Supplementary Material

for 'Uncertainty as Commitment'
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Appendix A. Proofs and Derivations

Appendix A.1. Derivation of Assumptions 3 and 4

If $\bar{t} < t^*$, value functions in equation (6) can be rewritten (net of the value of taking over another bank, V_{TO}) as

$$V(c|\bar{t} < t^*) = (P_0+P_1)[c+\rho_0+\rho_1]i(c)+P_1\rho_1\frac{c}{1-\rho_0}i(c)+P_2\left[c+(\rho_0+\rho_1-1)\frac{c}{1-\rho_0}\right]i(c)$$

when replacing $j(c) = (\bar{t}-1)i(c)$ by $j(c) = \frac{c}{1-\rho_0}i(c)$. Hence

$$V(c|\bar{t} < t^*) = (P_0+P_1)[c+\rho_0+\rho_1]i(c) + (P_1+P_2)\rho_1\frac{c}{1-\rho_0}i(c)$$

Since $ci = (\pi-1)i(c) + A$,

$$V(c|\bar{t} < t^*) = \left[P_0 + P_1 + (P_1+P_2)\frac{\rho_1}{1-\rho_0}\right][(\pi-1)i(c)+A]+(P_0+P_1)(\rho_0+\rho_1)i(c).$$

Taking derivatives with respect to c, $\frac{dV(c)}{dc} = \frac{\partial V(c)}{\partial i}\frac{\partial i}{\partial c}$ where

$$\frac{\partial V(c)}{\partial i} = (P_0+P_1)[\pi-1+\rho_0+\rho_1] + (P_1+P_2)\frac{\rho_1}{1-\rho_0}(\pi-1)$$

$$\frac{\partial i(c)}{\partial c} = -\frac{i^2(c)}{A} < 0$$

Then

$$\frac{dV(c)}{dc} = -C\frac{i^2(c)}{A}$$

where

$$C \equiv (P_0+P_1)\left[\rho_0+\rho_1+\pi-1\right] - (P_1+P_2)\frac{1-\pi}{1-\rho_0}\rho_1$$

Then, $\frac{dV(c)}{dc} > 0$ for $t < t^*$ if $C < 0$, as in Assumption 3.

If $\bar{t} > t^*$, value functions can be rewritten as

$$V(c|\bar{t} > t^*) = (P_0+P_1)[c+\rho_0+\rho_1]i(c)+P_1\rho_1\frac{c}{1-\rho_0}i(c)+P_2\left[c+\rho_0+\rho_1-(t^*-1)-\rho_0(2-t^*)\right]i(c)$$

A sufficient condition for this value function to decrease in c can be determined at $t^* = 2$, when the incentives to hold cash are maximized. Then

$$V(c|t^* = 2) = (P_0+P_1)[c+\rho_0+\rho_1]i(c)+P_1\rho_1\frac{c}{1-\rho_0}i(c)+P_2\left[c+\rho_0+\rho_1-1\right]i(c)$$

Again

$$\frac{dV(c)}{dc} = -D\frac{i^2(c)}{A}$$

where

$$D \equiv (P_0 + P_1 + P_2)\left[\rho_0 + \rho_1 + \pi - 1\right] - P_1 \frac{1-\pi}{1-\rho_0}\rho_1 - P_2$$

Then, $\frac{dV(c)}{dc} < 0$ for $t < t^*$ if $D > 0$, as in Assumption 4.

Appendix A.2. Proof of Lemma 1

Fix t^* and consider c such that $\frac{c}{1-\rho_0} < t^* - 1$. The bank's value function on this domain is

$$V(c) = \left[P_2(c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)) + (P_0 + P_1)(c + \rho_0 + \rho_1) + P_1\rho_1(\bar{t}(c) - 1)\right] i + P_1 V_{TO}$$

where $V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$ is independent of c. Since

$$ci = (\pi - 1)i + A$$

$V(c)$ can be written just in terms of $i(c)$, replacing ci into $j = (\bar{t}(c) - 1)i = \frac{ci}{1-\rho_0}$.

Since $\frac{dV(c)}{dc} = V'(i)i'(c)$ and $i'(c) = -\frac{A}{(1-\pi+c)^2} < 0$, then $\frac{dV(c)}{dc} > 0$ if and only if $V'(i) < 0$, which is the case if

$$V'(i) = (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - (P_1 + P_2)\frac{\rho_1}{1-\rho_0}(1-\pi) \underbrace{<}_{\text{Assumption 3}} 0.$$

Whenever c is too low to fully refinance at $R = 1$, it is optimal for the bank to increase cash holdings. For $1 > \frac{c}{1-\rho_0} > t^* - 1$, the bank always refinances fully on the market and the value function is

$$V(c) = \left[P_2(c + \rho_0 + \rho_1 - (t^* - 1) - \rho_0(2 - t^*)) + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1(\bar{t}(c) - 1)\right] i + P_1 V_{TO}.$$

Again, $\frac{dV(c)}{dc} = V'(i)i'(c)$ and $i'(c) < 0$, which implies that $\frac{\partial V(c)}{\partial c} < 0$ if and only if $V'(i) > 0$.

$$V'(i) = (P_0 + P_1 + P_2)(\pi - 1 + \rho_0 + \rho_1) - P_1\rho_1\frac{(1-\pi)}{1-\rho_0} - P_2((1-\rho_0)(t^* - 1) + \rho_0)$$

Then $V'(i) > 0$ if and only if

$$\begin{aligned}
&(P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1\frac{\rho_1}{1-\rho_0}(1-\pi) - P_2(\rho_0 + (1-\rho_0)(t^* - 1)) \\
&\quad > (P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1\frac{\rho_1}{1-\rho_0}(1-\pi) - P_2 \underbrace{>}_{\text{Assumption 4}} 0.
\end{aligned}$$

Hence, on this part of the domain, it is optimal to decrease cash holdings.

Finally, for $\frac{c}{1-\rho_0} > 1$, the value of the bank is

$$V(c) = (P_2 + P_1)(c + \rho_1 + \rho_0 - 1)i + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1 V_{TO},$$

taking the derivative with respect to i and considering $\frac{dV(c)}{dc} < 0$ if $V'(i) > 0$

$$V'(i) = (\pi + \rho_1 + \rho_0) - (1 + P_1 + P_2) \underbrace{>}_{\text{Assumption 1}} 0.$$

Therefore, here also it is optimal to decrease cash, which completes the proof. ■

Appendix A.3. Proofs of Propositions 1, 2 and 3

Follow directly from the arguments in the main text.

Appendix A.4. Proof of Proposition 4

Part (i) follows directly from the arguments in the main text. For part (ii), notice that (15) can be expressed as

$$-Z + \frac{\phi(0)}{\sqrt{2}\sigma_h} \frac{P_2 \rho_1}{(1-\rho_0)} (1 - \rho_0 - c)(1 - \pi + c) = 0$$

and is quadratic in c. Moreover, with $\sigma < \underline{\sigma}_h$, one root is strictly positive and in $[0, 1-\rho_0]$ (and a local max), and one is strictly negative (and a local min). What this means is that for any $\sigma_h < \underline{\sigma}_h$, there is exactly one root $c^*(\sigma_h) \in [0, 1-\rho_0]$ which solves (15) and the derivative is positive for all $0 \leq c < c^*(\sigma_h)$.

For the last part of the proof, notice that $c^*(\sigma_h)$ is decreasing in σ_h and as $\sigma_h \to 0$, the variable term in (15) goes to $+\infty$ and is always positive, hence giving a corner solution in the limit, such that $c^*(\sigma_h = 0) = 1 - \rho_0$.[18]

Appendix A.5. Proof of Lemma 2

For $N = 2$ the condition for not bailing out the first bank in distress is given by condition (8) with $P_2' = p_{(2,2)|1} = p$. In the case of $N = 3$, the social gains from bailing out the first bank in distress are

$$p3x + (1-p)\left[\frac{p_{(3,2)|1}}{(1-p)} 2x + \frac{p_{(3,1)|1}}{(1-p)} x\right],$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p2x + (1-p)\left[\frac{p_{(3,2)|1}}{(1-p)} (y + x) + \frac{p_{(3,1)|1}}{(1-p)} y\right]$$

[18] To complete the characterization, as $\sigma_h \to \underline{\sigma}_h$, the equilibrium cash holdings $c^*(\sigma_h)$ converges to zero if $\pi < \rho_0$ and to $\pi - \rho_0$ if $\pi > \rho_0$.

Since $\frac{p_{(3,2)|1}}{(1-p)} + \frac{p_{(3,1)|1}}{(1-p)} = 1$, the condition to delayed bailout is also $p_{(3,3)|1} = p < 1 - \frac{x}{y}$. More generally, for any arbitrary N, the social gains from bailing out the first bank in distress are

$$pNx + (1-p)\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p)} dx$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p(N-1)x + (1-p)\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p)} (y + (d-1)x).$$

Since $\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p)} = 1$, the condition to delayed bailout is again

$$p_{(N,N)|1} = p < 1 - \frac{x}{y}.$$

■

Appendix A.6. Proof of Lemma 3

For the $N = 3$ case, the delayed bailout condition is exactly as in the two bank case. For $N = 4$, the social gains from bailing out the first bank in distress are

$$p4x + (1-p)[x + q(p'2x + (1-p')x)]$$

while the social gains from not bailing out the first bank in distress are

$$p3x + (1-p)[y + q\max\{p'2x + (1-p')x, p'x + (1-p')y\}] \quad \text{(A.1)}$$

where $q = \frac{p_{(4,3)|1} + p_{(4,2)|1}}{(1-p)}$, $p' = p_{(4,3)|2,to} = \frac{p_{(4,3)|1}}{q(1-p)}$ and $(1-p') = p_{(4,2)|2,to} = \frac{p_{(4,2)|1}}{q(1-p)}$. Then it is optimal to not bail out the first bank in distress if

$$(1-p)(y-x) + (1-p)q\max\{-p'x + (1-p')(y-x), p'x - (1-p')(y-x)\} > px \quad \text{(A.2)}$$

The last term in equation (A.1) is the value of keeping the option of introducing the bailout at a later time, under new posterior p' which is necessarily more accurate. Since this number is at least weakly bigger than the value of immediate bailout, the actual cutoff value for delayed bailout condition $\bar{P}$ is weakly higher than for the two bank case. For values of p' for which it is optimal to not bail out the second bank in distress, the increase is strictly positive. This shows that the cutoff probability for delayed bailout in the $N = 4$ case is equal or larger than the two or three banks case.

For the more general N bank case the condition for delayed bailout is exactly (A.2) if we restrict p' and q to be the same across N and government policies to always bail out the third bank in distress. More generally, incentives to delay depend on the sequence of Bayesian updates in response to observing takeovers. Let $p'_{(N,N-k)|k,to}$ be the posterior probability that $N-k$ banks are in distress conditional on observing k takeovers (well defined for $2k < N$). Suppose that for two different numbers of banks in the economy, $M > N$, $p'_{(M,M-k)|k,to} = p'_{(N,N-k)|k,to}$, whenever both are well defined. In this case, giving the government more chances to introduce the bailout always has an effect of relaxing the delayed bailout condition. It is a sum of nonnegative numbers for both M and N, and these numbers are the same in both cases up to the point where you cannot delay further with N. In the M case, however, there are at least one more nonnegative term, as in the comparison between $N = 3$ and $N = 4$. Since the government will have at least weakly more chances for M than in N, the left hand side of the analog of (A.2) will be weakly larger.■

Appendix A.7. Proof of Lemma 4

The social gains from bailing out the first bank in distress is

$$p(N)Nx + (1 - p(N)) \sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p(N))} dx$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p(N)(N-1)x + (1 - p(N)) \sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p(N))} (y + (d-1)x)$$

Since $\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p(N))} = 1$, the condition to delay intervention is

$$p(N) < 1 - \frac{x}{y}$$

While the cutoff for delay does not change, the updated probability $P(N)$ is decreasing in N if the idiosyncratic shocks are independent, triggering strategic restraint more likely for larger number of firms. For example, if the probability of no aggregate shock is α and the probability of no idiosyncratic shock is λ, then under independence, the probability of no healthy banks after observing the first one is given by

$$p(N) \equiv p_{(N,N|1)} = \frac{(1-\alpha) + \alpha(1-\lambda)^N}{1 - \alpha\lambda},$$

clearly decreasing in N. ■

Appendix A.8. Proof of Proposition 5

Follows directly from the arguments in the main text.

Appendix B. Extensions

This section provides a discussion of the robustness of our results to modifications of the main assumptions of the model.

Timing of Moves. The benchmark model's setup assumes that the government decides whether to bail out a distressed bank or not *before* knowing if the non-distressed bank is willing to take over. Furthermore, this decision was assumed irreversible – the government cannot ex-post reverse the bailout, even if the true shock turns out to be idiosyncratic (i.e. the healthy bank would have been interested in taking over the distressed project). The setup in this section gives the first move opportunity to the healthy bank, before the bailout decision by the government.

Specifically, assume that in case of a bank's distress, the healthy bank moves first with a takeover bid, then the government decides to implement the bailout, and then the healthy bank moves again and has another chance to make a takeover bid. The idea is that, if a takeover bid is not made in the first move of the non-distressed bank, it can still be made ex-post, once the government's action is known.

The outcome of the model under the new assumption depends crucially on the outside investment opportunities of the healthy bank. Below, we sketch a modification of the model under which each bank has access to alternative investment opportunities, and the payoffs of these opportunities depend on the stability of the financial sector. This assumption captures the idea that, when a bank gets liquidated, there may be negative external effects on the investment opportunities of other banks, for example due to contagion, fire sales, runs, higher collateral requirements, tightening on regulation or government oversight, etc.

Formally, assume that i) banks have an additional investment opportunity to undertake at $t = 2$, and ii) such additional investment opportunity is more valuable when the projects of both banks come to fruition. More specifically, assume that the additional investment opportunity pays $\widehat{\rho} > 1$ per unit of investment if both banks' projects continue until $t = 2$, and zero otherwise. The healthy bank's payoffs from taking over are $\rho_0 + \rho_1 > 1$. In contrast, the healthy bank's payoffs if not taking over depend on the government's bailout choices. If there is a bailout, then the healthy bank can invest in the alternative project and gain $\widehat{\rho} > 1$ per unit of investment. If there is no bailout, the healthy bank still has a second chance to take over and to gain $\rho_0 + \rho_1 > 1$ per unit of investment.

Clearly, for $\rho_0 + \rho_1 \geq \widehat{\rho}$, the healthy bank always makes an immediate takeover bid, and the government gets to make the bailout decision under full information, bailing out only if there was no take over. In contrast, if $\rho_0 + \rho_1 < \widehat{\rho}$, the healthy bank makes a takeover bid only when there is no bailout ex-post. In this case, the government is uncertain when making its decision and the analysis of the benchmark model with imperfect information applies.

Note that so far the maintained assumption is of *targeted* bailouts. Hence, the type of the bailout policy is not crucial for our mechanism, but the ability and willingness of healthy banks to communicate truthfully their own realized shock is.

Non-targeted bailouts. In this section, the the scope of the bailout is increased so that the transfer is also available to the healthy banks, a case labeled *non-targeted bailout*. In this case, the expected social cost of bailouts may increase if there is a social cost to providing below-market-rate lending to healthy banks. Denoting that social cost as $\hat{x}$, the delayed bailout condition becomes

$$P_2' < \bar{P} \equiv 1 - \frac{x}{y - \hat{x}}$$

which is clearly more easily satisfied if $\hat{x} < 0$ and it is socially costly to transfer subsidized resources to otherwise safe banks.

With non-targeted policies it is easier that uncertainty acts as commitment for two reasons: it is less likely that healthy banks reveal the true state and it is more likely that governments delay interventions.

Non-targeted policies constitute a special case in which healthy banks prefer governments to introduce bailouts and obtain cheap funding than taking over distressed projects, revealing the true state to the government and giving up those subsidized funds. Using the previous notation, when the gains from alternative investment opportunities are $\hat{\rho}$, the social costs of bailing out healthy banks are $\hat{x} = \beta(\hat{\rho} - \rho_0) - (1 - \rho_0)$, which is negative as long as the welfare gains from the transfer to healthy banks, $\beta(\hat{\rho} - \rho_0)$, are smaller than the social costs from subsidized funds, $1 - \rho_0$.

Additionally, with non-targeted bailouts, healthy banks take over immediately and eliminate uncertainty when $\rho_0 + \rho_1 > \widehat{\rho} + (1 - \rho_0)$, which is more difficult to fulfill than with targeted policies.

This discussion implies that a government would like to implement ex-ante institutional arrangements that induce the use of non-targeted bailouts when they know they will face a bailout decision in the future. This is especially true if monitoring or ex-ante regulation is deemed ineffective or inefficient.

Gradual bailouts. The benchmark model's setup assumes that bailouts are discrete events – if the government does not provide the total amount needed to bring the project to fruition, the project is prematurely terminated.

This section considers an alternative technology that allows governments to inject funds gradually rather than completely, all the while keeping the project running. In principle a government would prefer this possibility because the nature of the shock may be eventually revealed, in which case the government would rather induce a take over from the healthy bank at least of a part of the distressed project.

The rest of the analysis in this section discusses two cases. First, if the government knows that will always provide a gradual bailout to keep the option value of learning in the future, then the analysis in the paper goes through without change. Intuitively, at the moment one bank shows distress, the government internalizes its own dynamic inconsistency and rationally chooses to not bail out at the moment of observing distress, inducing the distressed project to be terminated if the shock were aggregate. The delayed bailout condition in this case is identical to the one in the paper. The second case allows the government to provide a bailout between the first moment a bank shows distress, t, and $t' > t$, where t' is chosen optimally. The delayed bailout condition is derived for this case. The details of these two cases are fleshed out below.

Suppose first that the government is always tempted to provide a bailout for an extra 'second' while keeping the option of reversing the bailout later. In the meantime, a healthy bank does not have incentives to take over the distressed project (that is, if $\widehat{\rho} > \rho_0 + \rho_1$). At moment t, when a bank shows distress, a forward looking government internalizes these decisions, and hence the expected value of prolonging the bailout is actually bailing out until the true state is revealed to be aggregate – in which case the government bails out the second bank as well. The value of such strategy is

$$(2-t)ix + P_2(t)(2-\hat{t}(t))ix,$$

where $\hat{t}(t)$ is the expected time of distress of the second bank, conditional on an aggregate shock and conditional on the second bank not having showed distress until t when the shock is aggregate. Alternatively, if the government does not bail out the current distressed bank, which implies termination if the shock is aggregate, the value is

$$(1-P_2(t))(2-t)yi + P_2(t)(2-\hat{t}(t))xi.$$

The government, anticipating the behavior of future selves, choses do discontinue the bailout immediately if

$$(1-P_2(t))(2-t)y > (2-t)x \iff P_2(t) < 1 - \frac{x}{y}$$

which is exactly the delayed bailout condition (8). Hence, if the government knows it will be always tempted to provide gradual bailouts, and in case the idiosyncratic shock the healthy bank does not have incentives to take over, then it would rationally delay bailout under the same conditions as in the full model, and the same equilibrium analysis goes through.

Consider now a more general case, where there is an optimal time t' such that if until t' there is no failure of the second bank, then it is optimal to stop the bailout, but not before. Hence, t' is defined by indifference between continuing the bailout for another moment,

$$V_b(t) = ixdt + \frac{d}{dt}max[V_b(t), V_{nb}(t)]$$

and, as before, stopping has a value

$$V_{nb}(t) = (1-P_2(t))(2-t)yi + P_2(t)(2-\hat{t}(t))xi.$$

t' is then implicitly defined by

$$V_b(t') = V_{nb}(t').$$

Consider now this alternative strategy of choosing t' optimally. The value of keeping a bailout until t' is

$$(1-P_2(t))[(2-t')y+(t'-t)x]i+P_2(t)[(2-\hat{t})x+p(t_2>t')(t'-t)x+p(t_2\leq t')(2-t)x]i$$

where $p(t_2<t')$ is the probability that in case of an aggregate shock the second bank will show distress before the stopping time t',

$$p(t_2\leq t')=\frac{F(t')-F(t)}{1-F(t)}$$

where F is the cdf of the time of distress, conditional on surviving until t. The delayed bailout condition is now

$$(1-P_2(t))(2-t)yi+P_2(t)(2-\hat{t})xi-$$

$$[(1-P_2(t))[(2-t')y+(t'-t)x]i-P_2(t)[(2-\hat{t})x+p(t_2>t')(t'-t)x+p(t_2\leq t')(2-t)x]i>0$$

which simplifies to

$$(1-P_2(t))(2-t)yi-[(1-P_2(t))[(2-t')y+(t'-t)x]i-P_2(t)[p(t_2>t')(t'-t)x+p(t_2\leq t')(2-t)x]i>0$$

and hence

$$(1-P_2(t))(t'-t)(y-x)-P_2(t)[(t'-t)x+p(t_2\leq t')(2-t')x]>0$$

Which gives the new condition on the posterior $P_2(t)$:

$$P_2(t)<\frac{y-x}{y+x(2-t')\frac{p(t_2\leq t')}{t'-t}}$$

Clearly, when bailouts are once and for all, then $t'=2$ and the equation reduces to (8). In the general case, there is a new expression $x(2-t')\frac{p(t_2\leq t')}{t'-t}$ in the denominator, potentially making the cutoff smaller, i.e. less likely to be satisfied. This additional term depends on $(2-t')$ and

$$\frac{p(t_2\leq t')}{t'-t}=\frac{1}{1-F(t)}\frac{F(t')-F(t)}{t'-t}.$$

The constraint becomes tighter when the product of these expressions increase. While $(2-t')$ is clearly positive and decreasing in t', $\frac{p(t_2\leq t')}{t'-t}$ is bounded below by the value of the pdf evaluated at the unconditional mean. In fact, for small bailouts, i.e. t' close to t, the first term is proportional to the value of the pdf evaluated at t, then bounded for a fixed σ_h.

It is instructive to look at the limiting cases. For σ_h approaching zero, the upper bound of $\frac{p(t_2\leq t')}{t'-t}$ becomes arbitrarily large – the pdf at the mean goes to infinity, and we know that delay cannot be sustained if it is only momentary. The reason for that is that the government knows that it will learn right away if the shock was idiosyncratic or not, so gradual bailouts are very attractive. For

σ_h going to infinity, the pdf at any t approaches zero and the delayed bailout condition becomes (8). Effectively, in that case the shock h has a two point distribution – if the distress hasn't happened yet (i.e. it is not at the lower end) then it will probably not happen (i.e. the shock is at the high end of the distribution).

To summarize, the possibility of gradual bailouts affects the cutoff belief about an aggregate shock under which a government delays bailing out the first bank in distress, even gradually, but not the main mechanism of uncertainty as commitment. In particular, the cutoff is bounded away from zero for all positive values of the variance σ_h.

Even though our mechanism is robust to the introduction of gradual bailouts, these types of bailouts are usually not feasible for financial intermediaries with very high levels of leverage and short term debt. A bank facing a bank run, for example, requires a large immediate volume of funds to keep their assets without being liquidated at fire sale prices. In thinking about non-financial corporations, however, gradual bailouts seems more plausible, which is consistent with their ubiquitous implementation in non-financial activity, such as the reorganization of car companies or airlines, for example.

Equilibrium Crises and Bailouts. The benchmark model features banks holding enough cash reserves such that on the equilibrium path, there are never bank failures. As a result, effectively the government never has to decide about bailouts. However, this is just a specific feature of the benchmark, which is purposefully kept stark and simple to highlight the main mechanism. In particular, in our extension to the continuous case in Section 2.4, bank failures and bailouts do happen *on the equilibrium path.* With shocks to cash holdings, banks still hold more liquidity than in the non-commitment (or full information) equilibrium, but not enough to survive fully a refinancing shock.

Even in the limiting case when they hold full liquidity, shocks may cause them to run out of cash prematurely, and then the government makes a bailout decision in consistency with the delayed bailout condition (8). Hence, when uncertainty is large enough, with positive probability, a path is realized in which both banks are in distress, the first one fails and the second one is bailed out. If uncertainty is not large enough, then our mechanism is not at work, and both the first and second banks are bailed out.

There certainly exist a variety of extensions of our model which is also consistent with bank distress and bailouts on the equilibrium path. One example may be a case of imperfect observability of the government's type, parameterized by the weight on bankers in the welfare function β. Banks may have prior beliefs that β is high enough such that the delayed bailout condition is not satisfied, and hence take on excessive leverage (belief about β affects the right hand side of (8)). Ex-post, it may turn out otherwise and we may see the government not bailing out the first bank(s). Another example would be an environment with imperfect observability of the government's prior about the state of the economy (say P_2^g). This modification would affect the left hand side of (8), and again could lead to suboptimal behavior. Both of these modifications imply a dynamic response in bank beliefs to observed bailouts or lack thereof. For example, observing a bank failure would imply an update and potential increased discipline in bank risk-taking that develops with learning about government's types or beliefs over time.

Strategic Coordination and Too Many To Fail. Both Farhi and Tirole (2012) and Acharya and Yorulmazer (2007) stress the idea that governments are more prone to bail out when many banks are in distress. In Farhi and Tirole (2012), these considerations come into play because the cost of a bailout is independent of the size of the bailout and depends only on the bailout interest rate. In Acharya and Yorulmazer (2007), many banks in distress means that efficient takeovers are less likely, and government takeover is more efficient.

Strategic Complementarities. In our setup, the costs of a bailout are proportional to the required transfers, and hence the tradeoffs of the government are the same, independently of the number of banks in distress or the amount of money at stake. As a straightforward extension, we could introduce and additional cost of a bailout, independent of the size of the transfer, for example a constant L. This constant captures the idea of fixed costs, either bureaucratic or political, that governments face when implementing bailouts, regardless of their magnitude. In this case the delayed bailout condition is

$$P_2' \leq \frac{y - x + \frac{L}{2-t}}{y + \frac{L}{2-t}}.$$

First, notice that for $L = 0$, this simplifies to (8). Second, a higher L makes governments more prone to delay bailouts. In particular, the government is less likely to bail out if the distress comes late (high t). Intuitively, the size of the saved part of the project becomes relative small compared to the cost of bailout, which now includes the fixed constant L.

Finally, introducing L may give rise to multiplicity in the full information economy, just like in Farhi and Tirole (2012). In particular, there is a range of parameters for which it is optimal for governments to bailout two failing banks (this is, $2xi - L > 0$), but not a single failing bank (this is $xi - L < 0$). This effect creates strategic complementarity in bank actions – they want to go down together to maximize the likelihood of bailouts.

In our setting $xi - L > 0$ so the government has incentives to bail out a single bank, and hence unique equilibrium is guaranteed. Whichever the parameter case, however, imperfect information and strategic restraint give rise to bank actions being strategic substitutes (a very extreme one indeed in the benchmark case, where banks compete a la Bertrand), potentially flipping the strategic properties of the equilibrium actions by the banks.

Too Many To Fail. The too-many-to-fail case analyzed in Acharya and Yorulmazer (2007) can be mapped into our Too Big To Fail example in Section 3.2. What matters is the total amount of assets (projects) under distress, and the single big bank can be thought of as a subset of banks that fall in distress as the same time. In such instance, if *too many* banks show distress initially, i.e. the size of the distressed asset group is too large to be taken over, then the government may be more compelled to bail out.

Two sets of conclusions may be drawn from this analogy. First, too big too fail and too many to fail are the same problems in our economy. What really matters is whether there is enough liquidity in the system to refinance projects in distress. Second, this may give banks additional incentives to correlate risks and becoming too big too fail as a group. From Section 3.2, if such activity passes a certain threshold level of coordinated failures the economy switches from one extreme (best) equilibrium to the opposite (worst, high leverage) equilibrium.