standard errors clustered at the firm–product–country–currency level. *, **, *** significant at the 10%, 5%, and 1% levels.

2. *The corridor structure* (Section 3.2). The matching is bilateral: a firm that imports from country c in dollars is disproportionately likely to invoice its exports to that same country c in dollars. The pattern survives firm-by-time, country-by-time, and firm-by-country fixed effects. On the export side, the cross-corridor placebo, dollar import adoption on one corridor predicting dollar export adoption on others, is statistically indistinguishable from zero; the import-side placebo is negative.

Table 9: Import pass-through by export linkages and corridor match

	Export status		HS2 overlap		Linked inputs	
	$\Delta \ln p^M$	$\Delta \ln q^M$	$\Delta \ln p^M$	$\Delta \ln q^M$	$\Delta \ln p^M$	$\Delta \ln q^M$
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Euro-invoiced, euro–partner shock $\left(I^{€}_{c,t} \times \Delta e^{€,j}_{t}\right)$						
Group 1 × Corridor	0.1747***	−0.0500	0.1784***	−0.0509	0.1745***	−0.0404
	(0.0308)	(0.0575)	(0.0312)	(0.0583)	(0.0376)	(0.0723)
Group 1 × No corridor	0.0809***	−0.0830*	0.0806***	−0.0873*	0.0856***	−0.1149**
	(0.0217)	(0.0477)	(0.0220)	(0.0491)	(0.0232)	(0.0558)
Group 0 × Corridor	0.3083	−0.2152	0.1355	−0.0680	0.1791***	−0.0737
	(0.2106)	(0.3853)	(0.1151)	(0.1953)	(0.0479)	(0.0844)
Group 0 × No corridor	0.0861**	−0.0504	0.0859**	−0.0432	0.0776**	−0.0251
	(0.0368)	(0.0738)	(0.0342)	(0.0681)	(0.0304)	(0.0573)
Panel B. Dollar-invoiced, euro–dollar shock $\left(I^{\$}_{c,t} \times \Delta e^{€,\$}_{t}\right)$, the scale shock						
Group 1 × Corridor	0.6648***	0.3120***	0.6577***	0.3248***	0.6272***	0.3898***
	(0.0538)	(0.1052)	(0.0546)	(0.1065)	(0.0621)	(0.1241)
Group 1 × No corridor	0.5708***	0.3530***	0.5778***	0.3284***	0.5474***	0.4260***
	(0.0451)	(0.0972)	(0.0461)	(0.0993)	(0.0476)	(0.1069)
Group 0 × Corridor	0.7950***	−0.5220	0.8257***	−0.2652	0.7285***	0.1051
	(0.2537)	(0.4193)	(0.1645)	(0.2860)	(0.0882)	(0.1632)
Group 0 × No corridor	0.5563***	0.1496	0.5436***	0.2648*	0.5917***	0.1810
	(0.0790)	(0.1557)	(0.0705)	(0.1413)	(0.0660)	(0.1315)
Panel C. Dollar-invoiced, dollar–partner shock $\left(I^{\$}_{c,t} \times \Delta e^{\$,j}_{t}\right)$						
Group 1 × Corridor	0.0884*	−0.1157	0.0851*	−0.0977	0.0807	−0.2747***
	(0.0497)	(0.0877)	(0.0501)	(0.0880)	(0.0526)	(0.1066)
Group 1 × No corridor	0.0230*	−0.0407	0.0196	−0.0359	0.0284**	−0.0611*
	(0.0120)	(0.0322)	(0.0121)	(0.0323)	(0.0131)	(0.0340)
Group 0 × Corridor	0.0594	0.0317	0.0793	−0.0853	0.0885	0.0951
	(0.0630)	(0.1088)	(0.0743)	(0.1766)	(0.0745)	(0.0991)
Group 0 × No corridor	0.0489**	0.0169	0.0570***	−0.0100	0.0256	0.0347
	(0.0239)	(0.0665)	(0.0221)	(0.0619)	(0.0188)	(0.0521)
Sample	Full		Full		Full	
Firm×Product×Time FE	Y	Y	Y	Y	Y	Y
Firm×Country×Product FE	Y	Y	Y	Y	Y	Y
Area×Product×Time FE	Y	Y	Y	Y	Y	Y
Currency×Group×Corridor FE	Y	Y	Y	Y	Y	Y
R-squared	0.532	0.532	0.532	0.532	0.532	0.532
Identifying obs	4,811,929	4,811,938	4,811,929	4,811,938	4,811,929	4,811,938
Total sample	17,069,801	17,069,804	17,069,801	17,069,804	17,069,801	17,069,804

Notes: Unit of observation: firm–country–product–year. The dependent variable is the log change in import prices (odd columns) and import quantities (even columns). The corridor indicator equals one when the firm both exports to and imports from the same country in year t. Group 1 is defined separately across column pairs: *export status* (columns 1–2), firms above the 25th percentile of the export distribution in both t and $t-1$; *HS2 overlap* (columns 3–4), observations where the firm exports in the same HS2 class as the imported product and is above that threshold; *linked inputs* (columns 5–6), imports that are linked inputs for the firm's export basket under the top-20 product-association measure (Appendix C). $I^{€}_{c,t}$ and $I^{\$}_{c,t}$ denote euro- and dollar-invoiced transactions; $\Delta e^{€,j}_{t}$, $\Delta e^{€,\$}_{t}$, $\Delta e^{\$,j}_{t}$ are the euro–partner, euro–dollar, and dollar–partner exchange-rate changes. All specifications include firm–product–time, firm–country–product, area–product–time, and currency–group–corridor fixed effects. Standard errors, clustered at the firm–product–country–currency level, are in parentheses. *, **, *** denote significance at the 10%, 5%, and 1% levels.

3. *Variance reduction beyond aggregate exposure* (Section 4). Corridor alignment reduces the volatility of firm profits beyond what the firm's aggregate net dollar exposure can account for.

4. *The operational margin and the scale effect* (Sections 4 and 5). *(a)* In the net-shock specification, approximately three quarters of the transmission of exchange-rate movements to firm profits operates through the operational margin rather than balance-sheet revaluation. *(b)* Dollar-invoiced import quantities rise following a euro depreciation, with the response concentrated in inputs linked to the firm's export basket. Additionally, this scale effect is specific to linked inputs and unrelated to the corridor.

Finding 4(a) guides the choice of baseline: with the transmission to profits predominantly operational, the natural starting point is a model of the operational margin. Hence, we take as our baseline a partial-equilibrium simplification of the framework of Mukhin (2022), in which the currency of invoicing is chosen to minimize the variance of a firm's desired price and which nests the equivalence and pass-through results of Engel (2006) and Gopinath et al. (2010). Below, we show that the baseline model reproduces Finding 1, the firm-level co-movement of dollar import and export invoicing, and Finding 4(b), the scale effect. Without modification it cannot reproduce the remaining structure, the corridor matching of Finding 2 and the variance reduction of Finding 3. We then introduce additional structural features that bring the model in line with all four facts, in two extensions: a corridor production structure and risk preferences. Section 6.3 introduces a corridor production structure and addresses the corridor invariance of the scale effect. Section 6.4 organizes costs at the corridor level and delivers Finding 2. Section 6.5 introduces risk preferences, delivers Finding 3, and gives Finding 2 a second route.

Below, we set up the baseline model, derive predictions related to our four findings, and then add the extensions. The framework is deliberately kept in partial equilibrium, as our goal is to identify the mechanism and qualitative features that can potentially bring the model closer to the data patterns rather than providing a quantitative evaluation.

6.1 A baseline model of invoicing currency choice

Consider a home economy H (Italy) with a continuum of firms f selling to a set of destination markets $c \in \mathcal{C}$. The household and market structure follows Mukhin (2022).

Households supply labor, consume a composite of differentiated tradables defined by a Kimball aggregator, and have access to complete asset markets.

Firm f operates two production lines with separate constant-returns technologies. The export line combines home labor with a bundle of imported intermediates,

$$y_f = e^{z_f} M_f^{\Phi_f} L_f^{1-\Phi_f}, \tag{8}$$

where z_f is log productivity, M_f aggregates imported inputs across source countries, and Φ_f is the export line's intermediate share. A domestic line is analogous, selling at home, with its own bundle M_f^O and intermediate share Φ_f^O.

Throughout, we assume home firms have the choice of invoicing between the dollar and the euro, where the dollar factor is $s = e_{€,\$}$, the log price of the dollar in euros, normally distributed with $\mathbb{E}[s] = 0$ and $\mathrm{Var}(s) = \sigma_s^2 > 0$.[23] In general $e_{x,y}$ is the log price of one unit of currency y in units of x, so a y-price expressed in x adds $e_{x,y}$, and $e_{x,y} = -e_{y,x}$.

Prices are sticky, with Calvo-style resetting: the firm chooses its invoicing currency before shocks are realized, and in each period readjusts its price within that currency with an exogenous probability $1 - \lambda$. Let $\Pi_{f,c}(p)$ be the firm's profit from sales to c as a function of the destination-currency price p and let $\mathcal{M}_{t,t+j}$ be the stochastic discount factor between t and $t + j$. At a readjustment in period t the firm commits to a price $\bar{p}_{f,c}^k$ held constant in currency k, and maximizes the expected profit until its next readjustment,

$$\max_{k,\ \bar{p}_{f,c}^k} \ \mathbb{E}_t \sum_{j \geq 0} \lambda^j \, \mathcal{M}_{t,t+j} \, \Pi_{f,c}\Big(\bar{p}_{f,c}^k + e_{c,k,t+j}\Big).$$

We assume exchange-rate shocks are permanent, so every readjustment spell presents the same problem, giving the optimal currency choice k and price in that currency $\bar{p}_{f,c}^k$.

Let the desired price $\tilde{p}_{f,c}$ be the price that maximizes $\Pi_{f,c}$ state by state, i.e. the price the firm would set if it could reset freely. Then the following lemma holds.

[23] In what follows, we use capital letters for levels and lowercase letters for their logs, so that $p_c = \log P_c$. Quantities are written in levels, with hats reserved for log deviations. Pricing relations are log-linearized around the symmetric steady state, so constants, including the steady-state markup, are suppressed throughout.

Lemma 1 (Variance criterion) *To a second-order approximation, the optimal invoicing choice of firm f for destination c is the currency in which the desired price $\tilde{p}_{f,c}$, expressed in that currency, has the smallest variance, that is:*

$$k^*_{f,c} = \arg \min_{k \in \{€,\$\}} \quad \mathrm{Var}(\tilde{p}_{f,c} + e_{k,c}) \,. \tag{9}$$

Given $k^*_{f,c}$, the minimizing preset price is $\bar{p}^k_{f,c} = \mathbb{E}[\tilde{p}_{f,c} + e_{k,c}]$.

The proof is in Appendix A.1, which expands profit around the flexible optimum and shows that the survival weights and the discount factor drop out of the leading term. The desired price is written in the destination currency because that is the price its customer in c faces, and $e_{k,c}$ re-expresses it in currency k.

The export line's marginal cost is the input-share-weighted sum of the euro wage and the price of the aggregate imported-input bundle,

$$mc_f = (1 - \Phi_f)\, w_€ + \Phi_f\, p^M_f. \tag{10}$$

The Kimball aggregator implies a downward-sloping demand schedule for each quantity. In destination c,

$$q_{f,c} = D_c\, \Upsilon(P_{f,c}/P_c)\,, \qquad \Upsilon' < 0, \quad \Upsilon(1) = 1, \tag{11}$$

where D_c is real destination demand, $P_{f,c}$ the firm's destination-currency price, and P_c the destination price index. Away from the symmetric point the curvature of Υ makes the demand elasticity vary with the firm's relative price, so the firm's markup falls as it prices above its competitors. Appendix A.2 derives the resulting complementarity α from the curvature of Υ.

Given the demand schedule and marginal cost, the log-linearized desired price is the standard Kimball expression, with weight α on the destination price index p_c and $1-\alpha$ on marginal cost,

$$\tilde{p}_{f,c} = (1 - \alpha)\,(mc_f + e_{c,€}) + \alpha\, p_c, \qquad \alpha \;\equiv\; \frac{\partial \tilde{p}_{f,c}}{\partial p_c} \;\in\; (0,1). \tag{12}$$

Next, we impose a structure on the behavior of outside suppliers and destination market competitors, summarized by Assumption 1.

Assumption 1 (Suppliers and competitors) *(i) The euro price of a dollar-invoiced input has elasticity $\rho_M \in (0,1]$ to the dollar in every period. A euro-invoiced input has elasticity zero.*

(ii) Competitors selling in c invoice in dollars with share $\theta_c^{\$} \in [0,1]$ and in the euro with share $1-\theta_c^{\$}$; their prices are sticky in the currency of invoicing.

(iii) The home market is euro-priced, $\theta_H^{\$} = 0$.

Part (i) takes the response of an imported input's euro price to the dollar as exogenous, since the firm's suppliers are outside the model. Part (ii) defines the destination dollar share $\theta_c^{\$}$, which is one of the crucial quantities of this section.[24] Part (iii) is a restriction that governs the firm's domestic line: a euro-priced home market leaves that line's relative price unmoved by the dollar, so it carries input-cost exposure alone.

6.2 The baseline against the four findings

We now confront the baseline with the four findings. Two results organize the comparison. The first derives the sensitivity of the desired price to the dollar, and the second turns that sensitivity into an invoicing rule.

Lemma 2 (The aggregate dollar loading) *In the baseline model of Section 6.1 and under Assumption 1, let $\Phi_f^{\$} \leq \Phi_f$ be the dollar-invoiced share of the export line's input cost. Then the desired price for destination c, expressed in euros, is*

$$\tilde{p}_{f,c} + e_{\epsilon,c} \;=\; \Lambda_{f,c}\, s \;+\; u_{f,c}, \qquad u_{f,c} \perp s, \tag{13}$$

where $u_{f,c}$ collects the terms orthogonal to s and

$$\Lambda_{f,c} = (1-\alpha)\,\rho_M\,\Phi_f^{\$} + \alpha\,\theta_c^{\$}. \tag{14}$$

Proof. Substitute the marginal cost (10) into the desired price (12) and collect the terms in s. The euro wage loads zero, the input bundle loads $\rho_M \Phi_f^{\$}/\Phi_f$ by Assumption 1(i), and the destination-currency terms cancel against the euro conversion.[25] Appendix A.3 gives the details of the calculation. ■

Proposition 1 (Baseline invoicing) *Under the hypotheses of Lemma 2, the firm invoices its exports to c in dollars if and only if $\Lambda_{f,c} > \frac{1}{2}$.*

[24] Appendix A.3 admits a further share pricing in c's own currency. It widens $\theta_c^{\$}$ into an effective dollar share, and nothing below changes.

[25] That cancellation is why (14) carries no term in destination c's own currency; Appendix A.3 discusses it and the sense in which it extends beyond (14) (Remark A.1).

Proof. Compare the desired price expressed in the two candidate currencies, the euro and the dollar. The euro expression is (13). Expressing the same price in dollars subtracts the dollar appreciation, since $e_{\$,c} = e_{€,c} - s$, so its loading falls by one, $\tilde{p}_{f,c} + e_{\$,c} = (\Lambda_{f,c} - 1)\, s + u_{f,c}$. The component $u_{f,c}$ is common to both, so the two candidate variances,

$$\mathrm{Var}(\tilde{p}_{f,c} + e_{€,c}) = \Big(\Lambda_{f,c}\Big)^2 \sigma_s^2 + \mathrm{Var}(u_{f,c}),$$
$$\mathrm{Var}(\tilde{p}_{f,c} + e_{\$,c}) = \Big(\Lambda_{f,c} - 1\Big)^2 \sigma_s^2 + \mathrm{Var}(u_{f,c}),$$

differ only through the squared loading. By Lemma 1 the firm selects the smaller of the two. Dollar invoicing attains it precisely when $\Big(\Lambda_{f,c} - 1\Big)^2 < \Big(\Lambda_{f,c}\Big)^2$, that is, when $\Lambda_{f,c} > \frac{1}{2}$. ■

Finding 1 (firm-level co-movement) The two results deliver the first finding directly. When a firm begins or increases imports in dollars, it raises $\Phi_f^{\$}$ and hence $\Lambda_{f,c}$ toward the threshold of Proposition 1. If pushed over it, the firm switches its exports to dollar invoicing, reproducing the co-movement of Section 3.1.

Finding 2 (corridor matching and the cross-corridor placebo) The baseline cannot match invoicing corridor by corridor by construction, as the dollar input share enters only through the firm-aggregate $\Phi_f^{\$}$ in (14). Dollar imports arriving through *any* corridor raise that single number, and so raise the dollar incentive on *every* export corridor by the same amount. A firm that starts importing in dollars from c should therefore become more likely to invoice exports in dollars to every destination, with no additional incentive attaching to corridor c itself.

Finding 3 (variance reduction beyond aggregate exposure) The baseline is silent on the third finding, since the variance criterion (9) places no first-order weight on the variance of profit (Lemma 1). It ranks invoicing currencies by the stability of the desired price, not by the volatility of cash flow, so it cannot speak to whether corridor alignment lowers profit variance beyond what aggregate net exposure explains. Delivering that finding requires an objective in which profit volatility enters at first order. We explore this extension in Section 6.5.

Finding 4(b) (the scale effect on linked imports) The baseline predicts that dollar-invoiced import quantities rise after a euro depreciation when the destination market is dollar-priced enough relative to the firm's own dollar cost. Intuitively, part of the firm's costs are euro wages, which a depreciation leaves unchanged, while its dollar-pricing competitors' prices rise in euros. The firm's exports become relatively cheaper,

sales grow, and require more inputs even as those inputs cost more. Section 6.3 derives these results explicitly in an extended model with source-specific inputs. Those results trivially simplify to the more aggregated model of this section.

The two-sector structure determines which inputs react: an input in the export bundle carries the export-demand channel, an input in the domestic line's bundle does not, with no distinction in the model as to the specific source country of the input. That maps to the linked versus unlinked imports of Section 5, with no corridor effect. In this sense the baseline delivers Finding 4(b): the sign condition, the concentration on linked inputs, and the absence of any corridor differential.

6.3 Corridor invariance of the scale effect

In this section, we make inputs source-specific, which accomplishes two goals. The first is to investigate under which conditions the scale effect of Finding 4(b) does not depend on the corridor structure. The second is to supply the sourcing structure for the analysis of corridor invoicing of Section 6.5.

Specifically, we assume that the output firm f sells to destination c is produced with a constant-returns technology

$$y_{f,c} = e^{z_f}\, M_{f,c}^{\phi_{f,c}}\, L_{f,c}^{1-\phi_{f,c}}, \tag{15}$$

where

$$M_{f,c} = \prod_{c'} M_{f,cc'}^{\omega_{cc'}}, \qquad \sum_{c'} \omega_{cc'} = 1 \quad \text{for each destination } c. \tag{16}$$

Consequently, marginal production cost is

$$mc_{f,c} = (1-\phi_{f,c})\, w_{€} + \phi_{f,c} \sum_{c'} \omega_{cc'}\, p_{f,c'}^{M}, \tag{17}$$

where $\phi_{f,c}$ is the intermediate share of the output sold to c, the destination counterpart of Φ_f, and $p_{f,c'}^{M}$ is the price of inputs sourced from c'. Both indices of $\omega_{cc'}$ run over all countries, the first naming the destination and the second the source. A country the firm does not export to has no production behind it, hence a row of zeros; a country it does not buy from carries a zero in every row. Row sums are therefore one on the firm's destinations and zero elsewhere, and a source country may or may not be one of those destinations.

Let $d_{c'} \in \{0,1\}$ equal one if imports from c' are invoiced in dollars and zero if in euros, and $\bar{d}_{f,c} \equiv \sum_{c'} \omega_{cc'} d_{c'}$ for the dollar content of destination c's input bundle.

That bundle's price then loads on s with $\rho_M \bar{d}_{f,c}$, and destination c's marginal cost with $\rho_M \phi_{f,c} \bar{d}_{f,c}$. A given input can be used for several of the firm's destinations, so its demand aggregates across them. Under constant returns, intermediate inputs are the share $\phi_{f,c}$ of destination c's total cost, so its input spending is

$$X_{f,c} \equiv \phi_{f,c}\, mc_{f,c}\, q_{f,c}, \tag{18}$$

of which $\omega_{cc'} X_{f,c}$ goes to source c'. Let

$$s^{c'}_{f,c} = \frac{\omega_{cc'} X_{f,c}}{\sum_{\tilde{c}} \omega_{\tilde{c}c'} X_{f,\tilde{c}}}, \qquad \sum_c s^{c'}_{f,c} = 1, \tag{19}$$

be destination c's share of the firm's spending on inputs from c', and define

$$\bar{\theta}^{\$}_{f,c'} \equiv \sum_c s^{c'}_{f,c}\, \theta^{\$}_c, \qquad \bar{\Phi}^{\$}_{f,c'} \equiv \sum_c s^{c'}_{f,c}\, \phi_{f,c}\, \bar{d}_{f,c}, \tag{20}$$

the dollar pricing and the dollar cost share that input c' faces, each weighted by where that input is actually used. These are the same sourcing weights read down the column rather than across the row: $\omega_{cc'}$ fixes a destination and gives its source mix, while $s^{c'}_{f,c}$ fixes a source and gives its destination mix.[26]

Next, we evaluate the firm's demand for an imported input at a readjustment. The firm sets its price after observing s, so the shock passes into the price its buyers face and then into the quantity they demand. The price the firm posts at a readjustment is the average of the desired prices expected over the price's remaining life, and it differs from today's desired price by a term orthogonal to the dollar. What the comparative statics below use is that the two respond to the dollar identically under permanent shocks (for derivation, see Appendix A.1).[27]

Lemma 3 (Scale effect on linked imports) *Consider a dollar-invoiced input sourced from c' and used in the firm's export bundles. Under Assumption 1, evaluated*

[26] In the limit of a common intensity $\phi_{f,c} = \Phi_f$ and sourcing common across destinations, $\omega_{cc'} = \omega_{c'}$, the source weight cancels from (19) and $s^{c'}_{f,c}$ reduces to destination c's share of the firm's input spending, which at the symmetric point equals its export share $w^X_{f,c}$, the same for every c'. Both aggregates then collapse to the firm-level $\bar{\theta}^{\$}_f = \sum_d w^X_{f,d} \theta^{\$}_d$ and $\Phi^{\$}_f$, and (21) becomes the single-bundle expression of the baseline (8). That limit is how the current section's model nests the one in Section 6.1.

[27] Mukhin (2022) applies the same reset-price logic in his dynamic currency choice. That choice depends on the medium-run pass-through, the discounted response of the desired price to the exchange rate over the price's expected life, not the impact response.

at a readjustment, the elasticity of the firm's demand for the input with respect to a euro depreciation is

$$\frac{\partial \ln M^{\$}_{f,c'}}{\partial s} = \underbrace{\nu\,(1-\alpha)\left(\bar{\theta}^{\$}_{f,c'} - \bar{\Phi}^{\$}_{f,c'}\rho_M\right)}_{\textit{the gain: sales expansion}} - \underbrace{\rho_M\left(1-\bar{\Phi}^{\$}_{f,c'}\right)}_{\textit{the higher input price: substitution}}, \tag{21}$$

with the source-weighted aggregates of (20) *and* $\nu \equiv -\Upsilon'(1) > 1$ *the symmetric-point demand elasticity of* (11).

Sketch of proof. A euro depreciation raises destination c's marginal cost by $\phi_{f,c}\bar{d}_{f,c}\rho_M$, the euro price of the dollar-invoiced part of that destination's input bundle. The desired price passes this through with weight $1-\alpha$, while destination c's index rises by $\theta^{\$}_c$. The relative price in c therefore moves by $(1-\alpha)(\phi_{f,c}\bar{d}_{f,c}\rho_M - \theta^{\$}_c)$, and sales in c move against it with elasticity ν. Aggregation then delivers the equation (21). For derivations, see Appendix A.6. ■

The source country enters (21) only through the destination mix $s^{c'}_{f,\cdot}$, so two dollar-invoiced inputs respond identically whenever the destinations they feed are alike in dollar pricing and dollar cost share. Differences across groups of linked imports are therefore evidence about sourcing. Two comparisons organize what follows: the corridor comparison, which Section 5 runs by splitting the dollar-invoiced linked imports on whether the firm also exports to the source country, and the denomination comparison between dollar- and euro-invoiced linked imports, the subject of Corollary 2 below.

In order to determine these differences in the theory, it is useful to define several objects. Write

$$A_{f,c} \;\equiv\; \nu(1-\alpha)\,\theta^{\$}_c + \left[1-\nu(1-\alpha)\right]\rho_M\,\phi_{f,c}\,\bar{d}_{f,c}, \tag{22}$$

for the scale-effect incentive that destination c contributes, so that the response of a source of either denomination is

$$\frac{\partial \ln M_{f,c'}}{\partial s} \;=\; \sum_c s^{c'}_{f,c}\,A_{f,c} \;-\; \rho_M\,d_{c'}. \tag{23}$$

Setting $d_{c'} = 1$ and substituting the source aggregates of (20) recovers the dollar case, (21). Appendix A.6 derives (23). Each source is bought by several destinations, so summing that spending down a column gives what the firm pays for inputs from c',

$$X^S_{f,c'} \;\equiv\; \sum_c \omega_{cc'}X_{f,c}. \tag{24}$$

Splitting the sources into two groups $\mathcal{G}_1$ and $\mathcal{G}_0$, the coefficients the tests compare are the value-weighted means of the responses (23) over each group,

$$\beta_{\mathcal{G}_1} \equiv \frac{\sum_{c' \in \mathcal{G}_1} X^S_{f,c'}\, \partial \ln M_{f,c'}/\partial s}{\sum_{c' \in \mathcal{G}_1} X^S_{f,c'}}, \qquad \beta_{\mathcal{G}_0} \equiv \frac{\sum_{c' \in \mathcal{G}_0} X^S_{f,c'}\, \partial \ln M_{f,c'}/\partial s}{\sum_{c' \in \mathcal{G}_0} X^S_{f,c'}}. \tag{25}$$

With these objects in hand, Proposition 2 gives the difference $\beta_{\mathcal{G}_1} - \beta_{\mathcal{G}_0}$ for any split of the sources.

Proposition 2 (Source splits of the scale effect) *Under Assumption 1, split the sources into two disjoint groups $\mathcal{G}_1$ and $\mathcal{G}_0$ with positive spending on each, $D_i \equiv \sum_{c' \in \mathcal{G}_i} X^S_{f,c'} > 0$. Let $X^{\mathcal{G}}_{f,c} \equiv \sum_{c' \in \mathcal{G}_1 \cup \mathcal{G}_0} \omega_{cc'} X_{f,c}$ be destination c's spending on the compared sources, $\bar{g}_{f,c}$ the share of it going to $\mathcal{G}_1$, and $\bar{d}_{\mathcal{G}_i} \equiv \sum_{c' \in \mathcal{G}_i} d_{c'} X^S_{f,c'} \big/ D_i$ the groups' value-weighted dollar shares. Then*

$$\beta_{\mathcal{G}_1} - \beta_{\mathcal{G}_0} = \frac{\big(D_1 + D_0\big)^2}{D_1 D_0} \operatorname{Cov}^{X^{\mathcal{G}}}_c \Big(\bar{g}_{f,c},\, A_{f,c}\Big) \;-\; \rho_M \Big(\bar{d}_{\mathcal{G}_1} - \bar{d}_{\mathcal{G}_0}\Big), \tag{26}$$

where $\operatorname{Cov}^{X^{\mathcal{G}}}_c$ is the covariance across the firm's destinations under the weights $X^{\mathcal{G}}_{f,c} / \sum_d X^{\mathcal{G}}_{f,d}$, and

$$\operatorname{Cov}^{X^{\mathcal{G}}}_c \Big(\bar{g}_{f,c}, A_{f,c}\Big) = \nu(1-\alpha) \operatorname{Cov}^{X^{\mathcal{G}}}_c \Big(\bar{g}_{f,c}, \theta^{\$}_c\Big) + \Big[1 - \nu(1-\alpha)\Big] \rho_M \operatorname{Cov}^{X^{\mathcal{G}}}_c \Big(\bar{g}_{f,c}, \phi_{f,c} \bar{d}_{f,c}\Big).$$

Proof. In Appendix A.6. ■

Corollary 1 (The corridor comparison) *Let $\mathcal{G}_1$ collect the dollar-invoiced sources the firm exports to and $\mathcal{G}_0$ the dollar-invoiced sources it does not, with positive spending on each, and write $\beta^{1,1} \equiv \beta_{\mathcal{G}_1}$ and $\beta^{1,0} \equiv \beta_{\mathcal{G}_0}$. Then*

$$\beta^{1,1} - \beta^{1,0} = \frac{\big(D_1 + D_0\big)^2}{D_1 D_0} \operatorname{Cov}^{X^{\$}}_c \Big(\omega^{D\$}_{f,c},\, A_{f,c}\Big), \tag{27}$$

where D_1 and D_0 are the two groups' spending totals, $X^{\$}_{f,c} \equiv \bar{d}_{f,c} X_{f,c}$ is destination c's spending on dollar-invoiced sources, $\omega^{D\$}_{f,c}$ the share of that spending going to sources the firm exports to, and $\operatorname{Cov}^{X^{\$}}_c$ the covariance across the firm's destinations under the weights $X^{\$}_{f,c} / \sum_d X^{\$}_{f,d}$.

Proof. In Appendix A.6. ■

Corollary 2 (The denomination comparison) *Let $\mathcal{G}_1$ collect the dollar-invoiced sources and $\mathcal{G}_0$ the euro-invoiced ones, with positive spending on each, and write $\beta^{\$} \equiv \beta_{\mathcal{G}_1}$ and $\beta^{\epsilon} \equiv \beta_{\mathcal{G}_0}$. Then*

$$\begin{aligned} \beta^{\$} - \beta^{\epsilon} &= \frac{\left(S_{\$} + S_{\epsilon}\right)^2}{S_{\$} S_{\epsilon}} \operatorname{Cov}_c^X\left(\bar{d}_{f,c},\, A_{f,c}\right) \;-\; \rho_M, \\ S_{\$} &\equiv \sum_c \bar{d}_{f,c} X_{f,c}, \quad S_{\epsilon} \equiv \sum_c \left(1 - \bar{d}_{f,c}\right) X_{f,c}, \end{aligned} \tag{28}$$

where Cov_c^X is the covariance across the firm's destinations under the weights $X_{f,c}/\sum_d X_{f,d}$. If moreover $\bar{d}_{f,c} \in \{0,1\}$ for every destination and $\theta_c^{\$} = 0$ on every destination with $\bar{d}_{f,c} = 0$, then $\beta^{\epsilon} = 0$, and $\beta^{\$} > 0$ if and only if $\operatorname{Cov}_c^X(\bar{d}_{f,c}, A_{f,c}) > \rho_M\, \bar{w}(1-\bar{w})$, with $\bar{w} \equiv S_{\$}/(S_{\$} + S_{\epsilon})$.

Proof. In Appendix A.6. ∎

The corridor gap (27) vanishes exactly when $\operatorname{Cov}_c^{X^{\$}}(\omega_{f,c}^{D\$}, A_{f,c}) = 0$. In the decomposition, the dollar-pricing term carries the positive coefficient $\nu(1-\alpha)$ and the cost term a coefficient signed by whether $\nu(1-\alpha) \gtrless 1$; if the dollar cost intensity $\phi_{f,c}\bar{d}_{f,c}$ is common across destinations, the cost term drops. Hence, the equal responses that Section 5 estimates for corridor and non-corridor linked imports are a test of the covariance of the sourcing share $\omega_{f,c}^{D\$}$ with the scale-effect incentive $A_{f,c}$ of (22), not of the level of $\omega_{f,c}^{D\$}$. In particular, the firm's own export denomination does not appear either, as it drops out at reset prices.

In principle, the scale effect should be larger for euro-invoiced linked inputs. A euro depreciation makes dollar-invoiced inputs more expensive and leaves euro-invoiced ones alone, so on cost grounds the expansion should tilt toward the euro inputs, by the pass-through ρ_M. Corollary 2 says it can also be the other way. The two kinds of inputs need not serve the same markets. Sales expand where competitors price in dollars. If the dollar-invoiced inputs are the ones feeding those markets, demand pulls them up. The corollary states the gap accordingly: the dollar–euro difference equals the strength of this alignment minus the price penalty ρ_M. Corridor matching is what creates the alignment, since a matched firm buys in dollars on the corridors where it sells against dollar pricing. Under complete matching, the euro side shuts down entirely: a euro-invoiced input has a euro price, feeds destinations whose competitors also price in euros, and shares its bundle with no dollar-invoiced inputs, so the depreciation does not reach it from any side. Only the dollar-invoiced inputs then respond to the depreciation.

Finally, the model reproduces the scale effect being specific to linked imports. The linkage measures of Section 5 classify an input by whether it feeds the firm's exports. An unlinked input therefore feeds output the firm does not export, which is sold at home and euro-priced. That output's cost rises with a euro depreciation while the home index does not, so its relative price rises and its sales fall, giving a negative unlinked import response (this is given by (21) at $\bar{\theta}^{\$}_{f,c'} = 0$).

Remark 1 (Symmetric sourcing) *The corridor and non-corridor responses are equal whenever every destination splits its dollar-invoiced input spending in the same proportion between sources the firm exports to and its other sources, $\omega^{D\$}_{f,c} = \omega^{D\$}$ for every c with $\bar{d}_{f,c} > 0$: the covariance in (27) then has a constant argument and vanishes. This configuration leaves the own-corridor weights ω_{cc} unrestricted, so the estimated equality is consistent with the diagonal dominance of Assumption 2 below. It is likewise consistent with the hedging environment of Section 6.5, which restricts preferences and the corridor shock, not the sourcing split. It also leaves dollar pricing, the intermediate shares, and the dollar content of the bundles free to vary. A gap requires dispersion in $\omega^{D\$}_{f,c}$ itself, so the equality implies nothing about dollar pricing on its own.*

6.4 Corridor-level currency matching

Section 6.3 organized production by destination and left the sourcing weights $\omega_{cc'}$ unrestricted. We now derive the general loading of the desired price on the dollar, and show that corridor-level matching requires own-corridor sourcing to dominate, which we impose as Assumption 2.

Lemma 4 (The corridor loading) *In the model of Section 6.3, the desired price for destination c, expressed in euros, loads on the dollar with*

$$\Lambda_{f,c} = (1-\alpha)\,\rho_M\,\phi_{f,c}\sum_{c'}\omega_{cc'}\,d_{c'} + \alpha\,\theta^{\$}_{c}. \tag{29}$$

Sketch of proof. Repeat the proof of Lemma 2 with $mc_{f,c}$ of (17) in place of mc_f. Appendix A.4 sets out the steps. ■

This is the loading of Lemma 2, now determined by the destination-organized technology rather than by a single bundle, so (29) replaces (14) and the firm-aggregate share $\Phi^{\$}_f$ no longer enters. $d_{c'}$ above is the indicator that imports from c' are dollar-invoiced, and $\bar{d}_{f,c} = \sum_{c'}\omega_{cc'}d_{c'}$, the dollar content of destination c's input bundle, both from Section 6.3.

Under Lemma 4, the corridor's own sourcing changes the loading by $\partial\Lambda_{f,c}/\partial d_c = (1-\alpha)\rho_M\phi_{f,c}\omega_{cc}$ against $(1-\alpha)\rho_M\phi_{f,c}\omega_{cc'}$ for any other source country c'. Hence, the corridor's sourcing is dominant if the sourcing matrix features large diagonal elements. We impose that condition in Assumption 2 below.

Assumption 2 (Diagonal dominance) *The sourcing weights of* (16) *are diagonally dominant,* $\omega_{cc} > \omega_{cc'}$ *for every* $c' \neq c$.

Proposition 3 (Corridor matching) *In the model of Section 6.3, dollar invoicing on* c *attains the lower value of* (9) *if and only if* $\Lambda_{f,c} > \frac{1}{2}$, *where* $\Lambda_{f,c}$ *is the corridor loading* (29).

Proof. In Appendix A.4. ■

Finding 1 survives in this setup because it is a firm-level statistic. The firm trades on finitely many corridors, so corridor-level decisions pass through to firm-level aggregates. When imports from c switch to dollars, the destination-c loading (29) rises by $(1-\alpha)\rho_M\phi_{f,c}\omega_{cc}$, and when the rise clears the threshold the exports to c follow: at the firm level this is the co-movement of Finding 1. What comes closer to the data relative to the baseline is the cross-corridor pattern that accompanies it. In the data, a firm that begins invoicing its imports from c' in dollars does not change the currency of its exports to other destinations (Section 3.2), which the model reproduces when $\omega_{cc'}/\omega_{cc}$ is small.[28] Additionally, empirically, we find that a firm that begins invoicing its exports to c in dollars is less likely to begin invoicing its imports from other sources in dollars. On that the model is silent: import invoicing is a primitive here, not a decision, so there is nothing in it to respond. Delivering that pattern would require a general equilibrium extension including potentially limited substitutability on the sourcing side, which we do not pursue here.

6.5 Cash-flow hedging and corridor-specific risk

In this section, we further extend the model to explicitly incorporate risk preferences. This will allow it to speak to Finding 3, which the model so far was not equipped to do. Below, we lay out the details of the additional elements of the theory and then derive its predictions. The setup preserves all the results derived so far and is a strict extension of the model.

[28] The estimates of Section 3.2 put a number on how small: the cross-corridor coefficient is 0.003 of the same-corridor one, with a confidence band reaching approximately 0.07. Through the model, that ratio measures $\omega_{cc'}/\omega_{cc}$ for the corridors of firms that adopted the dollar elsewhere, the observations that identify the placebo coefficient. Appendix A.5 provides the derivations.

The hedging environment The firm ranks profit by the mean-variance criterion

$$\mathbb{E}[\Pi_f] \;-\; \frac{\gamma}{2}\,\mathrm{Var}(\Pi_f), \qquad \gamma \geq 0, \tag{30}$$

which replaces the expected-profit criterion that delivers (9).

As before, the firm chooses the price and the invoicing currency before shocks are realized. It now also chooses a dollar forward position at the same time, entered at (actuarially fair) zero cost. The forward rate is unbiased, so a notional h has the level payoff $h\big(e^s - \mathbb{E}[e^s]\big)$, which has mean zero by construction.

The corridor shock Let ξ_c be a corridor-specific real disturbance, a boom or a disruption in c's economy, normally distributed with mean zero and variance $\sigma^2_{\xi_c}$. It is independent of the dollar factor s and independent across corridors. It hits everything the firm has tied to c, raising the revenue on its exports to c and the cost of its imports from c. Specifically, on the export leg it shifts the demand schedule (11) within a price spell, so that, to second order in the firm's relative price, demand is

$$\hat{q}_{f,c} = \eta\,\xi_c - \nu\left(\hat{p}_{f,c} - \hat{p}_c\right) - \frac{\nu(\nu-1)\,\alpha}{2(1-\alpha)}\left(\hat{p}_{f,c} - \hat{p}_c\right)^2, \tag{31}$$

where hats denote log deviations from the symmetric point within the spell and $\eta \geq 0$ is the demand loading. The quadratic term is the order at which the elasticity of Υ moves with the relative price, which (48) ties to the complementarity α of (12). On the import leg, the country shock enters the supplier's price, expressed in euros:

$$\hat{p}^M_{f,c} = \kappa\,\xi_c + \rho_M\,d_c\,s, \tag{32}$$

where $\kappa \geq 0$ is the price loading.

Let $x_c \in \{0,1\}$ equal one if the firm presets its export price for destination c in dollars and zero if it presets it in euros. The firm's cash flow over one price spell is the sum of its destination profits, the domestic line's margin $\pi_{f,O}$, and the forward payoff:

$$\Pi_f = \sum_c \pi_{f,c} + \pi_{f,O} + h\big(e^s - \mathbb{E}[e^s]\big), \qquad \pi_{f,c} = R_{f,c} - TC_{f,c}, \tag{33}$$

with $R_{f,c}$ and $TC_{f,c}$ destination c's euro revenue and total cost over the spell. The lemma below expands Π_f to second order around the symmetric point, where destination revenue is $\bar{R}$, common across destinations, and the Kimball markup sets $\overline{TC} = \frac{\nu-1}{\nu}\bar{R}$.

Lemma 5 (Profit) *To second order in each shock, with $\Lambda_{f,c}$ the corridor loading (29), the mean and variance of the firm's cash flow are*

$$\mathbb{E}[\Pi_f] \;=\; -\frac{K}{2}\sum_c \Big(\Lambda_{f,c} - x_c\Big)^2 \sigma_s^2 \;+\; \mathbb{E}_f^{\circ}, \qquad K \;=\; \frac{(\nu-1)\,\bar{R}}{1-\alpha}, \tag{34}$$

$$\mathrm{Var}(\Pi_f) \;=\; \Big(\sum_c \beta_{f,c} + \beta_{f,O} + h\Big)^2 \sigma_s^2 \;+\; \sigma_s^2 \sum_{c'} \Psi_{f,c'}^2\, \sigma_{\xi_{c'}}^2 \;+\; 2\, G_f^2\, \sigma_s^4 \;+\; \mathrm{V}_f^{\circ}, \tag{35}$$

where

$$\begin{aligned} \beta_{f,c} &= \bar{R}\,\theta_c^{\$} - \rho_M\,\phi_{f,c}\overline{TC}\,\bar{d}_{f,c}, \\ \Psi_{f,c'} &= \mathbb{I}_{c'}\,\eta\,\beta_{f,c'} - \kappa\,\overline{TC}\sum_c \phi_{f,c}\,\omega_{cc'}\Big(\rho_M\,\phi_{f,c}\bar{d}_{f,c} - \nu(x_c - \theta_c^{\$})\Big) + \Psi_{f,c'}^{O}, \end{aligned} \tag{36}$$

with $\mathbb{I}_{c'}$ indicating that c' is one of the firm's destinations, and

$$G_f \;=\; \frac{h}{2} \;-\; \frac{K}{2}\sum_c \Big(\Lambda_{f,c} - x_c\Big)^2 \;+\; G_f^{\circ}. \tag{37}$$

The residuals $\mathbb{E}_f^{\circ}$, V_f°, and G_f°, the domestic exposure $\beta_{f,O}$, and the domestic term $\Psi_{f,c'}^{O}$ do not depend on the forward position h or on any export denomination x_c. For $\gamma > 0$ the optimal forward satisfies $\sum_c \beta_{f,c} + \beta_{f,O} + h^* = -G_f\,\sigma_s^2$.

Proof. In Appendix A.7. ∎

The two denominations do not enter the two exposures symmetrically. The export denomination is absent from $\beta_{f,c}$, as invoicing exports in dollars revalues euro revenue (by $\bar{R}$) and raises the firm's price relative to euro-pricing competitors when the dollar appreciates, losing ν times the margin $\bar{R}/\nu$: the two cancel exactly. What remains is a revenue exposure set by the destination competitors' dollar pricing rather than by the firm's own invoicing, held against the dollar-invoiced part of the input bill. The import leg is therefore the one that moves destination c's dollar position.

The interaction $\Psi_{f,c'}$ is the part of the dollar exposure scaled by the realized corridor shock, since a boom raises the volume carrying the denomination mismatch. It is affine in the export denomination, with slope

$$\frac{\partial \Psi_{f,c'}}{\partial x_c} = \nu\,\kappa\,\overline{TC}\,\phi_{f,c}\,\omega_{cc'} \;>\; 0, \tag{38}$$

for $\kappa > 0$, and the own-source term, $c' = c$, is the largest under diagonal dominance. The export denomination reaches $\Psi_{f,c'}$ only through κ: the export currency governs how far quantity moves with the dollar, quantity multiplies the intermediate bill, and the corridor shock moves the bill. The import denomination enters with the opposite sign, through $\bar{d}_{f,c'}$ in $\beta_{f,c'}$ and through $\bar{d}_{f,c}$ in the cost sum, so the two push $\Psi_{f,c'}$ in opposite directions and matching lets them offset.

At the optimal forward the aggregate dollar term is of order σ_s^6, since $\sum_c \beta_{f,c} + \beta_{f,O} + h^* = -G_f\sigma_s^2$ turns the first term of (35) into $G_f^2\sigma_s^6$, leaving

$$\mathrm{Var}(\Pi_f)\Big|_{h^*} \;=\; \sigma_s^2 \sum_{c'} \Psi_{f,c'}^2\, \sigma_{\xi_{c'}}^2 \;+\; 2\, G_f^2\, \sigma_s^4 \;+\; \mathrm{V}_f^{\circ} \;+\; O(\sigma_s^6). \tag{39}$$

The domestic line has no export denomination, so x_O is not a choice and matching runs on the export destinations alone.

Proposition 4 (Co-hedging) *Consider the model of Section 6.3 under Assumption 2, for $\gamma > 0$, at the optimal forward h^*, and maintaining*

$$\gamma\, \sigma_s^2 \Big(2G_f^{\circ} - \sum_c \beta_{f,c} - \beta_{f,O} \Big) \;<\; 1. \tag{40}$$

Write Ψ_0, Ψ_1 and G_0, G_1 for $\Psi_{f,c}$ and G_f at $x_c = 0$ and at $x_c = 1$. Then the following hold:

(i) Dollar invoicing on c is optimal if and only if $\Lambda_{f,c} > \Lambda_{f,c}^*$, where

$$\Lambda_{f,c}^* \;=\; \tfrac{1}{2} - \frac{\gamma}{2K_\gamma} \sum_{c'} \sigma_{\xi_{c'}}^2 \Big(\Psi_{f,c'}^2\Big|_{x_c=0} - \Psi_{f,c'}^2\Big|_{x_c=1} \Big), \qquad K_\gamma \;\equiv\; K\big[1 - \gamma\,(G_1 + G_0)\,\sigma_s^2\big], \tag{41}$$

where $\Lambda_{f,c}$ is the corridor loading (29). The $c' = c$ term of the sum is of order ω_{cc}, the remaining terms of order $\omega_{cc'}$.

(ii) Let corridor c move from mismatch to match, $x_c = 1 - d_c$ to $x_c = d_c$, with every other corridor held fixed. Then

$$\Delta\mathrm{Var}(\Pi_f) = \sigma_s^2 \sum_{c'} \Big(\Psi_{f,c'}^2\Big|_{x_c=d_c} - \Psi_{f,c'}^2\Big|_{x_c=1-d_c} \Big) \sigma_{\xi_{c'}}^2 \;+\; 2\,\sigma_s^2\,(G_1 + G_0)\,\Delta\mathbb{E}[\Pi_f], \tag{42}$$

where $\Delta\mathbb{E}[\Pi_f] = (2d_c - 1)K(\Lambda_{f,c} - \frac{1}{2})\sigma_s^2$. *The* $c' = c$ *term of the sum is*

$$\sigma_s^2\,\sigma_{\xi_c}^2\,(2d_c - 1)\Big(\Psi_1 + \Psi_0\Big)\,\frac{\partial \Psi_{f,c}}{\partial x_c}, \tag{43}$$

negative for $\kappa > 0$ *if and only if* $(1 - 2d_c)(\Psi_1 + \Psi_0) > 0$, *and the remaining terms of the sum are of order* $\omega_{cc'}$.

(iii) Let corridor c *move from mismatch to match,* $x_c = 1 - d_c$ *to* $x_c = d_c$, *with every other corridor held fixed. On the own corridor the two terms of* (42) *stand in the ratio*

$$\frac{2\,\sigma_s^2\,(G_1 + G_0)\,K\,(\Lambda_{f,c} - \frac{1}{2})}{\sigma_{\xi_c}^2\,(\Psi_1 + \Psi_0)\,\partial\Psi_{f,c}/\partial x_c}, \tag{44}$$

which is zero at $\Lambda_{f,c} = \frac{1}{2}$ *and of order* κ^{-1} *as* $\kappa \to 0$ *whenever* $\eta\,\beta_{f,c} \neq 0$, *and of order* κ^{-2} *when that product vanishes.*

Proof. In Appendix A.7. ■

Condition (40) bounds risk aversion relative to the variance of the dollar factor. Its left side carries no endogenous object: G_f° does not depend on h or on any x_c, and by Lemma 5 neither $\beta_{f,c}$ nor $\beta_{f,O}$ depends on any x_c, so the aggregate exposure is the same under every configuration of export denominations. Appendix A.7 shows that the bound implies $K_\gamma > 0$.

Intuitively, risk aversion reaches the cutoff of part (i) only through the corridor shock: it leaves the price-stability comparison of Proposition 3 in place and adds the cash-flow term of (41) beside it. That term is absent at $\gamma = 0$ and at $\kappa = 0$, where the cutoff is $\frac{1}{2}$. When it is present and $(1 - 2d_c)(\Psi_1 + \Psi_0) > 0$, the import denomination and the corridor shock move the two sides of the comparison in the same direction. On a dollar-import corridor, dollar imports raise $\Lambda_{f,c}$ in (29) and the shock lowers $\Lambda_{f,c}^*$ below one half through the corridor-c term of the sum, the remaining terms being of order $\omega_{cc'}$; on a euro-import corridor both move the other way. Either way the export leg is pushed toward the import leg's currency, and the push grows with $\sigma_{\xi_c}^2$.

The extension nests the baseline. Setting $\eta = \kappa = 0$ removes the corridor shock, so the invoicing enters the objective through G_f alone. The objective is increasing in G_f whenever $2\gamma G_f \sigma_s^2 < 1$, which is $K_\gamma > 0$ written out, so expected profit and (30) rank every configuration the same way, $\Lambda_{f,c}^*$ returns to one half, and Proposition 3 is recovered (Appendix A.7). The firm still weights variance, since $\gamma > 0$: the two objectives differ, but their predictions coincide.

The two terms of (42) in part (ii) arise for different reasons. The first is the corridor's own currency risk. The firm's dollar exposure is not the same in every state: a boom in c' raises both the volume the firm sells there and the bill it pays there, so the exposure is largest when the corridor is busiest, and $\Psi_{f,c'}$ is that state dependence. The forward is fixed before $\xi_{c'}$ is known and pays on s alone, so it is the same size in every state, while matching the corridor's two legs moves with the corridor and reduces the exposure where it is largest. Whether matching does reduce this term is the sign condition of (43), which Appendix A.7 bounds for each import denomination.

The second term is the variance that comes with moving expected profit. The misalignment $-\frac{K}{2}\sum_c(\Lambda_{f,c}-x_c)^2$, with $\Lambda_{f,c}$ the corridor loading (29), does two jobs at once: it lowers expected profit in (34), and it is the coefficient on s^2 in (37), the curvature of profit in the dollar, which carries variance of its own, the $2G_f^2\sigma_s^4$ of (35). A change in the misalignment therefore moves the mean and the variance together, the second by $2\sigma_s^2(G_1+G_0)$ times $\Delta\mathbb{E}[\Pi_f]$: matching raises the variance through this route where $(G_1+G_0)\,\Delta\mathbb{E}[\Pi_f]>0$ and lowers it where that product is negative. The misalignment is a pricing object, built from the two denominations and the forward, so this term operates at every κ, including $\kappa=0$.

The ratio (44) of part (iii) says which term dominates, the misalignment where its absolute value exceeds one and the interaction where it falls short. It is zero at a corridor loading (29) of $\Lambda_{f,c}=\frac{1}{2}$, where the mean does not move and matching acts through the interaction alone.[29] It is inversely proportional to $\sigma^2_{\xi_c}$, so the interaction dominates on volatile corridors and the misalignment on quiet ones. And it is of order κ^{-1} as $\kappa\to 0$, since $\partial\Psi_{f,c}/\partial x_c$ vanishes with κ while $\Psi_1+\Psi_0$ tends to $2\eta\,\beta_{f,c}$, which is nonzero unless the corridor carries no net dollar exposure: with no pass-through of the corridor shock into input prices there is nothing to co-hedge, and matching moves the mean alone.

The extended model against the findings The model now delivers all four findings, with the one exception on which the model is silent: the negative import-side placebo. Finding 1 survives the corridor structure as a firm-level statistic: dollar imports from c raise the destination-c loading (29) by $(1-\alpha)\rho_M\phi_{f,c}\omega_{cc}$, and the exports to c follow once that clears the threshold (Section 6.4). Finding 2 arrives twice, from the corridor-c desired price of Proposition 3 and again from part (i) here, each pushing the

[29] To the order of Lemma 5's expansion. For $\kappa>0$, third-order terms of the exact window flow contribute to the variance at the same $\sigma_s^2\sigma^2_{\xi_c}$ order as the interaction term and do not vanish at $\Lambda_{f,c}=\frac{1}{2}$, so the threshold shift in (41) is determined up to truncation terms of that order.

export leg toward the import leg's currency on the same corridors, the cross-corridor terms entering at order $\omega_{cc'}$. Finding 3 is the interaction term of (39): the residual variance that Section 4 regresses on matching quality is $\mathrm{Var}(\Pi_f)$ at the optimal forward, and matching lowers that term corridor by corridor under the condition of Proposition 4(ii). Finding 4(b) is the scale effect of Lemma 3 and Proposition 2, which lives at the reset-price margin and so stands apart from the preset window this subsection works in.

7 Conclusion

This paper shows that the invoicing currency is an active margin through which internationally engaged firms manage risk. Using the universe of Italian customs declarations matched to balance-sheet data, we document that firms dynamically match the dollar invoicing of their imports and exports, with adoption typically starting on the import side, and that the matching is organized at the level of the bilateral corridor: dollar adoption on one side of a corridor predicts adoption on the other side of that same corridor and not elsewhere. Corridor matching pays, as it reduces the residual profit variance beyond what the firm's aggregate net dollar position explains, while leaving the balance-sheet revaluation component untouched. Exchange-rate movements transmit to profits mainly through the operational margin, and a euro depreciation raises dollar-invoiced import quantities, with the increase concentrated among inputs linked to the firm's export basket, the scale effect on linked imports.

A model of invoicing currency choice with a rich sourcing and destination structure accounts for these patterns. Two elements of the environment do the work: the own-corridor weight in each destination's input mix, which delivers the corridor matching and, aggregated across corridors, the firm-level co-movement; and the dollar pricing of competitors relative to the firm's own dollar costs, which delivers the scale effect. Extending the model to a mean-variance objective with country-specific shocks delivers the variance reduction: matching the invoicing currencies of the two legs of a corridor offsets exposures that neither aggregate netting nor currency derivatives can reach. In this framework, invoicing currency choice is a real hedge against country risk, not only against currency risk.

These findings carry implications for the measurement of corporate exchange-rate exposure. Aggregate net foreign-currency positions overlook both the real hedges built within bilateral corridors and the production responses that exchange-rate movements set off, so information on invoicing currencies and on the country composition of a firm's

trade matters for assessing its vulnerability to external shocks. Our analysis also treats the invoicing currency as a one-sided choice, while in practice it is negotiated between two parties whose preferred currencies need not coincide. Studying how bargaining determines the chosen currency, and whether firms with greater bargaining power align their invoicing across imports and exports more fully, is a natural direction for future research.

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Online Appendix of

Corridor Invoicing: Real Hedging in International Trade

by Brancati E., Brianti M., Nosal J., and Schiantarelli F.

A Theoretical appendix: derivations for Section 6

This appendix derives the results stated in Section 6: the variance criterion (Lemma 1), the complementarity α, the dollar loadings (Lemmas 2 and 4) and corridor matching (Proposition 3), the placebo bound, the scale effect (Lemma 3, Proposition 2, and Corollaries 1 and 2), and the cash-flow hedging results (Lemma 5 and Proposition 4). The general-equilibrium closure (household optimization, asset markets, and the determination of exchange rates) is that of Mukhin (2022) and is taken as given.

A.1 The firm's problem and the variance criterion

Let $\Pi_{f,c}(p)$ be the firm's profit from sales to c as a function of the destination-currency price p, measured in real discounted units so that it carries the stochastic discount factor $\mathcal{M}$. With constant returns the problem is separable across destinations, and the flexible-price optimum

$$\tilde{p}_{f,c} = \arg\max_{p} \; \mathcal{M}\,\Pi_{f,c}(p) \tag{45}$$

holds state by state. Writing $\bar{p}^k$ for the preset price $\bar{p}^k_{f,c}$ of the main text, the firm's problem at a readjustment in period t is

$$\max_{k,\;\bar{p}^k} \; \mathbb{E}_t \sum_{j\geq 0} \lambda^j \,\mathcal{M}_{t,t+j}\,\Pi_{f,c}\Big(\bar{p}^k + e_{c,k,t+j}\Big), \tag{46}$$

with λ^j the probability that the price still holds j periods later, and exchange-rate shocks are permanent, $\mathbb{E}_t\, s_{t+1} = s_t$.

Lemma A.1 (Reduction to the one-period problem) *Under permanent shocks, the currency that minimizes the one-period expected loss from presetting, to second order in the gap $\bar{p}^k + e_{c,k} - \tilde{p}_{f,c}$, minimizes the discounted sum in (46).*

Proof. With permanent shocks every readjustment spell presents the same problem, and the loss from $\bar{p}^k + e_{c,k}$ straying from the desired price is second order in their gap in every period; the survival weights λ^j only discount these per-period gaps, so the ranking of currencies by the one-period loss is the ranking by the discounted sum. ■

Proof of Lemma 1. By Lemma A.1 it suffices to consider one period. Expand $\mathcal{M}\,\Pi_{f,c}(p)$ around the flexible optimum $\tilde{p}_{f,c}$ state by state:

$$\begin{aligned}\mathcal{M}\,\Pi_{f,c}(p) = \mathcal{M}\,\Pi_{f,c}(\tilde{p}_{f,c}) + \mathcal{M}\,\Pi'_{f,c}(\tilde{p}_{f,c})\,(p-\tilde{p}_{f,c}) \\ + \tfrac{1}{2}\,\mathcal{M}\,\Pi''_{f,c}(\tilde{p}_{f,c})\,(p-\tilde{p}_{f,c})^2 + O\Big((p-\tilde{p})^3\Big).\end{aligned}$$

By (45), $\Pi'_{f,c}(\tilde{p}_{f,c}) = 0$ in every state, so the linear term vanishes. The expected loss from presetting is therefore $\frac{1}{2}\,\mathbb{E}\Big[\mathcal{M}\,|\Pi''_{f,c}(\tilde{p}_{f,c})|\,(p-\tilde{p}_{f,c})^2\Big]$. To leading order the curvature $\mathcal{M}\,|\Pi''_{f,c}|$ equals its mean times a term of order one, and the covariance between its innovation and $(p-\tilde{p})^2$ is of higher order; the variation in $\mathcal{M}$ thus drops out of the leading term. Minimizing the expected loss is then equivalent to

$$\min_{\bar{p}^k}\ \mathbb{E}\Big[(\bar{p}^k + e_{c,k} - \tilde{p}_{f,c})^2\Big].$$

The optimal constant is $\bar{p}^k = \mathbb{E}[\tilde{p}_{f,c} - e_{c,k}] = \mathbb{E}[\tilde{p}_{f,c} + e_{k,c}]$, using $e_{c,k} = -e_{k,c}$, and the minimized value is $\mathrm{Var}(\tilde{p}_{f,c} - e_{c,k}) = \mathrm{Var}(\tilde{p}_{f,c} + e_{k,c})$. ■

The curvature of the expected loss, $K \equiv \mathbb{E}\Big[\mathcal{M}\,|\Pi''_{f,c}(\tilde{p}_{f,c})|\Big] > 0$, is the constant that carries into the invoicing threshold of Section 6.5. Under the mean-variance criterion (30) the mean carries no stochastic discount factor, and the same constant reads $K = \mathbb{E}\Big[|\Pi''_{f,c}(\tilde{p}_{f,c})|\Big] > 0$ up to a constant factor.

Lemma A.2 (The readjusted price) *Under permanent shocks, for either invoicing currency k, the price posted at a readjustment, expressed in the destination currency, responds to the dollar factor as the desired price does,*

$$\frac{d\Big(\bar{p}^k_t + e_{c,k,t}\Big)}{ds} = \frac{d\tilde{p}_{f,c}}{ds}, \tag{47}$$

and neither (47) nor the criterion of Lemma 1 involves λ.

Proof. The first-order condition for $\bar{p}^k$ in (46) sets it to a discounted average, over the expected life of the price, of the desired prices expected to prevail, each expressed in currency k. With permanent shocks every such desired price responds to the current shock as today's does, so $d\bar{p}^k_t/ds = d(\tilde{p}_{f,c} + e_{k,c})/ds$, and adding $e_{c,k,t}$ gives (47). The survival weights enter only as averaging weights, which cancel from the derivative, and the one-period ranking of Lemma A.1 does not involve them. ■

The comparative statics at a readjustment, the pass-through and the scale response of Lemma 3, are therefore those of the desired price, which is how Appendix A.6 computes them; to the extent that the annual horizon of Section 5 exceeds the mean price duration $1/(1-\lambda)$, the estimates speak to this readjusted response. The independence of λ is a partial-equilibrium property: the pass-throughs $\theta^{\$}_c$ and ρ_M are primitives here, whereas in general equilibrium they would carry the stickiness of competitors and suppliers.

A.2 The complementarity α

Write $\varrho = P_{f,c}/P_c$ for the firm's relative price, $\epsilon(\varrho) = -\varrho\,\Upsilon'(\varrho)/\Upsilon(\varrho)$ for the demand elasticity implied by (11), so that $\epsilon(1) = \nu$ at the symmetric point, $\sigma_\epsilon \equiv d\log\epsilon/d\log\varrho$ at $\varrho = 1$ for the superelasticity of demand, and $\Gamma \equiv d\log\Big[\epsilon/(\epsilon-1)\Big]/d\log\varrho$ at $\varrho = 1$ for the elasticity of the markup to the relative price.

Lemma A.3 (The complementarity) *Under Kimball demand, the log-linearized flexible price is* (12) *with the marginal cost* (10) *and*

$$\alpha \;=\; \frac{-\Gamma}{1-\Gamma} \;=\; \frac{\sigma_\epsilon}{\nu - 1 + \sigma_\epsilon}, \tag{48}$$

and $\alpha \in (0,1)$.

Proof. The first-order condition for the flexible price is the markup rule $P_{f,c} = \Big[\epsilon(\varrho)/(\epsilon(\varrho)-1)\Big]\,MC$, in which the markup moves with the relative price, and the euro wage $w_\text{€}$ is orthogonal to s. Log-linearizing around $\varrho = 1$ gives

$$\hat{p}_{f,c} = \widehat{mc} + \Gamma\left(\hat{p}_{f,c} - \hat{p}_c\right), \qquad \Gamma \equiv \left.\frac{d\log\Big[\epsilon/(\epsilon-1)\Big]}{d\log\varrho}\right|_{\varrho=1} = -\,\frac{\sigma_\epsilon}{\nu-1},$$

and solving for $\hat{p}_{f,c}$ returns (12) with the α of (48). Since $\sigma_\epsilon > 0$ and $\nu > 1$ under Kimball demand, $\alpha \in (0,1)$. ■

Under constant elasticity $\sigma_\epsilon = 0$, so $\alpha = 0$ and the firm prices off its own cost alone; as σ_ϵ grows the elasticity responds more sharply to the relative price, the markup absorbs more of any cost movement, and $\alpha \to 1$.

A.3 The dollar loading

The derivation needs the loading of destination c's own currency on the dollar factor, which we take to be

$$e_{\text{€},c} = a_{\$\to c}\,s + (\text{terms} \perp s), \qquad a_{\$\to c} \in [0,1], \tag{49}$$

the destination's dollar basket weight, one for a dollar peg and zero for a currency that moves with the euro.

Remark A.1 (Basket-weight irrelevance, partial equilibrium) *With inputs and competitors invoiced only in dollars or euros, the destination's dollar basket weight* $a_{\$\to c}$

cancels from every relative price, and hence from the loadings (14) *and* (29) *and the scale elasticity* (21)*. Currency choice and the scale response depend on dollar-versus-euro denomination alone, not on how tightly c's currency tracks the dollar.*

For any log variable v write $[v] \equiv \partial v/\partial s$ for its loading on the dollar factor, so that $v = [v]\, s + (\text{terms} \perp s)$ and loadings are additive across the linear expressions (12) and (10).

Proof of Lemma 2. The object is the loading of the desired price expressed in euros, $\tilde{p}_{f,c} + e_{€,c}$, which enters the invoicing comparison.

Step 1: loadings of the primitives. Under Assumption 1,

$$[w_€] = 0, \qquad [p_f^M] = \rho_M \frac{\Phi_f^\$}{\Phi_f},$$
$$[e_{€,c}] = a_{\$\to c}, \quad [e_{c,€}] = -a_{\$\to c}, \quad [p_c] = \theta_c^\$ (1 - a_{\$\to c}) + (1 - \theta_c^\$)(-a_{\$\to c}) = \theta_c^\$ - a_{\$\to c}.$$

The wage is in euros; the dollar-invoiced share of the bundle loads ρ_M and the euro-invoiced share zero; in the destination index, dollar-pricing competitors load $1 - a_{\$\to c}$ in c's currency and euro-pricing competitors $-a_{\$\to c}$.

Step 2: marginal cost. Substituting into (10),

$$[mc_f] = (1 - \Phi_f)\,[w_€] + \Phi_f\,[p_f^M] = \Phi_f \cdot \rho_M \frac{\Phi_f^\$}{\Phi_f} = \rho_M\, \Phi_f^\$, \tag{50}$$

the firm's dollar-invoiced input share; $a_{\$\to c}$ does not enter, since the euro cost of a dollar-priced input is governed by s alone.

Step 3: desired price in the destination currency. Substituting (50) and the primitive loadings into (12),

$$\begin{aligned}[\tilde{p}_{f,c}] &= (1-\alpha)\Big([mc_f] + [e_{c,€}]\Big) + \alpha\,[p_c] \\ &= (1-\alpha)\Big(\rho_M\, \Phi_f^\$ - a_{\$\to c}\Big) + \alpha\Big(\theta_c^\$ - a_{\$\to c}\Big) \\ &= (1-\alpha)\,\rho_M\, \Phi_f^\$ + \alpha\,\theta_c^\$ - a_{\$\to c}. \end{aligned} \tag{51}$$

Step 4: expressed in euros. Adding the conversion $e_{€,c}$, whose loading is $[e_{€,c}] = a_{\$\to c}$, the destination-currency term $-a_{\$\to c}$ cancels:

$$\Lambda_{f,c} \equiv [\tilde{p}_{f,c} + e_{€,c}] = (1-\alpha)\,\rho_M\, \Phi_f^\$ + \alpha\,\theta_c^\$. \tag{52}$$

which is the loading (14) of the main text. ■

If a local-currency competitor share θ_c^c is present, $[p_c] = \theta_c^{\$} - a_{\$\to c}(1-\theta_c^c)$ and the loading becomes $(1-\alpha)\rho_M\Phi_f^{\$} + \alpha(\theta_c^{\$} + \theta_c^c a_{\$\to c})$, the effective dollar share $\tilde{\theta}_c^{\$}$.

A.4 Corridor-level costs and Proposition 3

Proof of Lemma 4. With the destination-organized technology (15) in place of (8), the marginal cost of the goods sold to c is (17), drawing on all sources with the weights $\omega_{cc'}$. Repeating Steps 1–4 of Appendix A.3 with $mc_{f,c}$ in place of mc_f, and noting that $[mc_{f,c}] = \rho_M\phi_{f,c}\sum_{c'}\omega_{cc'}d_{c'} = \rho_M\phi_{f,c}\bar{d}_{f,c}$, the euro desired price for corridor c loads on the dollar with

$$\Lambda_{f,c} = (1-\alpha)\,\rho_M\,\phi_{f,c}\,\bar{d}_{f,c} + \alpha\,\theta_c^{\$}, \tag{53}$$

where $\bar{d}_{f,c} \equiv \sum_{c'}\omega_{cc'}d_{c'}$ is the dollar content of corridor c's input bundle; the firm-aggregate share $\Phi_f^{\$}$ has dropped out, and this is the corridor loading (29) of the main text. Assumption 2 enters none of these steps. ■

Proof of Proposition 3. The proof of Proposition 1 applies with (53) in place of (14). ■

A.5 The cross-corridor placebo under general sourcing

This appendix derives the placebo bound cited in Section 6.4. The corridor loading (29) has sensitivity $\partial\Lambda_{f,c}/\partial d_{c'} = (1-\alpha)\rho_M\phi_{f,c}\omega_{cc'}$ to sourcing elsewhere, which vanishes for every $c' \neq c$ only at $\omega_{cc} = 1$. The placebo of Section 3.2 regresses dollar export adoption on corridor c on dollar import adoption on the same corridor and on the firm's other corridors; the outcome is discrete, so the coefficients are crossing masses. With $\gamma > 0$ the corridor-c decision compares $\Lambda_{f,c}$ with the threshold (41), so the crossing variable is the gap $\tau_{f,c} \equiv \Lambda_{f,c} - \Lambda_{f,c}^*$ with boundary $\tau_{f,c} = 0$; let F be its distribution across firm-corridor cells and f the density.

Lemma A.4 (The placebo ratio) *Holding K_γ fixed, for $\kappa > 0$,*

$$\frac{\partial\tau_{f,c}}{\partial\bar{d}_{f,c}} = (1-\alpha)\,\rho_M\,\phi_{f,c} + \frac{\gamma\,\sigma_{\xi_c}^2}{K_\gamma}\,V_{f,c}\,|Z_{f,c}| \;\equiv\; g_{f,c} \;>\; 0, \tag{54}$$

with $V_{f,c} > 0$ and $Z_{f,c} < 0$ the slopes of (63) below. Same-corridor adoption shifts the gap by $\Delta_{\text{same}} = g_{f,c}\,\omega_{cc}$ and adoption on c' by $\Delta_{\text{plac}} = g_{f,c}\,\omega_{cc'}$ plus a nonnegative

correction of the same order that vanishes at $\gamma = 0$, and, to first order in the off-diagonal and domestic sourcing weights,

$$\frac{\textit{placebo coefficient}}{\textit{same-corridor coefficient}} = \frac{\omega_{cc'}}{\omega_{cc}} \cdot \frac{f(0)}{\bar{f}}, \qquad \bar{f} \equiv \frac{1}{\Delta_{\text{same}}} \int_{-\Delta_{\text{same}}}^{0} f(\tau)\, d\tau, \tag{55}$$

with equality at $\gamma = 0$ or $\kappa = 0$ and the left side larger otherwise.

Proof. Both terms of the gap move with $\bar{d}_{f,c}$: the loading through the markup channel $(1-\alpha)\rho_M \phi_{f,c}$, and the threshold through its own-corridor term, which Lemma A.10(iii) gives as $(\gamma\sigma^2_{\xi_c}/2K_\gamma)V_{f,c}(2J_{f,c}+2Z_{f,c}\bar{d}_{f,c}+V_{f,c})$, so that $\partial\Lambda^*_{f,c}/\partial\bar{d}_{f,c} = (\gamma\sigma^2_{\xi_c}/K_\gamma)V_{f,c}Z_{f,c} < 0$; the dependence of K_γ on $\bar{d}_{f,c}$ multiplies a term already of order γ and contributes at order γ^2. Same-corridor adoption moves $\bar{d}_{f,c}$ by ω_{cc} and adoption on c' by $\omega_{cc'}$, which gives the two shifts, the correction collecting the threshold's remaining routes, which are of the same order as $\omega_{cc'}$ and vanish at $\gamma = 0$. The placebo shift is small, so its crossing mass is $f(0)\,\Delta_{\text{plac}}$ to first order, while the same-corridor mass is the integral of f over the band of width Δ_{same}; cells away from the boundary contribute nothing, and the base rate of switching cancels in each coefficient, since each is a difference. Dividing gives (55). At $\gamma = 0$ or $\kappa = 0$ the slope $g_{f,c}$ cancels exactly; otherwise the omitted routes raise the left side. ■

On the export side, Section 3.2 reports a same-corridor coefficient of 1.486 and a placebo of 0.00447 (standard error 0.0507): the point ratio is 0.003, and the upper end of the placebo's confidence band puts it at approximately 0.07. Overturning the conclusion that the off-diagonal weights are small would require $f(0)$ to fall short of $\bar{f}$ by two orders of magnitude. Two caveats apply. After pooling, the estimated ratio is a density-weighted average of the cell-level ratios in which $\sigma^2_{\xi_c}$ re-enters through the weights, and with heterogeneity in $\phi_{f,c}$ and the weights across cells it recovers a weighted average of $\omega_{cc'}/\omega_{cc}$. And the empirical placebo regressor indicates dollar import adoption on at least one other country, so the shift it induces is $\sum_{c' \in \mathcal{A}} \omega_{cc'}$ over the adopting set $\mathcal{A}$: the ratio bounds that partial row sum, every weight in the adopting set at once, and falls short of the row total $1-\omega_{cc}$ whenever corridor c draws on a source outside the adopting set.

A.6 The scale effect: Lemma 3, Proposition 2, and its corollaries

Production is organized by destination as in (15), destination c draws the bundle (16), $d_{c'}$ indicates that imports from c' are dollar-invoiced, and $[v] = \partial v/\partial s$ is the loading

notation of Appendix A.3. Throughout we differentiate the desired price $\tilde{p}_{f,c}$, which by (47) has the same derivative in s as the price the firm posts at a readjustment.

Proof of Lemma 3. *Bundle price and marginal cost, destination by destination.* With the Cobb–Douglas weights of (16), the log price of destination c's bundle is $\bar{p}^M_{f,c} = \sum_{c'} \omega_{cc'}\, p^M_{f,c'}$. A dollar-invoiced source loads $[p^M_{f,c'}] = \rho_M$ by Assumption 1(i); a euro-invoiced source loads zero. Hence $[\bar{p}^M_{f,c}] = \rho_M\, \bar{d}_{f,c}$ and $[mc_{f,c}] = \phi_{f,c}\,[\bar{p}^M_{f,c}] = \rho_M\, \phi_{f,c}\, \bar{d}_{f,c}$.

Prices and quantities by destination. Repeating the steps of Appendix A.3 with $[mc_{f,c}] = \rho_M \phi_{f,c} \bar{d}_{f,c}$, the common $a_{\$\to c}$ cancels from each destination's relative price,

$$[\tilde{p}_{f,c} - p_c] = (1-\alpha)\,(\rho_M \phi_{f,c} \bar{d}_{f,c} - \theta^{\$}_c), \qquad [q_{f,c}] = \nu(1-\alpha)\,(\theta^{\$}_c - \rho_M \phi_{f,c} \bar{d}_{f,c}).$$

Input demand by source. Under Cobb–Douglas and cost minimization, the euro bill of source c' within destination c is a fixed share of that destination's cost, $p^M_{f,c'} M_{f,cc'} = \omega_{cc'}\, \phi_{f,c}\, mc_{f,c}\, q_{f,c} = \omega_{cc'} X_{f,c}$ in the input spending of (18). Summing over the destinations that use the input and log-differentiating,

$$[M_{f,c'}] = \sum_c s^{c'}_{f,c} \Big([mc_{f,c}] + [q_{f,c}] \Big) - [p^M_{f,c'}],$$

with the weights $s^{c'}_{f,c}$ of (19), which sum to one over c. The bracket equals $\nu(1-\alpha)\theta^{\$}_c + \Big(1 - \nu(1-\alpha)\Big)\rho_M \phi_{f,c} \bar{d}_{f,c}$, affine in $(\theta^{\$}_c, \phi_{f,c}\bar{d}_{f,c})$. A weighted average of an affine function is that function of the weighted averages, so for a dollar-invoiced source,

$$[M^{\$}_{f,c'}] = \rho_M \bar{\Phi}^{\$}_{f,c'} + \nu(1-\alpha)(\bar{\theta}^{\$}_{f,c'} - \rho_M \bar{\Phi}^{\$}_{f,c'}) - \rho_M = \nu(1-\alpha)(\bar{\theta}^{\$}_{f,c'} - \rho_M \bar{\Phi}^{\$}_{f,c'}) - \rho_M(1 - \bar{\Phi}^{\$}_{f,c'}),$$

which is (21). The source c' survives only through the weights $s^{c'}_{f,c}$, so it drops out whenever those weights are common across sources, and separately whenever the bracket $A_{f,c}$ is itself common across destinations, which two sources with the same destination mix share by construction. By (19) the first case holds exactly when $\omega_{cc'}$ factorizes into a destination term and a source term, hence, after row normalization, when sourcing is common across destinations. ■

Lemma A.5 (Unlinked inputs) *Let $M^{O\$}_{f,c'}$ be the domestic line's demand for a dollar-invoiced input from c' and $\Phi^{\$O}_f \equiv \phi^O_f \bar{d}^O_f$ the line's dollar cost intensity. Then*

$$[M^{O\$}_{f,c'}] = \nu(1-\alpha)\Big(\theta^{\$}_H - \rho_M \Phi^{\$O}_f\Big) - \rho_M\Big(1 - \Phi^{\$O}_f\Big),$$

and under Assumption 1(iii)

$$[M^{O\$}_{f,c'}] = -\rho_M\left(1-\Phi^{\$O}_f\right) - \nu(1-\alpha)\,\rho_M\Phi^{\$O}_f = -\rho_M\left[1+\Phi^{\$O}_f\left(\nu(1-\alpha)-1\right)\right] < 0.$$

Proof. The steps of the proof of Lemma 3 apply with the home market H in place of c and $[mc^O_f] = \rho_M\Phi^{\$O}_f$, which gives the first display, (21) at $\bar\theta^{\$}_{f,c'} = \theta^{\$}_H$ and $\bar\Phi^{\$}_{f,c'} = \Phi^{\$O}_f$. Assumption 1(iii) sets $\theta^{\$}_H = 0$, and the bracket is positive since $\Phi^{\$O}_f \in [0,1]$ and $\nu(1-\alpha) > 0$. ■

This is the unlinked prediction of Section 6.3.

Lemma A.6 (Monotonicity and the euro-invoiced comparison) *(i)* $\partial[M^{\$}_{f,c'}]/\partial\bar\theta^{\$}_{f,c'} = \nu(1-\alpha) > 0$.

(ii) For a euro-invoiced and a dollar-invoiced source that share a destination mix, $[M^{\epsilon}_{f,c'}] = [M^{\$}_{f,c'}] + \rho_M$.

(iii) (21) *is positive if and only if* $\bar\theta^{\$}_{f,c'} > \rho_M\bar\Phi^{\$}_{f,c'} + \rho_M(1-\bar\Phi^{\$}_{f,c'})/[\nu(1-\alpha)]$.

Proof. Part (i) differentiates (21). For part (ii), a euro-invoiced source has $[p^M_{f,c'}] = 0$ in the input-demand step of the proof of Lemma 3, so it carries the same demand expansion without the own-price rise; across sources the comparison holds up to differences in $s^{c'}_{f,\cdot}$. Part (iii) rearranges (21). ■

Proof of Proposition 2. Destination c spends $\omega_{cc'}X_{f,c}$ on source c', so by (24) the source weight is $s^{c'}_{f,c} = \omega_{cc'}X_{f,c}/X^S_{f,c'}$. The derivation of Lemma 3 carries a source of either denomination, with $[p^M_{f,c'}] = \rho_M d_{c'}$, which is (23). Multiplying by $X^S_{f,c'}$, $X^S_{f,c'}\,\partial \ln M_{f,c'}/\partial s = \sum_c \omega_{cc'}X_{f,c}A_{f,c} - \rho_M d_{c'}X^S_{f,c'}$. Summing over $c' \in \mathcal{G}_1$ collapses the sourcing weights, $\sum_{c'\in\mathcal{G}_1}\omega_{cc'}X_{f,c} = \bar g_{f,c}X^{\mathcal{G}}_{f,c}$, so $\beta_{\mathcal{G}_1} = \sum_c \bar g_{f,c}X^{\mathcal{G}}_{f,c}A_{f,c}/D_1 - \rho_M\bar d_{\mathcal{G}_1}$, and likewise $\beta_{\mathcal{G}_0} = \sum_c(1-\bar g_{f,c})X^{\mathcal{G}}_{f,c}A_{f,c}/D_0 - \rho_M\bar d_{\mathcal{G}_0}$. The two demand parts are weighted means of the same $A_{f,c}$ with weights summing to $X^{\mathcal{G}}_{f,c}$, and differencing them leaves the covariance term of (26); the substitution parts contribute $-\rho_M(\bar d_{\mathcal{G}_1} - \bar d_{\mathcal{G}_0})$. The decomposition follows from (22) and the bilinearity of the covariance. ■

Proof of Corollary 1. Both groups are dollar-invoiced, so $\bar d_{\mathcal{G}_1} = \bar d_{\mathcal{G}_0} = 1$ and the substitution term cancels. The compared spending is $X^{\mathcal{G}}_{f,c} = \sum_{c'}\omega_{cc'}d_{c'}X_{f,c} = \bar d_{f,c}X_{f,c} = X^{\$}_{f,c}$, and the group-one share of it is $\omega^{D\$}_{f,c}$. ■

Proof of Corollary 2. The groups exhaust the sources, so $X^{\mathcal{G}}_{f,c} = X_{f,c}$ and $\bar g_{f,c} = \bar d_{f,c}$, and the groups' dollar shares are one and zero, which gives (28). Under the condition of the second part, a euro-invoiced source feeds only destinations with $\bar d_{f,c} = 0$ and hence

$\theta_c^{\$} = 0$, so $A_{f,c} = 0$ on every destination it feeds by (22) and $\beta^{\epsilon} = 0$; (28) then reads $\beta^{\$} = \mathrm{Cov}_c^X(\bar{d}_{f,c}, A_{f,c}) \big/ [\bar{w}(1-\bar{w})] - \rho_M$, positive exactly under the stated bound. ■

A.7 Cash-flow hedging: Lemma 5 and Proposition 4

This appendix proves Lemma 5, states four auxiliary lemmas on the forward, the cutoff, and the own-corridor comparison, and proves Proposition 4 from them.

Lemma A.7 (Forward payoff) *For a notional h and the unbiased forward rate $\mathbb{E}[e^s]$,*

$$h\Big(e^s - \mathbb{E}[e^s]\Big) = h\,s + \tfrac{h}{2}\Big(s^2 - \sigma_s^2\Big) + \cdots, \tag{56}$$

where the remainder collects terms of third and higher order in s and σ_s, and each retained term has mean zero.

Proof. Expand e^s to second order and subtract $\mathbb{E}[e^s] = 1 + \frac{1}{2}\sigma_s^2 + \cdots$. Setting the rate at $e^{\mathbb{E}[s]}$ would instead leave a mean of $h\,(e^{\sigma_s^2/2} - 1)$. ■

Proof of Lemma 5. *Step 1: the window cash flow.* Let $\hat{q}_{f,c}$ be the demand schedule (31) at the relative price $(x_c - \theta_c^{\$})s$ and $\varepsilon_{q,c} \equiv -\nu(x_c - \theta_c^{\$})$ its loading on s: the buyer's preset price loads $x_c - a_{\$\to c}$ on s, the destination index $\theta_c^{\$} - a_{\$\to c}$, and the common $a_{\$\to c}$ cancels, while neither loads on ξ_c within the window. Destination c's profit in (33) is exactly

$$\pi_{f,c} = \bar{R}\, e^{\hat{q}_{f,c} + x_c s} - \overline{TC}\, e^{\hat{q}_{f,c} + \rho_M \phi_{f,c} \bar{d}_{f,c}\, s + \phi_{f,c}\kappa\, \bar{\xi}_{f,c}}, \qquad \hat{q}_{f,c} = \varepsilon_{q,c}\, s + \eta\, \xi_c - \frac{\nu\sigma_\epsilon}{2}\Big(x_c - \theta_c^{\$}\Big)^2 s^2, \tag{57}$$

with $\bar{\xi}_{f,c} \equiv \sum_{c'} \omega_{cc'} \xi_{c'}$ and $\sigma_\epsilon = \alpha(\nu - 1)/(1 - \alpha)$ by (48).

Step 2: loadings. Expanding both exponentials through second order in the shocks, each loading is $\bar{R}$ times the revenue exponent's coefficient less $\overline{TC}$ times the cost exponent's, and the interaction loading carries the product of the two coefficients:

$$\begin{aligned}
\beta_{f,c} &= \bar{R}\,(\varepsilon_{q,c} + x_c) - \overline{TC}\,(\varepsilon_{q,c} + \rho_M \phi_{f,c} \bar{d}_{f,c}), \\
\delta_{f,c} &= \bar{R}\,\eta - \overline{TC}\,(\eta + \phi_{f,c}\kappa\,\omega_{cc}), \\
\psi_{f,c} &= \bar{R}\,(\varepsilon_{q,c} + x_c)\,\eta - \overline{TC}\,(\varepsilon_{q,c} + \rho_M \phi_{f,c} \bar{d}_{f,c})\,(\eta + \phi_{f,c}\kappa\,\omega_{cc}).
\end{aligned} \tag{58}$$

Collecting on $\bar{\pi} \equiv \bar{R} - \overline{TC}$,

$$\begin{aligned}
\delta_{f,c} &= \bar{\pi}\,\eta - \phi_{f,c}\overline{TC}\,\kappa\,\omega_{cc}, \\
\psi_{f,c} &= \eta\,\beta_{f,c} - \phi_{f,c}\kappa\,\omega_{cc}\,\overline{TC}\,(\varepsilon_{q,c} + \rho_M\,\phi_{f,c}\bar{d}_{f,c}).
\end{aligned} \tag{59}$$

Substituting $\varepsilon_{q,c}$ and imposing $\overline{TC} = \bar{R}\,(\nu-1)/\nu$ returns the $\beta_{f,c}$ of (36), from which the export denomination has cancelled. The expansion retains the own quadratics as G_f and the cross-shock products as $Q_{f,c'c''}$, and drops only third-order terms. The level loading of $\xi_{c'}$ and the domestic objects are

$$\begin{aligned} D_{f,c'} &= \mathbb{I}_{c'}\,\tfrac{\bar{R}}{\nu}\,\eta - \kappa\,\overline{TC}\sum_{c}\phi_{f,c}\,\omega_{cc'} - \kappa\,\overline{TC}_O\,\phi_f^O\,\omega_{c'}^O, \\ \beta_{f,O} &= -\rho_M\,\phi_f^O\,\overline{TC}_O\,\bar{d}_f^O, \qquad \Psi_{f,c'}^O = -\kappa\,\overline{TC}_O\,\phi_f^O\,\omega_{c'}^O\,\rho_M\,\phi_f^O\,\bar{d}_f^O, \end{aligned} \tag{60}$$

where $\bar{R}_O$, $\overline{TC}_O$, ϕ_f^O, $\omega_{c'}^O$, and $\bar{d}_f^O = \sum_{c'}\omega_{c'}^O d_{c'}$ are the domestic line's symmetric-point revenue and cost, intermediate share, sourcing weights, and dollar content, and the domestic quantity loads zero on s, since the home market is euro-priced by Assumption 1(iii). The residuals of the lemma collect the rest: $\mathbb{E}_f^\circ$ the levels $\sum_c \bar{\pi} + \bar{\pi}_O$, $\bar{\pi}_O = \bar{R}_O - \overline{TC}_O$, and the means of the ξ blocks; V_f° the terms $\sum_{c'} D_{f,c'}^2\sigma_{\xi_{c'}}^2$ and $2\sum_{c'}\sum_{c''} Q_{f,c'c''}^2\sigma_{\xi_{c'}}^2\sigma_{\xi_{c''}}^2$.

Step 3: aggregation. Summing over destinations, the domestic line, and the forward, and grouping by shock,

$$\begin{aligned} \Pi_f = \sum_c \bar{\pi} + \bar{\pi}_O - \tfrac{h}{2}\sigma_s^2 + \Big(\sum_c \beta_{f,c} + \beta_{f,O} + h\Big)s + \sum_{c'} D_{f,c'}\,\xi_{c'} \\ + s\sum_{c'}\Psi_{f,c'}\,\xi_{c'} + G_f\,s^2 + \sum_{c'}\sum_{c''} Q_{f,c'c''}\,\xi_{c'}\xi_{c''}. \end{aligned} \tag{61}$$

The $\xi_{c'}$ loadings sum $\phi_{f,c}\omega_{cc'}$ over the destinations that buy from c', which gives the $\Psi_{f,c'}$ of (36) and the $D_{f,c'}$ of (60); $\delta_{f,c}$ and $\psi_{f,c}$ are their $c' = c$ summands, and the domestic line enters $\beta_{f,O}$ and the domestic term of each (Lemmas A.8 and A.10). The constant $-\frac{h}{2}\sigma_s^2$ is the forward's, by Lemma A.7, and cancels the $\frac{h}{2}$ in G_f when the mean is taken, so (34) is free of h.

Step 4: the quadratic terms. Writing $a_c = \varepsilon_{q,c} + x_c$ and $b_c = \varepsilon_{q,c} + \rho_M\phi_{f,c}\bar{d}_{f,c}$,

$$G_f = \frac{h}{2} + \sum_c\left[\tfrac{1}{2}\Big(\bar{R}\,a_c^2 - \overline{TC}\,b_c^2\Big) - \frac{(\nu-1)\alpha}{2(1-\alpha)}\,\nu\,\bar{\pi}\,\Big(x_c - \theta_c^{\$}\Big)^2\right] - \frac{\overline{TC}_O}{2}\Big(\rho_M\,\phi_f^O\,\bar{d}_f^O\Big)^2,$$

and substituting $\varepsilon_{q,c}$ under the markup condition collects every term in the export denominations into $-\frac{K}{2}\sum_c(\Lambda_{f,c} - x_c)^2$ with $K = (\nu-1)\bar{R}/(1-\alpha)$ and $\Lambda_{f,c}$ the loading (53), which is (37); the domestic term carries no export denomination and no forward and joins G_f°. K is the curvature that Lemma 1 leaves as $\mathbb{E}[|\Pi''_{f,c}(\tilde{p}_{f,c})|]$, and the complementarity enters here only, since the shock and the relative price enter $\log q_{f,c}$

additively. $Q_{f,c'c''}$ is built the same way from the cost exponent's ξ coefficients and carries no export denomination.

Step 5: the variance. With $\mathbb{E}[\xi_c] = \mathbb{E}[s] = 0$, independence, and normality,

$$\begin{aligned}&\mathrm{Cov}(s, \xi_c s) = \mathrm{Cov}(\xi_c, \xi_c s) = \mathrm{Cov}(s, s^2) = \mathrm{Cov}(\xi_{c'}, \xi_{c'}^2) = \mathrm{Cov}(s^2, \xi_{c'}\xi_{c''}) = 0,\\ &\mathrm{Cov}(\xi_{c'}, \xi_{c''}) = 0 \quad (c' \neq c''),\end{aligned}$$

the third and fourth by the vanishing third moments, and $\mathrm{Var}(s^2) = 2\sigma_s^4$ by $\mathbb{E}[s^4] = 3\sigma_s^4$, with the analogous coefficient in V_f°. The variance of (61) is then (35), exact for every ω; without the fourth covariance it would carry $2\sum_{c'} D_{f,c'} Q_{f,c'c'} \mathbb{E}[\xi_{c'}^3]$ as a further term.

Step 6: the forward. By Lemma A.7 the forward's payoff is $h\, s + \frac{h}{2} s^2$ to the order retained, less a constant that holds its mean at zero, so h is absent from (34) and reaches (35) through the first term and through G_f. Under (30),

$$\begin{aligned}\frac{\partial}{\partial h}\Big(\mathbb{E}[\Pi_f] - \tfrac{\gamma}{2}\mathrm{Var}(\Pi_f)\Big) &= -\gamma\Big[\Big(\sum_c \beta_{f,c} + \beta_{f,O} + h\Big)\sigma_s^2 + G_f\, \sigma_s^4\Big],\\ \frac{\partial^2}{\partial h^2}\Big(\mathbb{E}[\Pi_f] - \tfrac{\gamma}{2}\mathrm{Var}(\Pi_f)\Big) &= -\gamma\, \sigma_s^2\Big(1 + \tfrac{\sigma_s^2}{2}\Big),\end{aligned}$$

so for $\gamma > 0$ the objective is strictly concave in h and the first-order condition is $\sum_c \beta_{f,c} + \beta_{f,O} + h^* = -G_f \sigma_s^2$. ∎

Lemma A.8 (The optimal forward and K_γ) *For $\gamma > 0$, write $\hat{G}_f \equiv G_f - h/2$ and $B_f \equiv \sum_c \beta_{f,c} + \beta_{f,O}$. Then*

$$h^* = -\frac{B_f + \hat{G}_f\, \sigma_s^2}{1 + \sigma_s^2/2}, \qquad G_f\big|_{h^*} = \frac{\hat{G}_f - B_f/2}{1 + \sigma_s^2/2}, \tag{62}$$

the first term of (35) at h^* is of order σ_s^6, and under (40), $K_\gamma > 0$. The domestic exposure $\beta_{f,O} = -\rho_M\, \phi_f^O\, \overline{TC}_O\, \bar{d}_f^O \leq 0$ of (60) enters h^* one for one through B_f.

Proof. By Lemma 5 neither $\hat{G}_f$ nor B_f depends on h, and B_f does not depend on any x_c. The first-order condition $B_f + h^* = -G_f \sigma_s^2$ with $G_f = h/2 + \hat{G}_f$ gives (62), and the first term of (35) at h^* is $(G_f \sigma_s^2)^2 \sigma_s^2$. Since $\hat{G}_f = G_f^\circ - \frac{K}{2}\sum_c (\Lambda_{f,c} - x_c)^2$,

$$G_1 + G_0 = \frac{\hat{G}_1 + \hat{G}_0 - B_f}{1 + \sigma_s^2/2} \leq \frac{2G_f^\circ - \frac{K}{4} - B_f}{1 + \sigma_s^2/2},$$

the $K/4$ from $(\Lambda_{f,c}-1)^2+\Lambda_{f,c}^2 \geq \frac{1}{2}$. If $2G_f^\circ \leq B_f$ the right side is negative and $K_\gamma > K > 0$. Otherwise $G_1+G_0 \leq 2G_f^\circ - B_f$ and (40) gives $\gamma(G_1+G_0)\sigma_s^2 < 1$, which is $K_\gamma > 0$. The bound holds uniformly over the export denominations, since neither G_f° nor B_f moves with them. The last claim is the first-order condition with B_f written out. ■

Lemma A.9 (The cutoff under the mean-variance objective) *For one corridor c, let the gap between the preset and the desired price be $g = (x_c - \Lambda_{f,c})s + \bar{p}$ with $x_c \in \{0,1\}$, let profit be $\Pi = \Pi^{flex} - \frac{K}{2}g^2$ with Π^{flex} linear in s, and evaluate (30) at the preset price of Lemma 1. For every $\gamma \geq 0$ the difference of (30) between $x_c = 1$ and $x_c = 0$ is*

$$\tfrac{K}{8}\left(2\Lambda_{f,c}-1\right)\Big[2K\gamma\,\sigma_s^4\big(2\Lambda_{f,c}^2-2\Lambda_{f,c}+1\big)+4\sigma_s^2\Big],$$

whose sign is the sign of $2\Lambda_{f,c}-1$.

Proof. The preset price of Lemma 1 holds the mean of the gap at zero. Evaluating the mean and the variance of Π at the two denominations with $\mathbb{E}[s^3]=0$ and $\mathbb{E}[s^4]=3\sigma_s^4$ and differencing gives the display, and $2\Lambda^2-2\Lambda+1 = 2(\Lambda-\frac{1}{2})^2+\frac{1}{2} > 0$. Any distribution with $\mathbb{E}[s^3]=0$ and $\mathbb{E}[s^4] \geq \sigma_s^4$ delivers the sign, the second by Jensen's inequality. Without a vanishing third moment a term in $\mathbb{E}[s^3]$ survives, proportional to the net dollar exposure $\sum_c \beta_{f,c} + \beta_{f,O} + h$ and carrying the same factor $2\Lambda_{f,c}-1$, so the cutoff still does not move; at the optimal forward that exposure is $-G_f\sigma_s^2$ by (62), and the preset price optimal under (30) shifts off that of Lemma 1 by a term of order $\gamma\,\sigma_s^2$ times the same exposure, which moves the difference only at order σ_s^8. ■

Lemma A.10 (The own-corridor comparison) *Write $J_{f,c} \equiv \nu\theta_c^{\$}\delta_{f,c}$, $V_{f,c} \equiv \nu\,\phi_{f,c}\kappa\,\omega_{cc}\overline{TC}$, $\bar{d}_{f,c}^{-c} \equiv \sum_{c'\neq c}\omega_{cc'}d_{c'}$, and Ψ_0, Ψ_1 for $\Psi_{f,c}$ at $x_c=0$ and at $x_c=1$.*

(i) The own-corridor interaction is affine in the two denominations,

$$\psi_{f,c} \;=\; \nu\,\theta_c^{\$}\,\delta_{f,c} + V_{f,c}\,x_c + Z_{f,c}\,\bar{d}_{f,c}, \qquad Z_{f,c} \equiv -\rho_M\,\phi_{f,c}\overline{TC}\,(\eta+\phi_{f,c}\kappa\,\omega_{cc}), \quad (63)$$

and $\partial\Psi_{f,c'}/\partial x_c = \nu\kappa\overline{TC}\phi_{f,c}\omega_{cc'}$, whose $c'=c$ term is $V_{f,c}$.

(ii) Moving corridor c to match lowers $\psi_{f,c}^2$ on a euro-import corridor, where $\bar{d}_{f,c} = \bar{d}_{f,c}^{-c}$, if and only if $V_{f,c}(2J_{f,c}+2Z_{f,c}\bar{d}_{f,c}^{-c}+V_{f,c}) > 0$, and on a dollar-import corridor, where $\bar{d}_{f,c} = \bar{d}_{f,c}^{-c} + \omega_{cc}$, if and only if $V_{f,c}(2J_{f,c}+2Z_{f,c}\bar{d}_{f,c}^{-c}+2Z_{f,c}\omega_{cc}+V_{f,c}) < 0$. For $V_{f,c} > 0$ both hold if and only if $0 < 2J_{f,c}+2Z_{f,c}\bar{d}_{f,c}^{-c}+V_{f,c} < -2Z_{f,c}\,\omega_{cc}$, which at $\omega_{cc}=1$ reads $0 < 2J_{f,c}+V_{f,c} < -2Z_{f,c}$.

(iii) $\Psi_{f,c}$ *is affine in* x_c *with slope* $V_{f,c}$*, so* $\Psi^2_{f,c}|_{x_c=d_c} - \Psi^2_{f,c}|_{x_c=1-d_c} = (\Psi_1+\Psi_0)\,V_{f,c}\,(2d_c-1)$*, negative if and only if* $(1-2d_c)(\Psi_1+\Psi_0)>0$*; and at* $\omega_{cc}=1$ *the* $c'=c$ *term of* (41) *is* $(\gamma\sigma^2_{\xi_c}/2K_\gamma)\,V_{f,c}(2J_{f,c}+2Z_{f,c}\bar{d}_{f,c}+V_{f,c})$*.*

(iv) The domestic term $\Psi^O_{f,c}$ *of* (60) *is a constant in* x_c *with* $\Psi^O_{f,c}\le 0$*, so* $\partial\Psi_{f,c}/\partial x_c$ *and the difference of squares of part (iii) are as without the domestic line,* Ψ_0 *and* Ψ_1 *each include it, and* $\Psi_1+\Psi_0$ *is lower by* $2|\Psi^O_{f,c}|$ *than without it.*

Proof. Substituting $\varepsilon_{q,c} = -\nu(x_c-\theta^{\$}_c)$ into (59) gives (63) with export slope $\eta\,(\bar{R}-\nu\bar{\pi})+\nu\,\phi_{f,c}\kappa\,\omega_{cc}\overline{TC}$, whose first term vanishes by the markup condition $\bar{\pi}=\bar{R}/\nu$; differentiating the $\Psi_{f,c'}$ of (36) in x_c gives the slope of part (i), with $c'=c$ term $V_{f,c}$. For part (ii), difference $\psi^2_{f,c}$ across x_c at $\bar{d}_{f,c}=\bar{d}^{-c}_{f,c}$ and at $\bar{d}_{f,c}=\bar{d}^{-c}_{f,c}+\omega_{cc}$, since flipping the import leg moves $\bar{d}_{f,c}$ by ω_{cc}; the intersection of the two conditions for $V_{f,c}>0$ is the two-sided bound, and $\omega_{cc}=1$ sets $\bar{d}^{-c}_{f,c}=0$. For part (iii), $\Psi_1-\Psi_0=V_{f,c}$ by part (i), which gives the difference of squares, and at $\omega_{cc}=1$, where $\Psi_{f,c}=\psi_{f,c}$, the $c'=c$ summand of (41) is $-(\gamma\sigma^2_{\xi_c}/2K_\gamma)(\Psi_0^2-\Psi_1^2)$ with $\Psi_0^2-\Psi_1^2=-V_{f,c}(2\Psi_0+V_{f,c})$ and $\Psi_0=J_{f,c}+Z_{f,c}\bar{d}_{f,c}$. For part (iv), $\Psi^O_{f,c}$ carries no x_c and every factor in it is nonnegative, so it adds the same constant to Ψ_0 and Ψ_1; $\Psi_1^2-\Psi_0^2$ equals $2\Psi_0V_{f,c}+V^2_{f,c}$ at that base, and $\Psi_1+\Psi_0$ shifts by $2\Psi^O_{f,c}$. ■

The domestic line therefore enters the hedging results through two constants. Its exposure $\beta_{f,O}$ moves the level of the optimal forward and nothing else, by Lemma A.8. Its interaction term $\Psi^O_{f,c}\le 0$ lowers $\Psi_1+\Psi_0$ on every corridor c from which the domestic line sources in dollars, by Lemma A.10(iv), which lowers the own-corridor term of the cutoff (41) toward dollar invoicing of the exports to c and tilts the sign condition of Proposition 4(ii) toward holding on dollar-import corridors and against holding on euro-import corridors. The corridor loading (29) does not involve the domestic line, and (35) is exact with it, since independence is a property of the shocks; the domestic line matters for which corridors satisfy the condition, not for the mechanism.

Proof of Proposition 4. *Part (i).* Flipping x_c moves every $\Psi_{f,c'}$ that destination c sources, by (38), leaves V°_f untouched, and moves G_f. Here and below G_1 and G_0 are evaluated at the optimal forward of the respective denomination, so by (62) and (37) $G_1-G_0=K(\Lambda_{f,c}-\frac{1}{2})/(1+\sigma_s^2/2)$, which is $K(\Lambda_{f,c}-\frac{1}{2})$ to the order retained in (39); every step below holds to that order. The mean and the variance differences accrue over the same preset window, whose expected duration multiplies the first once and the second twice; it rescales γ and is normalized to one. Adding the differences of (34) and (39) with the weights of (30) and solving for indifference, σ_s^2 cancels, and dividing by

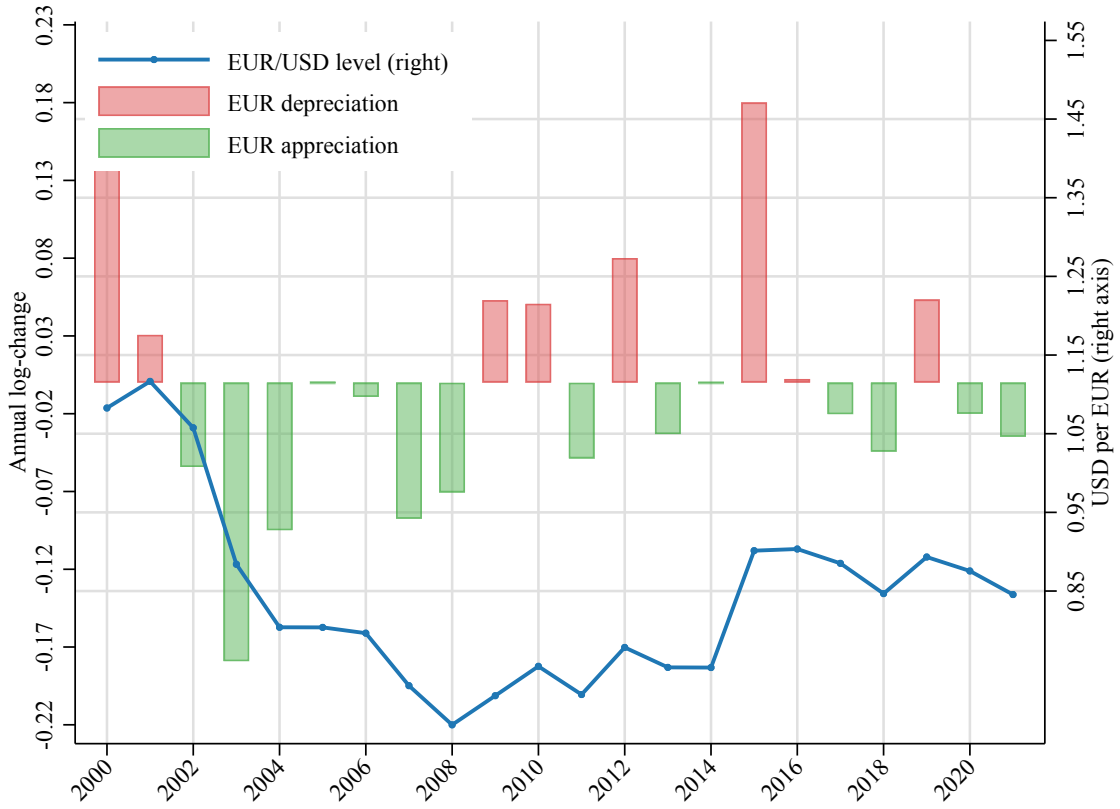
$K_\gamma > 0$ (Lemma A.8) gives (41); its $c' = c$ term is of order ω_{cc} and the others of order $\omega_{cc'}$ by (38). By Lemma A.10(iii) the $c' = c$ term is $(\gamma\sigma_{\xi_c}^2/2K_\gamma)\, V_{f,c}(2J_{f,c} + 2Z_{f,c}\bar{d}_{f,c} + V_{f,c})$ at $\omega_{cc} = 1$, so that term carries $\Lambda^*_{f,c}$ below $\frac{1}{2}$ on dollar-import corridors and above it on euro-import corridors precisely under the condition of part (ii).

Part (ii). At h^* the first term of (35) is of order σ_s^6 (Lemma A.8) and V_f° carries no export denomination, so differencing (39) across the two denominations leaves the Ψ terms and $2(G_1^2 - G_0^2)\sigma_s^4 = 2(G_1 + G_0)(G_1 - G_0)\sigma_s^4$; with $\Delta\mathbb{E}[\Pi_f] = (2d_c - 1)K(\Lambda_{f,c} - \frac{1}{2})\sigma_s^2$ from (34) and $G_1 - G_0$ from part (i), this is (42). Its $c' = c$ term is (43) by Lemma A.10(iii), negative for $\kappa > 0$ if and only if $(1 - 2d_c)(\Psi_1 + \Psi_0) > 0$, and the remaining terms are of order $\omega_{cc'}$ by (38).

Part (iii). Dividing the second term of (42) by (43), with $\Delta\mathbb{E}[\Pi_f] = (2d_c - 1)K(\Lambda_{f,c} - \frac{1}{2})\sigma_s^2$, the factor $2d_c - 1$ and one power of σ_s^2 cancel, which leaves (44), zero at $\Lambda_{f,c} = \frac{1}{2}$. G_f and K carry no κ, $\partial\Psi_{f,c}/\partial x_c$ carries one factor of κ by (38), and $\Psi_{f,c'} = \mathbb{I}_{c'}\,\eta\beta_{f,c'} + O(\kappa)$ by (36), so $\Psi_1 + \Psi_0 \to 2\eta\,\beta_{f,c}$ as $\kappa \to 0$: the ratio is of order κ^{-1} when $\eta\,\beta_{f,c} \neq 0$ and of order κ^{-2} when that product vanishes. ∎

B Additional tables

Figure B1: Annual EUR/USD Exchange Rate



Notes: The bars plot the annual log change in the euro price of the dollar, so that positive values denote a depreciation of the euro against the dollar and negative values an appreciation. The line plots the level of the exchange rate in dollars per euro on the right axis. The sample spans 2000–2021. The largest depreciation episodes occur in 2000, 2009–2010, 2012, and 2015, corresponding to the early years of the euro, the aftermath of the global financial crisis, and the European sovereign debt crisis and subsequent ECB quantitative easing, respectively. Source: Federal Reserve, EUR/USD exchange rate.

Table B1: Descriptive Statistics

	Mean	SD	P25	P50	P75	Min	Max	N
				Panel A. General Characteristics				
Employees	29.349	466.986	2.000	7.000	16.000	0.000	174553.000	3,647,082
Age	17.819	14.010	6.000	15.000	27.000	0.000	208.000	3,644,827
Sales (thousand EUR)	9,835	144,708	326	1,191	4,016	0	59,037,236	3,642,990
Profits (thousand EUR)	286	17,402	-2	14	99	-3,076,992	7,161,469	2,538,845
N countries export	5.210	10.525	0.000	1.000	4.000	0.000	145.000	3,648,669
N products export	5.557	16.964	0.000	1.000	5.000	0.000	3357.000	3,648,669
N countries import	2.149	3.672	0.000	1.000	2.000	0.000	107.000	3,648,669
N products import	6.378	21.190	0.000	1.000	4.000	0.000	3153.000	3,648,669
				Panel B. Main Regression Variables				
Currency choice: Country-product level								
$\mathbb{I}\{M^{\$}_{c,t}=1\}$	0.118	0.323	0.000	0.000	0.000	0.000	1.000	50,309,896
Start $M^{\$}_{c,t}$	0.066	0.248	0.000	0.000	0.000	0.000	1.000	50,309,896
Stop $M^{\$}_{c,t}$	0.002	0.040	0.000	0.000	0.000	0.000	1.000	50,309,896
$\mathbb{I}\{X^{\$}_{c,t-1}=1\}$	0.058	0.234	0.000	0.000	0.000	0.000	1.000	50,309,896
Start $X^{\$}_{c,t}$	0.034	0.181	0.000	0.000	0.000	0.000	1.000	50,309,896
Stop $X^{\$}_{c,t}$	0.002	0.046	0.000	0.000	0.000	0.000	1.000	50,309,896
Currency choice: Country level								
$\mathbb{I}\{M^{\$}_{c,t}=1\}$	0.122	0.327	0.000	0.000	0.000	0.000	1.000	17,145,782
Start $M^{\$}_{c,t}$	0.038	0.190	0.000	0.000	0.000	0.000	1.000	16,299,397
Stop $M^{\$}_{c,t}$	0.005	0.070	0.000	0.000	0.000	0.000	1.000	16,299,397
$\mathbb{I}\{X^{\$}_{c,t-1}=1\}$	0.070	0.255	0.000	0.000	0.000	0.000	1.000	17,145,782
Start $X^{\$}_{c,t}$	0.020	0.141	0.000	0.000	0.000	0.000	1.000	16,299,397
Stop $X^{\$}_{c,t}$	0.006	0.075	0.000	0.000	0.000	0.000	1.000	16,299,397
Profitability and exposure								
π_t/A_{t-1}	0.009	0.155	-0.002	0.012	0.050	-1.779	0.504	230,695
NFXI_t/A_{t-1}	-0.000	0.004	-0.001	0.000	0.000	-0.009	0.007	230,681
$X^{\$}_{t-1}/A_{t-1}$	0.035	0.037	0.003	0.016	0.075	0.000	0.093	230,724
$M^{\$}_{t-1}/A_{t-1}$	0.079	0.113	0.003	0.021	0.106	0.000	0.349	230,724
$(X^{\$}_{t-1}-M^{\$}_{t-1})/A_{t-1}$	-0.039	0.116	-0.061	-0.001	0.037	-0.332	0.067	230,724
$\Delta e^{€,\$}_t \times X^{\$}_{t-1}/A_{t-1}$	-0.000	0.000	-0.001	-0.000	0.000	-0.001	0.000	230,724
$\Delta e^{€,\$}_t \times M^{\$}_{t-1}/A_{t-1}$	-0.001	0.003	-0.001	-0.000	0.000	-0.008	0.003	230,724
$\Delta e^{€,\$}_t \times (X^{\$}_{t-1}-M^{\$}_{t-1})/A_{t-1}$	0.000	0.004	-0.001	0.000	0.001	-0.007	0.009	230,724
Pass-through								
$\Delta \ln p^X$	0.027	0.875	-0.180	0.012	0.227	-19.274	19.210	34,745,708
$\Delta \ln q^X$	-0.014	1.572	-0.693	0.000	0.693	-17.395	19.079	34,745,708
$\Delta \ln p^M$	0.026	0.989	-0.202	0.009	0.248	-18.368	16.874	17,071,068
$\Delta \ln q^M$	-0.019	1.603	-0.693	0.000	0.692	-20.377	18.154	17,071,070

Table B2: Trade by macroarea

Rank	Macroarea	% of total exports	% of total USD exports
Panel A. Exports			
1	Euro Area	44.77	0.40
2	Non-Euro Europe	24.85	8.57
3	North America	9.40	46.83
4	Asia	8.99	19.58
5	Middle East & North Africa	6.86	15.12
6	Latin America & Caribbean	2.36	6.18
7	Sub-Saharan Africa	1.75	2.30
8	Oceania	0.97	0.87
9	Other	0.05	0.16
Panel B. Imports			
Rank	Macroarea	% of total imports	% of total USD imports
1	Euro Area	50.06	0.06
2	Non-Euro Europe	20.61	15.63
3	Asia	13.88	36.19
4	Middle East & North Africa	6.62	21.23
5	North America	4.01	13.17
6	Latin America & Caribbean	2.47	7.73
7	Sub-Saharan Africa	1.97	4.93
8	Oceania	0.38	1.03
9	Other / Territories	0.01	0.03

Table B3: Top 50 countries across alternative trade rankings

	By total exports		**By total USD exports**		**By total imports**		**By total USD imports**	
Rank	ISO	%	ISO	%	ISO	%	ISO	%
1	DEU	13.31	USA	41.85	DEU	17.72	CHN	21.98
2	FRA	11.40	CHN	5.49	FRA	9.43	USA	11.16
3	USA	7.70	HKG	3.18	CHN	6.91	RUS	6.97
4	ESP	5.84	TUR	3.17	NLD	5.93	LBY	6.18
5	GBR	5.73	MEX	3.01	ESP	5.03	AZE	4.15
6	CHE	4.23	ARE	2.46	BEL	4.33	SAU	3.98
7	BEL	2.85	SAU	2.09	GBR	3.37	BRA	3.43
8	NLD	2.57	KOR	2.06	USA	3.32	IRQ	2.64
9	POL	2.53	BRA	2.05	CHE	2.56	IND	2.26
10	CHN	2.33	CAN	1.79	AUT	2.46	IRN	2.21
11	AUT	2.27	EGY	1.67	RUS	2.08	DZA	1.82
12	TUR	2.14	SGP	1.57	POL	2.03	TWN	1.65
13	RUS	1.93	GIB	1.56	TUR	1.81	KAZ	1.63
14	SVN	1.53	LBY	1.52	LBY	1.60	UKR	1.56
15	GRC	1.47	TUN	1.33	JPN	1.37	KOR	1.46
16	JPN	1.36	IND	1.30	CZE	1.20	EGY	1.43
17	ROU	1.31	CHE	1.29	ROU	1.20	CAN	1.38
18	HKG	1.16	JPN	1.09	SWE	1.16	IDN	1.22
19	CZE	1.15	LBN	0.96	IND	1.15	ARG	1.21
20	SWE	1.06	TWN	0.94	HUN	1.10	VNM	1.15
21	PRT	1.02	ISR	0.90	SVN	1.09	ZAF	1.15
22	HUN	0.97	DZA	0.90	IRL	1.05	BGD	1.02
23	ARE	0.96	MYS	0.89	BRA	1.02	NGA	1.01
24	BRA	0.92	RUS	0.89	KOR	0.99	TUR	0.96
25	KOR	0.89	AUS	0.75	SAU	0.97	THA	0.85
26	CAN	0.83	ARG	0.75	AZE	0.92	ARE	0.77
27	AUS	0.83	ZAF	0.70	SVK	0.78	AUS	0.69
28	MEX	0.82	VNM	0.59	DZA	0.69	CHE	0.67
29	SAU	0.81	THA	0.59	DNK	0.67	SYR	0.65
30	IND	0.77	NGA	0.59	TUN	0.64	MYS	0.63
31	DNK	0.72	IDN	0.56	UKR	0.63	MEX	0.63
32	TUN	0.71	MAR	0.54	GRC	0.62	CHL	0.59
33	DZA	0.67	VEN	0.47	IRQ	0.60	COL	0.53
34	HRV	0.67	JOR	0.44	IRN	0.60	AGO	0.52
35	EGY	0.65	QAT	0.43	TWN	0.58	JPN	0.51
36	SVK	0.56	CHL	0.42	FIN	0.55	PER	0.46
37	KEN	0.55	IRN	0.40	VNM	0.50	NOR	0.46
38	ISR	0.54	IRQ	0.40	PRT	0.50	PAK	0.44
39	SGP	0.49	SYR	0.37	EGY	0.46	TUN	0.43
40	FIN	0.45	PHL	0.36	CAN	0.46	CMR	0.43
41	IRN	0.43	PER	0.36	ZAF	0.43	ISR	0.41
42	IRL	0.43	COL	0.34	HRV	0.43	ECU	0.36
43	ZAF	0.43	PAN	0.30	THA	0.40	HKG	0.34
44	BGR	0.40	DOM	0.29	KAZ	0.40	GNQ	0.32
45	NOR	0.40	KWT	0.28	IDN	0.40	VEN	0.24
46	MAR	0.40	HRV	0.28	BGR	0.40	MAR	0.24
47	LBY	0.39	ALB	0.26	ARG	0.34	GHA	0.23
48	UKR	0.36	UKR	0.26	KEN	0.33	PRY	0.22
49	MLT	0.34	PAK	0.24	CHL	0.31	SGP	0.21
50	TWN	0.32	OMN	0.22	LUX	0.31	PHL	0.18

Table B4: Bilateral invoicing matching and profit volatility: leave-one-out volatility measure

	$(NFXI_{i,t} - \overline{NFXI}_{i,-t})^2$			$(\pi_{i,t} - \overline{\pi}_{i,-t})^2$		
	(1)	(2)	(3)	(4)	(5)	(6)
$MQ_{i,t}$	0.000000229	0.000000286		−0.00237***	−0.00251***	
	(0.000000230)	(0.000000235)		(0.000202)	(0.000209)	
$MQ_{i,t-1}$			−0.000000131			−0.000524**
			(0.000000257)			(0.000237)
Sample	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	Switchers	Switchers	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	Switchers	Switchers
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
R-squared	0.499	0.467	0.464	0.560	0.515	0.512
Observations	195,461	144,356	148,234	195,461	144,356	148,234

Notes: Unit of observation: firm-year. The dependent variable is the squared deviation of the outcome from its firm-specific leave-one-out mean $\overline{y}_{i,-t}$, the average of the outcome for the same firm over all other years. Columns (1)–(3) use the foreign-currency profit component (NFXI) scaled by lagged total assets; columns (4)–(6) use total profits scaled by lagged total assets. Higher values correspond to larger deviations from the firm's typical outcome and therefore to higher within-firm volatility.
$MQ_{i,t}$ is the share of dollar-active corridors (i.e. corridors in which the firm uses USD on at least one side) in which both exports and imports are invoiced in USD. Switchers are firms that cross the pooled-sample median of $MQ_{i,t}$ at least once over the observation window.
Column (6) provides the main quantitative specification. Moving from the 25th to the 75th percentile of lagged matching quality among switchers is associated with a 4.0% reduction in within-firm total-profit volatility; the corresponding estimate for NFXI volatility in column (3) is small and statistically insignificant.
All specifications include firm and year fixed effects. Standard errors are clustered at the firm level. *, **, *** denote significance at the 10%, 5%, and 1% levels.

Table B5: Dollar invoicing and switching within pre-existing product-country relationships

	$\mathbb{I}\{X^{\$}_{c,p,t}\}$			$\mathbb{I}\{M^{\$}_{c,p,t}\}$		
	(1)	(2)	(3)	(4)	(5)	(6)
Start $M^{\$}_{c,t}$	1.105***	1.576***	0.376***			
	(0.0462)	(0.0371)	(0.0694)			
$\mathbb{I}\{X^{-\$}_{c,p,t-}>0\}$	0.0513***					
	(0.00326)					
Start $M^{\$}_{c,t} \times \mathbb{I}\{X^{-\$}_{c,p,t-}>0\}$	0.545***					
	(0.0549)					
Start $X^{\$}_{c,t}$				0.297***	1.845***	0.272***
				(0.00749)	(0.105)	(0.00712)
$\mathbb{I}\{M^{-\$}_{c,p,t-}>0\}$				0.0183***		
				(0.00272)		
Start $X^{\$}_{c,t} \times \mathbb{I}\{M^{-\$}_{c,p,t-}>0\}$				1.193***		
				(0.0377)		
Sample	$X^{\$}_{c,p,t-}=0$ & $M^{\$}_{c,t-}=0$ & $X^{-\$}_{c,p,t-}>0$		$X^{\$}_{c,p,t-}=0$ & $M^{\$}_{c,t-}=0$ & $X^{-\$}_{c,p,t-}=0$	$M^{\$}_{c,p,t-}=0$ & $X^{\$}_{c,t-}=0$ & $M^{-\$}_{c,p,t-}>0$		$M^{\$}_{c,p,t-}=0$ & $X^{\$}_{c,t-}=0$ & $M^{-\$}_{c,p,t-}=0$
Country × Product × Time FE	Y	Y	Y	Y	Y	Y
Firm × Product × Time FE	Y	Y	Y	Y	Y	Y
Firm × Country × Product FE	Y	Y	Y	Y	Y	Y
R-squared	0.709	0.585	0.940	0.779	0.775	0.776
Identifying obs	16,441,857	8,997,491	723,068	16,468,272	476,134	14,306,551

Notes: The unit of observation is the firm-country-product-year. $t-$ refers to either $t-1$ or $t-2$. In columns (1)–(3), the key regressor is Start $M^{\$}_{c,t}$ and the dependent variable is the indicator for starting dollar export invoicing. In columns (4)–(6), the roles are reversed. $\mathbb{I}\{X^{-\$}_{c,p,t-}>0\}$ equals one if the firm was already exporting the same product to the same country in any non-dollar currency in either of the two preceding years; $\mathbb{I}\{M^{-\$}_{c,p,t-}>0\}$ is defined analogously for imports. Dependent dummy variables are rescaled by their unconditional means. Standard errors clustered at the firm-country-product level. *, **, *** significant at 10%, 5%, 1%.

Table B6: Dollar adoption and bilateral trade activity: extensive-margin evidence

	Panel A: Effect of Start $M^{\$}_{c,t}$ on export-side activity		Panel B: Effect of Start $X^{\$}_{c,t}$ on import-side activity	
	(1)	(2)	(3)	(4)
	$\mathbb{I}\{X^{-\$}_{c,p,t}>0\}$	$\mathbb{I}\{X_{c,p,t}>0\}$	$\mathbb{I}\{M^{-\$}_{c,p,t}>0\}$	$\mathbb{I}\{M_{c,p,t}>0\}$
Start $M^{\$}_{c,t}$	−0.0699***	−0.0427***		
	(0.000879)	(0.000693)		
Start $X^{\$}_{c,t}$			−0.00842***	+0.0148***
			(0.000718)	(0.000777)
Sample	$X^{\$}_{c,p,t-}$=0 & $M^{\$}_{c,t-}$=0		$M^{\$}_{c,p,t-}$=0 & $X^{\$}_{c,t-}$=0	
Country × Product × Time FE	Y	Y	Y	Y
Firm × Product × Time FE	Y	Y	Y	Y
Firm × Country × Product FE	Y	Y	Y	Y
R-squared	0.888	0.910	0.850	0.854
Observations	16,441,857	16,441,857	16,468,272	16,468,272

Notes: The unit of observation is the firm–country–product–year. Panel A measures the effect of import-side dollar adoption (Start $M^{\$}_{c,t}$) on the extensive margin of export activity: column (1) reports the effect on the probability of any non-dollar export activity on the same product–country pair, and column (2) on any export activity in any currency. Panel B measures the corresponding effects of export-side dollar adoption (Start $X^{\$}_{c,t}$): column (3) on non-dollar import activity, and column (4) on any import activity. Import-side dollar adoption restructures the export relationship entirely toward dollars (Panel A: both margins fall). Export-side dollar adoption reduces non-dollar import activity while opening new import links (Panel B: column 3 negative, column 4 positive). Standard errors clustered at the firm–country–product level. *, **, *** significant at 10%, 5%, 1%.

Table B7: USD invoicing in exports and imports: Firm-country level evidence excluding USD-currency countries

	$\mathbb{I}\{X^{\$}_{c,t}\}$ (1)	$\mathbb{I}\{X^{\$}_{c,t}\}$ (2)	$\mathbb{I}\{X^{\$}_{c,t}\}$ (3)	$\mathbb{I}\{X^{\$}_{c,t}\}$ (4)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (5)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (6)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (7)	$\mathbb{I}\{M^{\$}_{c,t}\}$ (8)
$\mathbb{I}\{M^{\$}_{c,t-1}=1\}$	0.429***	0.951***						
	(0.0119)	(0.0264)						
Start $M^{\$}_{c,t}$			4.181***					
			(0.0615)					
Stop $M^{\$}_{c,t}$				−0.0526***				
				(0.00641)				
$\mathbb{I}\{X^{\$}_{c,t-1}=1\}$					0.199***	0.476***		
					(0.00609)	(0.0144)		
Start $X^{\$}_{c,t}$							1.922***	
							(0.0284)	
Stop $X^{\$}_{c,t}$								−0.0509***
								(0.00494)
Sample	Full	$X^{\$}_{c,t-}$=0	$X^{\$}_{c,t-}$=0 & $M^{\$}_{c,t-}$=0	$X^{\$}_{c,t-}$=1	Full	$M^{\$}_{c,t-}$=0	$M^{\$}_{c,t-}$=0 & $X^{\$}_{c,t-}$=0	$M^{\$}_{c,t-}$=1
Country × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Country FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.666	0.419	0.441	0.666	0.807	0.515	0.557	0.731
Identifying obs	12,574,003	11,419,071	10,429,732	614,917	12,574,003	11,045,655	10,429,732	719,478
Total sample	15,213,884	14,056,466	13,082,490	806,439	15,213,884	13,668,914	13,082,490	1,193,991

Notes: The unit of observation is the firm-country-year. The sample excludes countries in which the US dollar is the national currency. $t-$ refers to both $t-1$ and $t-2$. Dependent dummy variables are rescaled by their unconditional means; coefficients measure percentage changes relative to the baseline adoption probability. Additional controls include the share of exports invoiced in USD by other Italian exporters in the same HS4 industry–destination and a proxy for firm market share in the destination. Standard errors clustered at the firm-country level. *, **, *** significant at 10%, 5%, 1%.

Table B8: USD invoicing in exports and imports: Firm-country-product level evidence excluding USD-currency countries

	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (1)	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (2)	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (3)	$\mathbb{I}\{X^{\$}_{c,p,t}\}$ (4)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (5)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (6)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (7)	$\mathbb{I}\{M^{\$}_{c,p,t}\}$ (8)
$\mathbb{I}\{M^{\$}_{c,t-1}=1\}$	0.0970***	0.254***						
	(0.0141)	(0.0166)						
Start $M^{\$}_{c,t}$			1.655***					
			(0.0322)					
Stop $M^{\$}_{c,t}$				−0.178***				
				(0.0150)				
$\mathbb{I}\{X^{\$}_{c,t-1}=1\}$					0.0762***	0.207***		
					(0.00508)	(0.00671)		
Start $X^{\$}_{c,t}$							0.452***	
							(0.00941)	
Stop $X^{\$}_{c,t}$								−0.165***
								(0.0124)
Sample	Full	$X^{\$}_{c,p,t-}$=0	$X^{\$}_{c,p,t-}$=0 & $M^{\$}_{c,t-}$=0	$X^{\$}_{c,p,t-}$=1	Full	$M^{\$}_{c,p,t-}$=0	$M^{\$}_{c,p,t-}$=0 & $X^{\$}_{c,t-}$=0	$M^{\$}_{c,p,t-}$=1
Country × Product × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Product × Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm × Country × Product FE	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.813	0.688	0.704	0.818	0.913	0.736	0.780	0.827
Identifying obs	20,164,039	18,938,337	16,017,377	298,642	20,164,039	18,469,716	16,025,732	383,899
Total sample	46,450,633	45,252,142	39,078,795	1,198,491	46,450,633	44,016,771	40,184,351	2,433,862

Notes: The unit of observation is the firm-country-product-year. The sample excludes countries in which the US dollar is the national currency. $t-$ refers to both $t-1$ and $t-2$. Dependent dummy variables are rescaled by their unconditional means; coefficients measure percentage changes relative to the baseline adoption probability. Standard errors clustered at the firm-country-product level. *, **, *** significant at 10%, 5%, 1%.

Table B9: USD invoicing shares in exports and imports: Intensive-margin evidence

	Firm-country level				**Firm-country-product level**			
	$X^{\$}_{c,t}/X_{c,t}$ (1)	$\Delta\left(X^{\$}_{c,t}/X_{c,t}\right)$ (2)	$M^{\$}_{c,t}/M_{c,t}$ (3)	$\Delta\left(M^{\$}_{c,t}/M_{c,t}\right)$ (4)	$X^{\$}_{c,p,t}/X_{c,p,t}$ (5)	$\Delta\left(X^{\$}_{c,p,t}/X_{c,p,t}\right)$ (6)	$M^{\$}_{c,p,t}/M_{c,p,t}$ (7)	$\Delta\left(M^{\$}_{c,p,t}/M_{c,p,t}\right)$ (8)
$M^{\$}_{c,t-1}/M_{c,t-1}$	0.03245***				0.003379***			
	(0.001639)				(0.0003411)			
$\Delta\left(M^{\$}_{c,t}/M_{c,t}\right)$		0.1381***				0.01099***		
		(0.002931)				(0.0006463)		
$X^{\$}_{c,t-1}/X_{c,t-1}$			0.06742***				0.009173***	
			(0.002525)				(0.0005589)	
$\Delta\left(X^{\$}_{c,t}/X_{c,t}\right)$				0.1942***				0.03063***
				(0.003974)				(0.001004)
Sample	Full	Continuing	Full	Continuing	Full	Continuing	Full	Continuing
Country × Time FE	Y	Y	Y	Y				
Firm × Country FE	Y	Y	Y	Y				
Firm × Time FE	Y	Y	Y	Y				
Country × Product × Time FE					Y	Y	Y	Y
Firm × Product × Time FE					Y	Y	Y	Y
Firm × Country × Product FE					Y	Y	Y	Y
R-squared	0.800	0.365	0.845	0.340	0.810	0.571	0.862	0.448
Identifying obs	987,728	602,558	983,111	602,558	4,165,236	1,976,776	8,619,640	4,697,304
Total sample	1,501,846	891,566	1,494,438	891,566	11,217,394	4,961,228	16,478,303	8,339,486

Notes: The unit of observation is the firm-country-year in columns (1)–(4) and the firm-country-product-year in columns (5)–(8). $X^{\$}_{c,t}/X_{c,t}$ and $M^{\$}_{c,t}/M_{c,t}$ are the shares of the firm's exports to and imports from country c invoiced in U.S. dollars in year t. In columns (5)–(8) the dependent variables are the corresponding shares at the firm-country-product-year level, while the explanatory variable remains the firm-country-year share. Columns (1), (3), (5), and (7) relate the current USD share on one side of a corridor to the one-year-lagged USD share on the opposite side, on the sample (*Full*) in which the dependent variable, the lagged opposite-side share, and the controls are available. Columns (2), (4), (6), and (8) are first-difference specifications on the sample (*Continuing*) in which both shares are observed in consecutive periods, and at the product level the product relationship as well, so that they are identified from changes in invoicing shares within pre-existing trade relationships rather than from entry or exit. Country-level specifications include country-by-time, firm-by-country, and firm-by-time fixed effects and the same additional controls as in Table 2. Product-level specifications include country-by-product-by-time, firm-by-product-by-time, and firm-by-country-by-product fixed effects. Standard errors are clustered at the firm-country level in columns (1)–(4) and at the firm-country-product level in columns (5)–(8). *, **, *** significant at 10%, 5%, 1%.

Table B10: Currency shocks and profits

	$\frac{NFXI_t}{A_{t-1}}$				$\frac{\pi_t}{A_{t-1}}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta e_t^{€,\$} \times \frac{X^{\$}_{t-1}}{A_{t-1}}$	0.602***		0.672***		−2.852***		0.121	
	(0.0219)		(0.0313)		(0.760)		(0.871)	
$\Delta e_t^{€,\$} \times \frac{M^{\$}_{t-1}}{A_{t-1}}$		−0.0384***	−0.0633***			−0.428***	−0.249**	
		(0.00220)	(0.00491)			(0.0623)	(0.105)	
$\Delta e_t^{€,\$} \times \frac{X^{\$}_{t-1}-M^{\$}_{t-1}}{A_{t-1}}$				0.0586***				0.241***
				(0.00268)				(0.0484)
$\frac{X^{\$}_{t-1}}{A_{t-1}}$	0.000233		0.000477		0.0321***		0.0480***	
	(0.000367)		(0.000497)		(0.0118)		(0.0135)	
$\frac{M^{\$}_{t-1}}{A_{t-1}}$		−0.000915***	−0.00105***			0.0477***	0.0397***	
		(0.0000925)	(0.000236)			(0.00364)	(0.00674)	
$\frac{X^{\$}_{t-1}-M^{\$}_{t-1}}{A_{t-1}}$				0.000554***				−0.0261***
				(0.000213)				(0.00593)
Sample	$X^{\$}_{t-1}>0$	$M^{\$}_{t-1}>0$	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$		$X^{\$}_{t-1}>0$	$M^{\$}_{t-1}>0$	$X^{\$}_{t-1}>0$ & $M^{\$}_{t-1}>0$	
Firm FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y	Y	Y
R-squared	0.289	0.251	0.287	0.288	0.663	0.607	0.673	0.673
Observations	314,627	750,670	208,028	208,028	314,623	750,645	208,028	208,028

Notes: The dependent variable in columns (1)–(4) is net foreign exchange income (*utili e perdite su cambi*) scaled by lagged total assets. The dependent variable in columns (5)–(8) is pre-tax profits scaled by lagged total assets. $\Delta e_t^{€,\$}$ is the log change in the EUR/USD rate (positive = euro depreciation). $X^{\$}$ ($M^{\$}$) are lagged dollar-invoiced exports (imports); A is lagged total assets. Columns (5)–(8) additionally restrict to firms with non-missing NFXI to ensure comparability across panels. All specifications include firm and year fixed effects. Standard errors clustered at the firm level. *, **, *** significant at 10%, 5%, 1%.

C Construction of Linked Import Products

This section describes the construction of the *linked import products* measure used in the main analysis. The goal is to identify, for each exported product, the set of imported products that are disproportionately associated with that export activity within firms, while giving greater weight to economically important links and excluding overly ubiquitous imported goods.

Let i index firms, t years, p exported products, and m imported products, all defined at the HS4 level. The use of HS4 strikes a balance between granularity and statistical power: it is sufficiently detailed to capture meaningful input–output relationships across products, while avoiding the sparsity and measurement noise that arise at finer levels of disaggregation. Let $X_{i,t,p}$ denote exports of product p by firm i in year t, and let $M_{i,t,m}$ denote imports of product m.

We first collapse the transaction-level data to the firm–year–product level by summing export and import values within each firm–year–product cell. All variables are therefore defined at the HS4 product level. We retain only strictly positive trade flows.

For each firm-year (i, t), we construct all product pairs (p, m) such that firm i exports product p and imports product m in year t. Operationally, this is obtained by matching the export and import product-level datasets at the firm–year level. The resulting dataset records, for each firm-year, the joint presence of exported and imported HS4 products.

For each pair (p, m), we compute the total import value of product m across all firm-years in which product p is exported:

$$\text{ImportMass}_{pm} = \sum_{i,t:\, X_{i,t,p}>0} M_{i,t,m}.$$

We also compute the total import mass across all imported products among firm-years exporting p:

$$\text{TotalImportMass}_{p} = \sum_{m} \text{ImportMass}_{pm}.$$

This allows us to define the conditional import share

$$s_{pm} \equiv \Pr(m \mid p\text{-exporter}) = \frac{\text{ImportMass}_{pm}}{\text{TotalImportMass}_{p}}.$$

Next, for each imported product m, we compute its share in aggregate imports:

$$s_m \equiv \Pr(m) = \frac{\sum_{i,t} M_{i,t,m}}{\sum_{i,t,m} M_{i,t,m}}.$$

This captures how common product m is in the overall import distribution.

We then define the baseline association between exported product p and imported product m as

$$\text{Assoc}_{pm} = \frac{s_{pm}}{s_m}.$$

This ratio measures whether m is disproportionately imported by firm-years exporting p, relative to the unconditional prevalence of m in the import basket. Values above one indicate that m is overrepresented among exporters of p.

Some imported products are extremely common across firms and sectors. As a result, they tend to co-occur mechanically with a large number of exported products, generating high association scores that reflect their ubiquity rather than a meaningful economic link. This is particularly relevant for generic inputs (e.g., basic intermediates or widely used components) that enter a broad range of production processes. To isolate product-specific linkages, we exclude imported products in the upper tail of the aggregate import-share distribution before ranking product pairs. In the baseline specification, we drop imported products whose aggregate share s_m lies above the 95th percentile of the cross-product distribution. This restriction removes products whose high prevalence would otherwise inflate co-occurrence measures without providing information about the structure of firms' export-related input use. By filtering out these ubiquitous goods, the linked-products measure more cleanly captures differential input usage across export activities, rather than common inputs shared across the entire economy.

To further sharpen the measure, among the remaining (p, m) pairs we retain only pairs with non-negligible import mass and rank them using a penalized association score:

$$\text{Assoc}^{\text{pen}}_{pm} = \text{Assoc}_{pm} \times \ln\Big(\text{ImportMass}_{pm}\Big).$$

This ranking preserves the relative logic of the baseline association measure, but gives greater weight to links that are not only disproportionately represented among exporters of p, but also economically important in value terms. Intuitively, this helps distinguish strong and quantitatively relevant links from links that are highly specific but based on very small trade values.

For each exported product p, we rank all candidate imported products m in descending order of $\text{Assoc}_{pm}^{\text{pen}}$ and retain the top K products. In the baseline specification, we set $K = 20$. We denote the resulting set by

$$\mathcal{L}(p) = \left\{ m : \text{rank of } \text{Assoc}_{pm}^{\text{pen}} \leq K \right\}.$$

Thus, $\mathcal{L}(p)$ collects the imported products that are most strongly and most economically relevantly associated with exports of product p in the data. Our results are robust to more restrictive thresholds (e.g., $K = 10$ and $K = 5$), which yield qualitatively similar patterns.

For each firm-year (i, t), we define the set of linked import products as the union of the linked sets corresponding to all products exported by the firm in that year:

$$\mathcal{L}_{i,t} = \bigcup_{p:\, X_{i,t,p}>0} \mathcal{L}(p).$$

An imported product therefore enters the firm-year linked set if it belongs to the top-ranked associated import products of at least one exported product in the firm's contemporaneous export basket.

For each firm–country–product–year import observation, we define the indicator

$$\mathbb{I}\{X_{i,t}^{\text{Linked}(p)} > 0\} = \begin{cases} 1 & \text{if } m \in \mathcal{L}_{i,t}, \\ 0 & \text{otherwise.} \end{cases}$$

That is, the indicator equals one if the imported product in the observation belongs to the set of products most strongly linked, in the data, to the firm's export basket in that year.

Interpretation. The linked-products measure is a revealed-complementarity measure constructed directly from within-firm trade patterns. It does not rely on external input-output tables, engineering classifications, or predefined technological correspondences. Instead, it uses the joint distribution of the universe of imports and exports within firms to identify imported goods that are systematically and disproportionately associated with particular export products.

The additional penalization by import mass is important. A pure association score may rank highly a product pair that is unusually specific but quantitatively negligible. By contrast, the penalized score favors links that are both specific and economically

meaningful. As a result, the final measure is best interpreted as identifying imported products that are plausibly relevant inputs or complementary goods for a firm's export activity, rather than merely products that co-occur sporadically with exports.

Filtering and robustness. To reduce noise, we exclude pairs with negligible import mass before ranking, and we remove highly ubiquitous imported goods from the candidate set. The main results are robust to alternative but closely related implementations of the linked-products measure. In particular, the qualitative pattern is unchanged when we vary the stringency of the ubiquity filter (for example, excluding only the top 2 percent or instead the top 10 percent of imported products by aggregate share), when we vary the number of retained linked products per exported product (for example, top 5 or top 10 instead of top 20), and when we impose an additional within-firm-year restriction based on the best rank attained by a linked product. Across these variants, the exact magnitude and precision of the estimates naturally vary, but the economic message remains the same: import quantity responses are stronger for imported goods that are more tightly connected to firms' export baskets.