

Investor Sophistication and Capital Income Inequality*

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Abstract

Capital income inequality is large and growing fast, accounting for a significant portion of total income inequality. We study its determinants in a general equilibrium portfolio choice model with endogenous information acquisition and heterogeneity across household sophistication and asset riskiness. The model implies capital income inequality that grows with aggregate information technology. Investors differentially adjust both the size and composition of their portfolios, as unsophisticated investors retrench from trading risky securities and shift their portfolios toward safer assets. Technological progress also reduces aggregate returns and increases the volume of transactions, features that are consistent with recent U.S. data.

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1 Introduction

The rise in income and wealth inequality has been among the most hotly discussed topics in academic and policy circles.¹ Among the various possible explanations, heterogeneity in the returns on savings—due to differences in rates of return or in the composition of the risky portfolio—has been highlighted as an important driver of the distribution of both income and wealth in the data. This factor has emerged in empirical work that has studied the entire wealth distribution, such as Fagereng, Guiso, Malacrino, and Pistaferri (2016b, 2016a), as well as in research that has focused on the very top of the wealth distribution (Benhabib, Bisin, and Zhu (2011)), and on inequality due to entrepreneurship forces (Pástor and Veronesi (2016)).² However, as noted by De Nardi and Fella (2017), more work is needed to understand the determinants of such heterogeneity.

This paper studies a portfolio choice model in which information frictions drive capital income inequality. When investors differ in their capacity to process news about risky asset payoffs, both rates of return on assets and the composition of the risky portfolio differ across investors. Moreover, progress in aggregate information technology can exacerbate inequality and this effect can be economically large, as less sophisticated investors get priced out of high-return assets.

At the core of our model is each investor’s decision of how much to invest in assets with different risk characteristics. This decision is shaped by the investor’s capacity to process information about asset payoffs. We model the learning choice using the theory of rational inattention of Sims (2003). In this framework, investors endowed

¹See Piketty and Saez (2003); Atkinson, Piketty, and Saez (2011). A comprehensive discussion is also offered in the 2013 Summer issue of the *Journal Economic Perspectives* and in Piketty (2014).

²See also the review by Benhabib and Bisin (2017). Saez and Zucman (2016) emphasize the role of differential savings rates, rather than differential rates of return, in generating wealth inequality. However, the capitalization method these authors use imposes homogeneity on the rates of return within asset classes, thereby ruling out one mechanism over the other. Fagereng, Guiso, Malacrino, and Pistaferri (2016b) show that imposing this homogeneity assumption can overstate the degree of wealth inequality when it is violated.

with a fixed capacity to learn about different asset payoffs decide how to allocate their capacity: which assets to learn about, how much information about them to process, and based on that information, how much to invest.³ The rational inattention framework is a tractable way to model agents’ optimal choice of information when information processing is constrained. While stylized, the framework captures several appealing aspects of learning. First, getting information about one’s investments requires expending resources. Second, learning about more volatile assets consumes more resources. Lastly, investors can allocate their information capacity optimally across different types of assets, depending on their objective and the characteristics of the assets they are investing in.⁴

Our theoretical framework generalizes existing models—Van Nieuwerburgh and Veldkamp (2010) in particular—by considering heterogeneously informed agents investing in multiple heterogeneous assets. We analytically characterize three channels of how investor heterogeneity generates capital income inequality: investors with higher information capacity hold larger portfolios on average, tilt their average holdings towards riskier assets within the risky portfolio, and adjust their investments more aggressively in response to changes in payoffs. Hence, *portfolio size*, *portfolio composition*, and *rates of return* differ across investors. These patterns are consistent with the empirical literature on portfolio composition differences between wealthy and less wealthy investors, going back to Greenwood (1983), Kessler and Wolff (1991), and Mankiw and Zeldes (1991), and shown more recently by Fagereng, Guiso, Malacrino, and Pistaferri (2016b). Bach, Calvet, and Sodini (2015) also document portfolio

³In the model, we endow each investor with a particular level of information processing capacity. However, this capacity should be interpreted more broadly, as a stand-in for the individual’s ability to access high quality investment advice, not limited to his or her own knowledge of or ability to invest in financial markets.

⁴In finance, rational inattention models have been used successfully to address underdiversification puzzles, price volatility and comovement puzzles, overconfidence, and the home bias, among other applications. References include Peng (2005), Peng and Xiong (2006), Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010). See also Maćkowiak and Wiederholt (2009, 2015), Matějka (2015), and Stevens (2018) for applications in macroeconomics. Our application to inequality is new, to our knowledge.

composition heterogeneity, using Swedish data, and they show that this heterogeneity is a major contributor to the financial wealth inequality in their data. Moreover, when we decompose the sources of return differentials across investors into compensation for skill—readjusting the portfolio in response to shocks—and compensation for risk—holding on average different portfolios, our numerical work indicates that 75% of the return differential between sophisticated and unsophisticated investors can be attributed to compensation for skill.

The key dynamic prediction of the model is that growth in aggregate information capacity, interpreted as a general progress in information-processing technologies, disproportionately benefits the initially more skilled investors and leads to growing capital income inequality. The growth in inequality is driven by a disproportionate expansion of ownership among the sophisticated investors, both overall and especially in high-return assets. As aggregate capacity to process information grows, all investors would like to grow their portfolios, to take advantage of their newfound knowledge. However, in equilibrium, prices increase in response to the excess demand, and as a result, only the sophisticated investors are able to benefit from the higher capacity. They expand their ownership share, while the less sophisticated investors reduce their holdings. The increased gap in ownership generates an increase in capital income inequality. This result holds regardless of the learning technology assumed, and the specific functional form for information acquisition only affects the magnitude of the gap.

The above mechanism is amplified in a setting with heterogeneous assets, because the shifts in ownership shares occur asymmetrically across assets. Allowing investors to choose how to learn about different assets is critical here: with endogenous information choice, the sophisticated ownership share grows most for the most volatile assets, which are precisely the assets that generate the largest capital income gains. As a result, the model with multiple risky assets generates more inequality growth compared with a model with one risky asset. This outcome reflects two characteris-

tics of learning in equilibrium. First, learning exhibits *preference for volatility*: All else equal, individuals choose to learn about more volatile assets because they offer the highest returns to information. Second, there is *strategic substitutability* in learning: The value of learning diminishes as more individuals learn about a given asset, through a general equilibrium effect on prices. As a result, symmetric growth in capacity leads to an expansion of sophisticated ownership across asset classes, starting with the most volatile and continuing to lower-volatility assets. Simultaneously, the less sophisticated individuals are priced out of investing in high-risk, high-return assets, and retrench to safer assets.

To provide some guidance regarding the magnitudes of the effects identified in our model, we conduct a set of numerical experiments in a parameterized economy. We show that a 5.1% growth in aggregate information capacity⁵ generates an economically significant rise in capital income inequality of 42% over 24 years. Calibrating the information capacity growth is challenging because the information that investors have when they make their investment decisions is not observable. However, we find that for a range of plausible values of the information capacity growth, inequality growth ranges from 24% to 60%. The corresponding number in the SCF for the 1989-2013 period would be 87%. In contrast, the economy with a single risky asset implies that the same information capacity growth of 5.1% generates only 20% growth in capital income inequality. This result underscores the crucial role of endogenous information and diverging portfolio composition in driving capital income inequality, also identified in empirical literature. In terms of aggregates, general progress in information technology also generates lower market returns, higher market turnover, and larger and more volatile portfolios. These trends in the model are broadly consistent with the data on turnover and ownership from CRSP and Morningstar on stocks and mutual funds over the last 25 years.

⁵This growth rate in annual information processing capacity is chosen to generate an average market return of 7% in the model. We discuss the parameterization of the model in detail in Section 4.

The last few decades have witnessed significant financial innovation and the introduction of complex securities. To explore the role that financial innovation might play in the dynamics of income inequality, we consider an expansion of the assets available for investment. For illustrative purposes, we consider a version of the benchmark experiment in which we add more high-risk assets as time goes by. We find that doing so slows down the decline in market returns and the growth in capital income inequality, which declines to 17%. Hence, the model predicts an interesting interplay between innovation in investors' ability to process news about assets on the one hand, and financial institutions' innovation in designing new securities on the other hand, as both these kinds of innovation are predicted to have a significant impact on the dynamics of capital income inequality.

Our findings connect to the idea that matching the inequality in outcomes observed in the data requires linking rates of return to wealth, which is our indicator for access to better information on investment strategies.⁶ This idea has a long history, going back to Aiyagari (1994), who discusses the wide disparities in portfolio compositions across the wealth distribution, focusing on the fact that rich households are much more likely to hold risky assets, such as equities and risky debt instruments. Subsequently, Krusell and Smith (1998) suggest that the data requires that wealthy agents have higher propensities to save, generate higher returns on savings, or both. Benhabib, Bisin, and Zhu (2011) develop this relationship theoretically. Recent quantitative contributions include Favilukis (2013) and Cao and Luo (2017). We contribute to this literature by focusing on the within-period portfolio problem with many risky assets, rather than the dynamic savings decision with a single risky asset.

Our work contributes to a broader literature that has sought to generate inequality in capital income, including the work on bequests by Cagetti and De Nardi (2006), on limited stock market participation by Guvenen (2007, 2009), on heterogeneous dis-

⁶This choice is inspired by work that has linked trading strategy sophistication to asset prices, wealth and income levels, such as Calvet, Campbell, and Sodini (2009), Chien, Cole, and Lustig (2011), and Vissing-Jorgensen (2004).

count factors by Krusell and Smith (1998), on financial literacy by Lusardi, Michaud, and Mitchell (2017), and on entrepreneurial talent by Quadrini (1999). Our approach to generating this heterogeneity via differences in information offers a novel mechanism relative to this literature, building on the insights of Arrow (1987). Our focus on skill rather than differences in risk aversion is supported by portfolio-level evidence provided by Fagereng, Guiso, Malacrino, and Pistaferri (2016a), who find that the majority of difference in returns is due to either better timing or composition within risky portfolio, rather than differing overall exposure to financial markets. See Pástor and Veronesi (2016) for a one-asset model with heterogeneity in risk aversion and exogenous entrepreneurial skill differences. Another related paper is Peress (2004) who examines the role of wealth and decreasing absolute risk aversion in investors' information acquisition and investment in a single risky asset. Lastly, relative to the literature focused on the tails of the income distribution, such as Gabaix, Lasry, Lions, and Moll (2016), we provide a mechanism that works on the entirety of the distribution and is not solely operational asymptotically.

The rest of the paper is organized as follows. Section 2 presents the theory. Section 3 derives analytic predictions, which we quantify in Section 4. Section 5 presents auxiliary supporting empirical evidence for our framework, and Section 6 concludes. All proofs and derivations are in the Appendix.

2 Theoretical Framework

Below, we set up a rational inattention portfolio choice model in which investors are constrained in their capacity to process information about asset payoffs, and in which both asset characteristics and investors are heterogeneous. In the presence of this heterogeneity, we derive the optimal mass of agents learning about each asset and characterize the gains from learning in equilibrium.

2.1 Setup

A continuum of atomless investors of mass one, indexed by j , solve a sequence of portfolio choice problems, so as to maximize mean-variance utility over wealth W_j in each period, given risk aversion coefficient $\rho > 0$. The financial market consists of one risk-free asset, with price normalized to 1 and payoff r , and $n > 1$ risky assets, indexed by i , with prices p_i , and independent payoffs $z_i = \bar{z} + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2)$.⁷ The risk-free asset has unlimited supply, and each risky asset has fixed supply, \bar{x} . For each risky asset, non-optimizing “noise traders” trade for reasons orthogonal to prices and payoffs (e.g., liquidity, hedging, or life-cycle reasons), such that the net supply available to the (optimizing) investors is $x_i = \bar{x} + \nu_i$, with $\nu_i \sim \mathcal{N}(0, \sigma_x^2)$, independent of payoffs and across assets.⁸ Following Admati (1985), we conjecture (and later verify) that prices are $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$, for some coefficients $a_i, b_i, c_i \geq 0$.

Investors know the distributions of asset payoffs and noise shocks, but they are uncertain about the realized shocks each period. Prior to making their portfolio decisions, investors can obtain information about some or all of the risky assets, in the form of signals. The informativeness of these signals is constrained by each investor’s capacity to process information. A higher capacity can be interpreted as having more resources that can be devoted to gathering and processing news about different assets, and it will be shown to translate into signals that track the realized payoffs with higher precision. Hence, a limit on this capacity limits investors’ ability to reduce posterior uncertainty about payoffs. Given this constraint, investors choose how to allocate their capacity to acquire signals about different assets. Following Sims (2003), we use the reduction in the entropy of the payoffs conditional on the signals as a measure of how much capacity the chosen signals consume. A signal with higher precision,

⁷Under certain simplifying assumptions about the investors’ learning technology (discussed further below), assuming independent payoffs is without loss of generality. See Van Nieuwerburgh and Veldkamp (2010) for a discussion of how to orthogonalize correlated assets under such assumptions.

⁸For simplicity, we introduce heterogeneity only in the volatility of payoffs, although the model can easily accommodate additional heterogeneity in supply and in mean payoffs.

which reduces conditional entropy by more, uses up more of the investor’s capacity. Entropy reduction is a commonly used measure of information in information theory and statistics, and has been also applied in a variety of finance settings.⁹ Entropy has a number of appealing properties as a measure of uncertainty. For example, for normally distributed random variables, it is linear in variance. Moreover, the entropy of a vector independent random variables is the sum of the entropies of the individual variables (Shannon (1948)). While stylized, this learning process captures the key trade-offs investors face when deciding how to allocate their limited capacity across multiple investment decisions, as a function of their objective and of the risks they face.

Lastly, we consider two investor types: mass $\lambda \in (0, 1)$ of investors, labeled *sophisticated*, have high capacity to process information, K_1 , and mass $1 - \lambda$, labeled *unsophisticated*, have low capacity, K_2 , with $0 < K_2 < K_1 < \infty$.

2.2 Investor optimization

Optimization occurs in two stages. In the first stage, investors solve their information acquisition problem: they choose the distribution of their individual signals in order to maximize expected utility, subject to their information capacity. In the second stage, investors draw signals, update their beliefs about the payoffs and choose their portfolio holdings to maximize utility. We first describe the optimal portfolio choice in the second stage, for a given signal choice. We then solve for the ex-ante optimal signal choice.

⁹Peng (2005), Peng and Xiong (2006), Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010).

Portfolio choice Given equilibrium prices and posterior beliefs, each investor's portfolio problem is standard. The investor solves

$$U_j = \max_{\{q_{ji}\}_{i=1}^n} E_j(W_j) - \frac{\rho}{2} V_j(W_j) \quad (1)$$

$$s.t. \quad W_j = r \left(W_{0j} - \sum_{i=1}^n q_{ji} p_i \right) + \sum_{i=1}^n q_{ji} z_i, \quad (2)$$

where E_j and V_j denote the mean and variance conditional on investor j 's information set, and W_{0j} is initial wealth. Optimal portfolio holdings are given by

$$q_{ji} = \frac{\hat{\mu}_{ji} - r p_i}{\rho \hat{\sigma}_{ji}^2}, \quad (3)$$

where $\hat{\mu}_{ji}$ and $\hat{\sigma}_{ji}^2$ are the mean and variance of investor j 's posterior beliefs about the payoff z_i .

Information acquisition choice Each investor j can choose to receive a separate signal s_{ji} on each of the asset payoffs z_i . Given the optimal portfolio holdings, the distribution of signals maximizes ex-ante expected utility,

$$E_{0j}[U_j] = \frac{1}{2\rho} E_{0j} \left[\sum_{i=1}^n \frac{(\hat{\mu}_{ji} - r p_i)^2}{\hat{\sigma}_{ji}^2} \right]. \quad (4)$$

The choice of the vector of signals $s_j = (s_{j1}, \dots, s_{jn})$ about the vector of payoffs $z = (z_1, \dots, z_n)$ is subject to the information capacity constraint $I(z; s_j) \leq K_j$, where K_j is the investor's capacity for processing news about the assets and $I(z; s_j)$ quantifies the reduction in the entropy of the payoffs, conditional on the vector of signals (defined below).

For analytical tractability, we assume that the signals s_{ji} are independent across assets and investors. Then, the total quantity of information obtained by an investor is the sum of the quantities of information obtained for each asset. The information

constraint becomes $\sum_{i=1}^n I(z_i; s_{ji}) \leq K_j$, where $I(z_i; s_{ji})$ measures the information conveyed by the signal s_{ji} about the payoff of asset i .

We can think of the information problem as a decomposition of each payoff into the signal component and a residual component that represents the information lost because of the investor's capacity constraint, and we can write this decomposition as $z_i = s_{ji} + \delta_{ji}$. If we assume that the signal and the residual are independent, then the Gaussianity of the payoffs implies that posterior beliefs are also normally distributed random variables, with mean $\hat{\mu}_{ji} = s_{ji}$ and variance $\hat{\sigma}_{ji}^2 = \sigma_{\delta_{ji}}^2$.¹⁰ Using the definition of entropy for normally distributed random vectors, the information constraint becomes

$$I(z; s_j) = \frac{1}{2} \sum_{i=1}^n \log \left(\frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \right) \leq K_j. \quad (5)$$

The investor's information problem is then choosing the precision of posterior beliefs for each asset to solve

$$\max_{\{\hat{\sigma}_{ji}^2\}_{i=1}^n} \sum_{i=1}^n G_i \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \quad s.t. \quad \prod_{i=1}^n \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \leq e^{2K_j}, \quad (6)$$

where G_i represents the expected utility gain from learning about asset i ,

$$G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\bar{z} - ra_i)^2}{\sigma_i^2}. \quad (7)$$

The gains from learning are a function of equilibrium prices and asset characteristics only, and hence they are common across investor types and taken as given by each investor.¹¹

Lemma 1. *The solution to the capacity allocation problem (6)-(7) is a corner: each*

¹⁰Since z_i is normally distributed, the independence assumption implies that s_{ji} and δ_{ji} are also normally distributed. By Cramer's Theorem, $s_{ji} \sim \mathcal{N}(\bar{z}, \sigma_{s_{ji}}^2)$ and $\delta_{ji} \sim \mathcal{N}(0, \sigma_{\delta_{ji}}^2)$ with $\sigma_i^2 = \sigma_{s_{ji}}^2 + \sigma_{\delta_{ji}}^2$.

¹¹The investor's objective omits terms from the expected utility function that do not affect the optimization. See the Appendix for detailed derivations.

investor allocates her entire capacity to reducing posterior uncertainty for a single asset from the set of assets with maximal gains from learning. For all other assets, the investor's optimal portfolio holdings are determined by her prior beliefs.

For each investor j learning about asset $l_j \in \arg \max_i G_i$, the posterior beliefs are normally distributed, with mean and variance given by

$$\hat{\mu}_{ji} = \begin{cases} s_{ji} & \text{if } i = l_j \\ \bar{z} & \text{if } i \neq l_j \end{cases} \quad \text{and} \quad \hat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j} \sigma_i^2 & \text{if } i = l_j \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases} \quad (8)$$

For $i = l_j$, conditional on the realized payoff z_i , the signal is normally distributed with mean $E(s_{ji}|z_i) = \bar{z} + (1 - e^{-2K_j}) \varepsilon_i$, and variance $V(s_{ji}|z_i) = (1 - e^{-2K_j}) e^{-2K_j} \sigma_i^2$.

The linear objective function and the convex constraint imply that each investor specializes, learning about a single asset. This result extends the specialization results of Van Nieuwerburgh and Veldkamp (2010) to the case of heterogeneous investors.¹² Moreover, it shows that regardless of her level of sophistication, each investor chooses to learn about one asset from among the assets with the highest gains from learning. Hence, in equilibrium, all assets that are learned about will have the same gains. Which assets these are is determined in equilibrium.

2.3 Equilibrium

Equilibrium prices Given the solution to each investor's optimization problem, equilibrium prices are linear combinations of the shocks, with coefficients that depend on the aggregate information capacity in this economy.

¹²Investors hold all assets, but invest relatively more in the asset they learn about. Hence, the model generates underdiversification of individual portfolios, consistent with the empirical evidence (e.g., Vissing-Jorgensen (2004) and references therein)

Lemma 2. *The price of asset i is given by $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, with*

$$a_i = \frac{1}{r} \left[\bar{z} - \frac{\rho\sigma_i^2\bar{x}}{(1 + \Phi_i)} \right], \quad b_i = \frac{\Phi_i}{r(1 + \Phi_i)}, \quad c_i = \frac{\rho\sigma_i^2}{r(1 + \Phi_i)}, \quad (9)$$

$$\Phi_i \equiv m_{1i}\lambda(e^{2K_1} - 1) + m_{2i}(1 - \lambda)(e^{2K_2} - 1), \quad (10)$$

where Φ_i measures the information capacity allocated to learning about asset i in equilibrium, and $m_{1i}, m_{2i} \in [0, 1]$ are the fractions of sophisticated and unsophisticated investors who choose to learn about asset i .

The price of an asset reflects the asset's payoff and the supply shocks, with relative importance determined by the mass of investors learning about the asset. If there is no information capacity in the economy ($K_1 = K_2 = 0$), or for assets that are not learned about ($m_{1i} = m_{2i} = 0$), the price only reflects the supply shock ν_i . As the capacity allocated to an asset increases, the asset's price co-moves more strongly with the underlying payoff (c_i decreases and b_i increases, though at a decreasing rate). In the limit, as $K_j \rightarrow \infty$, the price approaches the discounted realized payoff, z_i/r , and the supply shock becomes irrelevant for price determination.

Equilibrium learning Using equilibrium prices, we now determine the assets that are learned about and the mass of investors learning about each asset. Without loss of generality, let assets be ordered such that $\sigma_i > \sigma_{i+1}$ for all $i \in \{1, \dots, n-1\}$. Let $\xi_i \equiv \sigma_i^2(\sigma_x^2 + \bar{x}^2)$ summarize the properties of asset i .

Lemma 3. *The gain from learning about asset i is given by*

$$G_i = \frac{1 + \rho^2\xi_i}{(1 + \Phi_i)^2}. \quad (11)$$

The allocation of information capacity across assets, $\{\Phi_i\}_{i=1}^n$, is uniquely pinned down by equating the gains from learning among all assets that are learned about, and by

ensuring that all assets not learned about have strictly lower gains:

$$G_i = \max_{h \in \{1, \dots, n\}} G_h, \quad \forall i \in \{1, \dots, k\}, \quad (12)$$

$$G_i < \max_{h \in \{1, \dots, n\}} G_h, \quad \forall i \in \{k + 1, \dots, n\}, \quad (13)$$

where k denotes the endogenous number of assets with strictly positive learning mass.

Let m_i denote the total mass of investors learning about asset i and let $c_{i1} \equiv \sqrt{\frac{1+\rho^2\xi_i}{1+\rho^2\xi_1}} \leq 1$ denote the exogenous value of learning about asset i relative to asset 1. In a symmetric equilibrium in which $m_{1i} = m_{2i} = m_i$, the masses $\{m_i\}_{i=1}^n$ are given by

$$m_i = \frac{c_{i1}}{C_k} + \frac{1}{\phi} \left(\frac{kc_{i1}}{C_k} - 1 \right), \quad \forall i \in \{1, \dots, k\}, \quad (14)$$

$$m_i = 0, \quad \forall i \in \{k + 1, \dots, n\}, \quad (15)$$

where $C_k \equiv \sum_{i=1}^k c_{i1}$, and $\phi \equiv \lambda(e^{2K_1} - 1) + (1 - \lambda)(e^{2K_2} - 1)$ is a measure of the total capacity for processing information available in the economy, so that $\Phi_i = \phi m_i$.

The model uniquely pins down the total capacity allocated to each asset, Φ_i , but it does not separately pin down m_{1i} and m_{2i} . Since the asset-specific gain from learning is the same for both types of investors, we assume that the participation of sophisticated and unsophisticated investors in learning about each asset is proportional to their mass in the population. In turn, this implies a unique set of masses $\{m_i\}_{i=1}^n$.

3 Model Predictions

In this section, we present analytic results implied by our information friction. We identify the channels through which heterogeneity and growth in information capacity generate capital income inequality growth. We find that heterogeneity in information implies differences in portfolio sizes, different composition of the risky portfolio across

investors, and also implies that investors are able to adjust their holdings in response to payoff shocks more effectively if their capacity is higher. These predictions are consistent with prior empirical evidence on household portfolio heterogeneity.¹³

3.1 Learning dynamics

We begin by characterizing how investors learn about the different assets in equilibrium and over time, which in turn determines their investment choices.

Learning in the cross section First, learning in the model exhibits preference for volatility (high σ_i^2) and strategic substitutability (low m_i). Furthermore, the value of learning about an asset also falls with the aggregate amount of information in the market (ϕ), since higher capacity overall increases the co-movement between prices and payoffs, thereby reducing expected excess returns:

$$\frac{\partial G_i}{\partial \sigma_i^2} > 0, \quad \frac{\partial G_i}{\partial m_i} < 0, \quad \frac{\partial G_i}{\partial \phi} < 0.$$

These properties imply that with a sufficiently low information capacity, all investors learn about the same asset, namely the most volatile one: for $\phi \in (0, \phi_1]$, $m_1 = 1$ and $m_i = 0$ for all $i > 1$, where

$$\phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1. \tag{16}$$

This threshold endogenizes single-asset learning as an optimal outcome for low enough information capacity relative to asset dispersion. For higher capacity levels, strategic substitutability in learning pushes some investors to learn about less volatile assets. For sufficiently high information capacity (or alternatively, for low enough dispersion in assets volatilities), all assets are actively traded, thus endogenizing the assumption employed in models with exogenous signals. Here asset heterogeneity is critical: even

¹³Fagereng, Guiso, Malacrino, and Pistaferri (2016a), Bach, Calvet, and Sodini (2015).

if capacity is high enough so that multiple assets are learned about, not all assets are learned about with the same intensity, so that holdings differ across assets, as we show below.

Learning over time We now study how learning changes in response to changes in aggregate capacity in the economy. It is useful to define the thresholds for learning as follows:

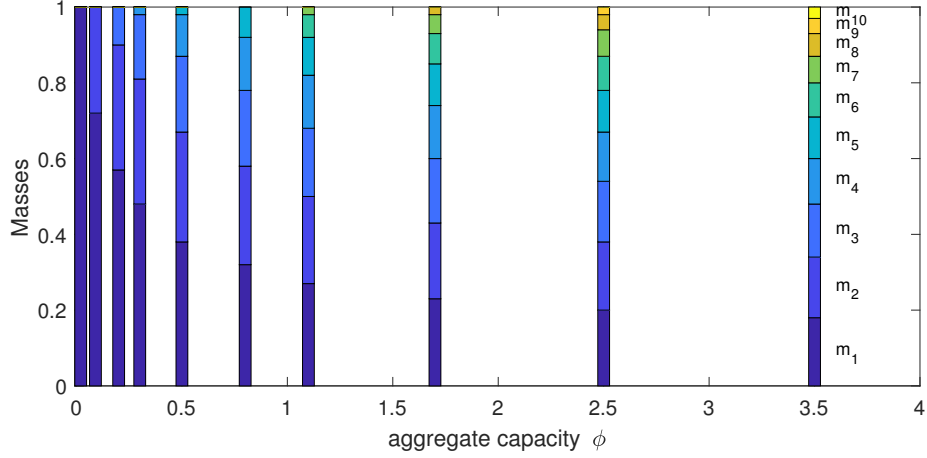
Definition 1. Let the aggregate market capacity ϕ_k be such that for any $\phi \leq \phi_k$, at most the first k assets are actively traded (learned about) in equilibrium, while for $\phi > \phi_k$, at least the first $k + 1$ assets are actively traded in equilibrium.

Lemma 3 implies that the threshold values of aggregate information capacity are monotonic: $0 < \phi_1 < \phi_2 < \dots < \phi_{n-1}$.

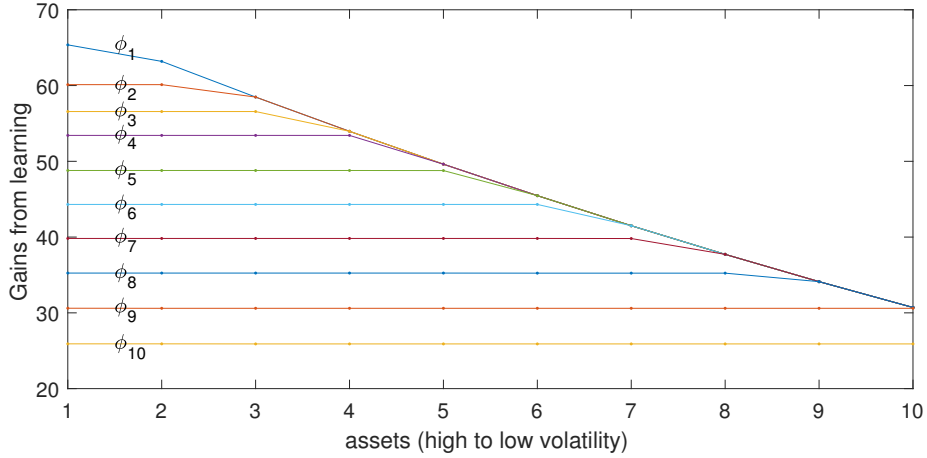
Proposition 1 (Learning). *Let $\phi \in (\phi_{k-1}, \phi_k]$ such that $k > 1$ assets are actively traded. Consider an increase in ϕ such that $k' \geq k$ is the new number of actively traded assets.*

- (i) *There exists a threshold asset $\bar{i} < k'$, such that m_i is strictly decreasing in ϕ for all $i \in \{1, \dots, \bar{i} - 1\}$ and strictly increasing in ϕ for all $i \in \{\bar{i} + 1, \dots, k'\}$.*
- (ii) *The quantity (ϕm_i) is increasing in ϕ for all assets $i \in \{1, \dots, k'\}$.*
- (iii) *For an increase in ϕ generated by a symmetric growth, $K'_j = (1 + \gamma) K_j$, with $\gamma \in (0, 1)$, the quantity $m_i(e^{2K_j} - 1)$, $j \in \{1, 2\}$, is increasing in K_j at an increasing rate, for $i \in \{\bar{i} + 1, \dots, k'\}$. For $i \in \{1, \dots, \bar{i}\}$, $m_i(e^{2K_1} - 1)$ grows while $m_i(e^{2K_2} - 1)$ grows by less, or even falls if capacity dispersion is large enough.*

Proposition 1 shows the diversification in learning effect. First, as the amount of aggregate capacity increases, some investors shift to learning about less volatile assets, and the mass of investors learning about the most volatile assets decreases. The



(a)



(b)

Figure 1: (a) The masses of investors learning about each asset m_i as a function of aggregate capacity ϕ . At very low aggregate capacity, all investors learn about the most volatile asset, $m_1 = 1$. As capacity increases, investors diversify to lower volatility assets, until all assets have positive investor mass. (b) The gains from learning about each asset, as a function of aggregate capacity ϕ . Gains are higher for higher volatility assets. As capacity increases, gains fall. Gains are equated for all assets that are learned about in equilibrium. ϕ_k indicates the level of aggregate capacity for which k assets are learned about in equilibrium. On the x-axis, assets are ordered from most (1) to least (10) volatile.

threshold \bar{v} determines this turning point in the distribution of assets. Figure 1 shows this effect numerically in panel (a), as the aggregate information capacity increases from ϕ_1 , the level of capacity for which only a single asset is learned about, to ϕ_{10} , the level for which ten assets are learned about.¹⁴ Nevertheless, the total amount of capacity allocated to each asset (ϕm_i) strictly increases, such that all gains from learning decline and are equated at a new, lower level for all assets that are learned about, as shown in panel (b) of the figure. Most importantly, the increase in aggregate capacity benefits the sophisticated group disproportionately more because across all actively traded assets, this group allocates relatively more capacity to each asset. We now use this result to derive analytic predictions for the patterns of investment and inequality.

3.2 Capacity dispersion

Let q_{1i} and q_{2i} denote the average per-capita holdings of asset i for sophisticated and unsophisticated investors, respectively. They are given by

$$q_{1i} = \left(\frac{\bar{z}_i - rp_i}{\rho\sigma_i^2} \right) + m_i (e^{2K_1} - 1) \left(\frac{z_i - rp_i}{\rho\sigma_i^2} \right), \quad (17)$$

and q_{2i} defined analogously. Equation (17) shows that per-capita holdings are given by the quantity that would be held under the investors' prior beliefs plus a quantity that is increasing in the realized excess return. The weight on the realized excess return is asset and investor specific, and it is given by the amount of information capacity allocated to this asset by this investor group. Hence, for actively traded assets, heterogeneity in capacities generates differences in ownership across investor

¹⁴One could also let the degree of dispersion in asset payoff volatilities vary, which will imply that learning also varies, with periods with high dispersion being characterized by more concentrated learning, and periods with low dispersion characterized by more diversified learning (and hence portfolios).

types at the asset level:

$$q_{1i} - q_{2i} = m_i (e^{2K_1} - e^{2K_2}) \left(\frac{z_i - rp_i}{\rho\sigma_i^2} \right). \quad (18)$$

Integrating over the realizations of the state (z_i, x_i) , the expected per-capita ownership difference, as a share of the supply of each asset, is also asset specific,

$$\frac{E[q_{1i} - q_{2i}]}{\bar{x}} = (e^{2K_1} - e^{2K_2}) \frac{m_i}{1 + \phi m_i}. \quad (19)$$

Hence, the portfolio of the sophisticated investor is not simply a scaled up version of the unsophisticated portfolio. Rather, the portfolio weights *within* the class of risky assets also differ across the two investor types. The proposition below summarizes our results regarding ownership differences across investor types.

Proposition 2 (Ownership). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then, for $i \in \{1, \dots, k\}$,*

- (i) $E[q_{1i} - q_{2i}] / \bar{x} > 0$;
- (ii) $E[q_{1i} - q_{2i}] / \bar{x}$ is increasing in $E[z_i - rp_i]$ and in σ_i^2 ;
- (iii) $q_{1i} - q_{2i}$ is increasing in $z_i - rp_i$.

Proposition 2 states that the average sophisticated investor (i) holds a larger portfolio of risky assets on average, (ii) tilts her portfolio towards more volatile assets with higher expected excess returns, and (iii) adjusts ownership, state by state, towards assets with higher *realized* excess returns. These results identify the channels through which capital income differs across investor types.

To see the effects of the portfolio scale and composition differences on capital income, we define the capital income of an investor of type j as $\pi_{ji} \equiv q_{ji} (z_i - rp_i)$. For actively traded assets, heterogeneity in ownership generates heterogeneity in capital

income across investor types at the asset level:

$$\pi_{1i} - \pi_{2i} = m_i (e^{2K_1} - e^{2K_2}) \frac{(z_i - rp_i)^2}{\rho\sigma_i^2}. \quad (20)$$

Integrating over the realizations of (z_i, x_i) , the expected capital income difference is

$$E[\pi_{1i} - \pi_{2i}] = \frac{1}{\rho} m_i G_i (e^{2K_1} - e^{2K_2}), \quad (21)$$

where G_i is the gain from learning about asset i . The next proposition summarizes our analytic predictions regarding the sources of cross-sectional capital income inequality.

Proposition 3 (Capital Income). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then, for $i \in \{1, \dots, k\}$,*

- (i) $\pi_{1i} - \pi_{2i} \geq 0$, with a strict inequality in states with non-zero realized excess returns;
- (ii) $E[\pi_{1i} - \pi_{2i}]$ is increasing in asset volatility σ_i .

The average sophisticated investor realizes larger profits in states with positive excess returns, and incurs smaller losses in states with negative excess returns, because her holdings co-move more strongly with the realized state. Importantly, the biggest difference in profits, on average, comes from investment in the more volatile, higher expected excess return assets. It is these volatile assets that drive inequality because they generate the biggest gain from learning, and hence the biggest advantage from having relatively high learning capacity. Hence, holding capacity constant, the more volatile the asset market, the more unequal will be the distribution of capital income.

Finally, the next proposition shows that increasing dispersion in capacities while keeping aggregate capacity unchanged implies further polarization in holdings, which in turn leads to a growing capital income inequality. However, while this increase generates higher inequality, it has no effect on financial markets.

Proposition 4 (Capacity Dispersion). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Consider an increase in*

capacity dispersion of the form $K'_1 = K_1 + \Delta_1 > K_1$, $K'_2 = K_2 - \Delta_2 < K_2$, with Δ_1 and Δ_2 chosen such that the total information capacity ϕ remains unchanged. Then, for $i \in \{1, \dots, k\}$,

- (i) Asset prices and excess returns remain unchanged.
- (ii) The difference in ownership shares $(q_{1i} - q_{2i})/\bar{x}$ increases.
- (iii) Capital income gets more polarized as π_{1i}/π_{2i} increases state by state.

Intuitively, greater dispersion in information capacity implies that sophisticated investors receive relatively higher-quality signals about the fundamental payoffs, which enables them to respond more strongly to realized state. However, since aggregate capacity remains unchanged, both the number of assets learned about and the mass of investors learning about each asset remain unchanged. Hence, the adjustment reflects a pure transfer of income from the relatively unsophisticated investors (who now have even lower capacity) to the more sophisticated investors (who now have even higher capacity) without any general equilibrium effects.

3.3 Capacity growth

To capture recent trends in financial markets, we next consider growth in aggregate capacity. We find that in the presence of initial capacity dispersion, growth in information capacity, interpreted as general progress in information-processing technologies, leads to a disproportionate increase in the ownership of risky assets by the sophisticated investors, and to a growing capital income polarization, while simultaneously affecting asset prices.

Proposition 5 (Symmetric Growth). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Consider an increase in ϕ generated by a symmetric growth in capacities to $K'_1 = (1 + \gamma)K_1$ and $K'_2 = (1 + \gamma)K_2$, $\gamma \in (0, 1)$. Let $k' \geq k$ denote the new equilibrium number of actively traded assets. Then, for $i \in \{1, \dots, k'\}$,*

- (i) Average asset prices increase and average excess returns decrease.
- (ii) Average ownership share of sophisticated investors $E[q_{1i}]/\bar{x}$ increases and average ownership share of unsophisticated investors $E[q_{2i}]/\bar{x}$ decreases, and the gap is increasing in asset volatility.
- (iii) Average capital income gets more unequal, as $E[\pi_{1i}]/E[\pi_{2i}]$ increases, and the inequality is higher for the more volatile assets.

Higher capacity to process information means that investors have more precise news about the realized payoffs. Hence, their demand for assets co-moves more closely with the realized state, which implies that prices contain a larger amount of information about the fundamental shocks. As a result, the equilibrium implies lower average returns, larger and more volatile positions.

As asset prices increase reflecting the additional information content, only the sophisticated investors are able to benefit from the growth in capacity. They increase their ownership share at the expense of the less sophisticated investors, who retreat. This result holds regardless of the learning technology assumed, since it is driven by the fact that posterior variance is lower for the sophisticated investors, and hence they will wish to hold a larger quantity than the unsophisticated investors. Moreover, the increase in ownership occurs more for the high volatility assets that have higher gains from learning in equilibrium. These assets generate the higher expected returns, hence this differential ownership growth generates higher capital income inequality than if sophisticated investors had uniformly increased their holdings across all assets.

3.4 Trading volume

The differential response to shocks of the two investor types also implies differences in trading intensity, which provides an additional set of testable implications. We divide the investors into 3 groups: (i) sophisticated investors who learn about asset i , with per capita average volume \bar{V}_i^{SL} ; (ii) unsophisticated investors who learn about asset i , with per capita average volume \bar{V}_i^{UL} ; and (iii) investors who do not learn about

asset i , with per capita average volume \bar{V}_i^{NL} . For assets that are not learned about, volume is denoted by \bar{V}_i^{ZL} . Hence, the total volume generated by the optimizing investors at the asset level is¹⁵

$$V_i = \begin{cases} \lambda m_i \bar{V}_i^{SL} + (1 - \lambda) m_i \bar{V}_i^{UL} + (1 - m_i) \bar{V}_i^{NL} & \text{if } i \text{ is learned about} \\ \bar{V}_i^{ZL} & \text{if } i \text{ is not learned about.} \end{cases} \quad (22)$$

We derive an analytic expression for the average per capita volume across states for each asset and investor group, given by

$$\bar{V}_i^g = \frac{1}{\sqrt{\pi}} \left(\sigma_{qi}^g + \sqrt{(\sigma_{qi}^g)^2 + (\sigma_{\mu i}^g)^2} \right), \quad (23)$$

where σ_{qi}^g is the cross-sectional standard deviation of holdings across investors in group g and $\sigma_{\mu i}^g$ is the variability of that group's mean holdings across states. Intuitively, trading volume is higher the more disagreement there is in the cross-section of investors and the more the group responds to shocks over time.

In turn, the degree of cross-sectional disagreement depends on how much capacity investors allocate to learning about that asset, with

$$(\sigma_{qi}^g)^2 = \begin{cases} \frac{e^{2K_g} - 1}{\rho^2 \sigma_i^2} & \text{if } i \text{ is learned about \& } g = \text{SL, UL} \\ 0 & \text{if } i \text{ is learned about \& } g = \text{NL} \\ 0 & \text{if } i \text{ is not learned about.} \end{cases}$$

while the degree to which investors adjust holdings over time depends on how much learning is allocated to the asset, both by the particular investor group and by the

¹⁵The average volume of the noise traders is exogenous, given by the standard deviation of the noise shock. Among optimizing investors, we assume that investors do not change groups over time. When we take the volume predictions to the data, we compute turnover, which is given by $T_i \equiv V_i/\bar{x}$.

market overall:

$$(\sigma_{\mu i}^g)^2 = \begin{cases} \left(\frac{e^{2K_g}}{1+\phi m_i}\right)^2 \sigma_x^2 + \left(\frac{e^{2K_g}-1-\phi m_i}{1+\phi m_i}\right)^2 \frac{1}{\rho^2 \sigma_i^2} & \text{if } i \text{ is learned about \& } g = \text{SL, UL} \\ \left(\frac{1}{1+\phi m_i}\right)^2 \sigma_x^2 + \left(\frac{\phi m_i}{1+\phi m_i}\right)^2 \frac{1}{\rho^2 \sigma_i^2} & \text{if } i \text{ is learned about \& } g = \text{NL} \\ \sigma_x^2 & \text{if } i \text{ is not learned about.} \end{cases}$$

These expressions enable us to derive a set of testable implications summarized below.

Proposition 6 (Volume). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then for assets that are learned about, $i \in \{1, \dots, k\}$, average volume is increasing in investor sophistication and is higher for investors who actively trade the asset: $\bar{V}_i^{SL} > \bar{V}_i^{UL} > \bar{V}_i^{NL}$.*

Hence, sophisticated investors generate more asset turnover, since having higher capacity to process information enables them to take larger and more volatile positions, relative to unsophisticated investors. Moreover, assets that are actively traded, in turn, have higher volumes compared with assets that are passively traded (based only on prior beliefs).

4 Quantitative Analysis

So far, we have found that progress in information technology can qualitatively generate growing capital income inequality. In this section, we parameterize the model to provide some guidance for the magnitudes implied by the proposed mechanism. We combine data on household capital income from the SCF with data on the financial market from CRSP. We parameterize the model based on data from the first half of the SCF sample (1989-2000), and then we consider an experiment in which aggregate information capacity in the economy grows deterministically at a constant rate, in order to generate predictions for the second half of the sample (2001-2013).

4.1 Parameterization

We have two sets of parameters: those relating to the financial market and those characterizing investor preferences and information processing ability. Table 1 presents the complete list of parameter values and targets for the baseline quantitative exercise.

Table 1: Parameter Values in the Baseline Model

Parameter	Symbol	Value	Target (1989-2000 averages)
Risk-free rate	r	2.5%	3-month T-bill – inflation = 2.5%
Number of assets	n	10	Normalization
Vol. of asset payoffs	σ_i	$\in [1, 1.59]$	p90/p50 of idio. return vol = 3.54
Vol. of noise shocks	σ_{xi}	0.4 for all i	Average turnover = 9.7%
Mean payoff, supply	\bar{z}_i, \bar{x}_i	10, 5 for all i	Normalization
Risk aversion	ρ	1.032	Average return = 11.9%
Information capacities	K_1, K_2	0.37, 0.0037,	Sophisticated share = 69%
and investor masses	λ	0.675	Share actively traded = 50%
			Sophisticated return = 13.1%

Note: Data are from CRSP for idiosyncratic stock return volatility, turnover, and average return and from SCF for return spread and sophisticated ownership share. Targets are for the 1989-2000 period.

Financial Market Data The parameters characterizing the financial market are the risk free rate, $r = 2.5\%$, which matches the 3-month T-bill rate net of inflation over the period, the number of risky assets, n , which we set to 10 arbitrarily, and the means and volatilities of payoffs and noise shocks. In the absence of detailed information regarding holdings of different types of securities at the household level, the baseline parameterization targets volatility moments from the U.S. equities market. Specifically, we set the dispersion in the volatilities of asset payoffs σ_i to target a dispersion in idiosyncratic return volatilities of 3.54, as measured by the the ratio of the 90th percentile to the median of the cross-sectional idiosyncratic volatility of

stock returns.¹⁶ We set the volatility of shocks from noise traders to $\sigma_x = 0.4$ to target an average monthly turnover (defined as the total monthly volume divided by the number of shares outstanding), equal to 9.7%. We also normalize the level of prices by normalizing the mean payoff and the mean supply for each asset to $\bar{z}_i = 10$, $\bar{x}_i = 5$.¹⁷

Investor Data The investor-level parameters we need to pin down are the risk aversion coefficient ρ , the information capacities of the two investor types (K_1, K_2), and the fraction of sophisticated investors in the population (λ). We select those parameters to target the average return on risky assets equal to the market return of 11.9% (corresponding to 1989-2000 average); the fraction of assets that investors learn about, which, in the absence of empirical guidance, we set to 50%; the equity ownership share of sophisticated investors, and the return spread between sophisticated and unsophisticated households, which we compute using data from the SCF.

Although not as comprehensive as tax records data, the SCF provides detailed information about the balance sheets of a representative sample of U.S. households.¹⁸ While the SCF data may not necessarily capture the levels of returns, we use these data to capture the dispersion in returns among market participants. We define as *participants* households that report holding stocks, bonds, mutual funds, receiving dividends, or having a brokerage account. On average, 34% of households participate.¹⁹

¹⁶We normalize the lowest volatility to $\sigma_n = 1$, and we set $\sigma_i = \sigma_n + \alpha(n - i)/n$, which implies the volatility distribution is linear. The dispersion target generates a slope coefficient $\alpha = 0.65$.

¹⁷The crucial parameter from the financial market is the dispersion in payoff volatilities, which has a large impact on the value of capacity to process news and hence reduce uncertainty. We discuss the role of this dispersion in more detail in the next subsection. Changing the number of assets in the parameterization does not have a major impact on our results. The normalization of the means follows Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016).

¹⁸See Saez and Zucman (2016) for a detailed comparison of the SCF to U.S. administrative tax data. In short, they find that the SCF is representative of trends and levels of inequality in the U.S., but understates inequality inside the top 1% of the wealth distribution.

¹⁹As a robustness check, we also consider a broader measure of participation that includes all households with equity in a retirement account. This inclusion raises the participation rates, but does not alter our main findings.

To define investor sophistication in the SCF, we assume that an investor’s capacity is a function of his or her initial wealth, and we classify as sophisticated investors the participants in the top decile of the wealth distribution, and relatively unsophisticated investors as the remaining 90% of participants. Our model predicts that investors with higher capacities hold larger portfolios of risky assets on average and moreover, *within* the portfolio of risky assets, they invest more in the riskier assets. In the data, these characteristics are correlated with initial wealth.²⁰

Using this definition, the equity ownership share of sophisticated investors is 69%.²¹ In order to quantify the return heterogeneity, we proceed as follows. First, we calculate the holdings of risky securities for each household, comprising of holdings of stocks, bonds, and mutual funds. These are the holdings that match well with the sources of capital income in the SCF. Next, we compute the return on risky holdings as capital income divided by holdings of risky securities, and then compute the median return in the top 10% and the bottom 90% of the wealth distribution of participants. We then use these measures to capture the heterogeneity in rates of return among the two household groups. In particular, we compute the ratio of the return of the unsophisticated (bottom 90%) relative to the sophisticated households (top 10%), over the first half of our sample, which gives us that unsophisticated households earned 69.2% of the return of the sophisticated households. We use this gap to obtain targets for the levels of returns of each household type, given the market return. The weights used in computing the aggregate return are the fraction of risky securities held by each type of household in the SCF (31% versus 69%). That gives us the target for sophisticated return of 13.1% and for the unsophisticated return of 9.1%. We then map these two returns into the sophisticated portfolio and the unsophisticated+noise

²⁰In the online appendix, we also show that higher-wealth individuals use more sophisticated investment instruments and invest a lower proportion of their assets in money-like instruments. Additional evidence that links wealth to investment sophistication includes Calvet, Campbell, and Sodini (2009) and Vissing-Jorgensen (2004).

²¹To compute the number, we first compute the dollar value of the risky part of the financial holdings of households (stocks, bonds, non-money market funds, and other financials) for each decile of the wealth distribution. Then, we compute the share of these risky assets held by the top decile.

trader portfolio in the model.²²

We perform a detailed grid search over parameters until all the simulated moments are within a 10% distance from target. That gives sophisticated ownership within 0.7%, sophisticated and unsophisticated returns within 7%, ratio of volatilities within 2% and all other targets matched exactly.

4.2 Technological Progress and Inequality Growth

We now discuss the model's quantitative predictions for the evolution of capital income inequality, in response to aggregate annual growth in information technology. Results are presented in Table 2. We consider the response of the model to an aggregate capacity growth of 5.1% over 25 years, which generates a decline in market return to 2.6%, bringing the average return for the entire period to 7%, as in the data. The model predicts that capacity growth generates a decline in returns, as more information is reflected in equilibrium prices, and an increase in trading volume, as better informed investors adjust their holdings more aggressively. Quantitatively, turnover increases from 9.7% in the first half of the sample to 16.8% in the second half, versus 16.0% in the data. This process leads to higher capital income inequality, which increases 42% over the period. This figure suggests that aggregate capacity growth is quite potent in generating capital income inequality growth. For reference, in the corresponding period capital income inequality growth in the SCF equals 87%.²³

The growth in capital income inequality in the benchmark model arises due to two effects: (i) larger relative exposure of sophisticated investors to the asset market, marked by higher ownership shares across all assets, and (ii) a shift of sophisticated investors towards high risk, high return assets and that of unsophisticated investors towards lower risk and lower return assets. The table reports the ownership share of

²²Noise traders are also market participants from the perspective of the model, and hence would be captured by the SCF.

²³We compute this inequality growth as follows. For each survey year, we sort the sample of participants by the level of total wealth, and we calculate inequality as the ratio of average capital income of the top 10% to that of the bottom 90% of participants.

Table 2: Aggregate Capacity Growth Outcomes

Statistic	Baseline	Low growth	High growth	Data
Capacity growth	5.1	4	8	
Average market return	7	8	5	7
Capital income ineq. growth	42	24	60	87
Sophis end own share of top	1.22	1.16	1.43	
Sophis end own share of bottom	1.15	1.08	1.41	
	One asset	New assets		Asym. growth
Capacity growth	6.7	5.1	7.4	5.1
Average market return	7	10	7	5.6
Capital income ineq. growth	20	28	85	54
Sophis end own share of top	-	1.17	1.34	1.24
Sophis end own share of bottom	-	1.08	1.30	1.17

Note: All other statistics are non-targeted. All numbers are percentages. Average market return is the market return over the entire 1989-2013 period. “Sophis own share” represents the ownership share of sophisticated investors, relative to their population share, for the assets that are above the median in terms of volatility (“top”) and for assets that are below the median in terms of volatility (“bottom”), at the end of the simulation period.

sophisticated investors, relative to their population share, for the assets that are above the median in terms of volatility (“top”) and for assets that are below the median in terms of volatility (“bottom”). For both types of assets, sophisticated owners are over-represented relative to their size in the population (both numbers are greater than 1), reflecting their larger overall exposure. But the difference is larger for the more volatile assets: at the end of the simulation period, sophisticated investors hold 22% more of high-risk assets relative to their population weight, compared to 15% more for low-risk assets. This gap demonstrates the retrenchment of unsophisticated investors from the most profitable assets that fuels inequality growth.

Calibrating the information capacity growth is challenging because the information that investors have when they make their investment decisions is not observable. Hence, our strategy is to set capacity growth so as to match the decline in market

returns seen in the data, and to complement these results with robustness checks on this growth rate. We consider two alternative annual growth rates: 4% and 8%, based on the number of stocks actively analyzed by the financial industry, which grew at 4% annually, and the number of analysts per stock in the financial industry, which grew 8% annually over our sample period. Our information friction implies that growth in information capacity translates into growth in actively analyzed stocks, and also more information capacity allocated per stock, consistent with these growth trends. These growth rates imply 24% and 60% inequality growth. Although the results are sensitive to the growth rate of information capacity, for both specifications the model generates a quantitatively significant rise in capital income inequality relative to the data values.

The Role of Asset Heterogeneity To isolate the effects due to portfolio composition and volatility dispersion, we solve and parameterize our model with just one risky asset.²⁴ In such a setup, risky portfolio composition is identical across investors by construction, and all differences in capital income inequality come from only differing exposure to the one risky asset. The bottom panel of Table 2 presents results from this alternative specification of the model. The one-asset economy generates growth in capital income inequality that is less than half of the growth generated by the benchmark model. Hence, risky portfolio differences and rebalancing play a crucial role in driving capital income inequality in the model. The model generates higher payoffs from learning and larger effects on the retrenchment of unsophisticated

²⁴In terms of the parameterization, the model with one risky asset takes away three targets from the benchmark model: heterogeneity in asset volatility, fraction of actively traded assets, and the return of sophisticated investors. We keep the value of the risk aversion coefficient the same as in the benchmark model and pick the volatility of the single asset payoff equal to the median payoff volatility of the benchmark model (equal to 1.295). That leaves three remaining parameters: volatility of the noise trader demand σ_x , and the two capacities of sophisticated and unsophisticated investors. We choose these to match: the average market return (11.9%), average asset turnover (9.7%), and sophisticated ownership (69%). That gives $(K_1, K_2, \sigma_x) = (0.0544, 0.0163, 0.37)$. In simulating the model, we pick the growth rate of aggregate capacity just like in the benchmark model—to match the decline in the market return. That implies the growth rate of technology of 6.7%, which is actually higher than the one implied by our benchmark specification.

investors from risky asset markets. In the benchmark exercise, as aggregate capacity increases, less sophisticated investors are priced out of trading the more risky assets and shift their portfolio weights towards less risky, lower-return assets. The one-asset economy shuts down this mechanism, and hence generates less divergence between investors.

Expansion of Asset Space The last several decades have been marked by significant financial innovation and the introduction of complex securities. To explore the role that financial innovation might play in the dynamics of income inequality, we consider an expansion of the assets available for investment. For illustrative purposes, we introduce new assets on the high end of the volatility spectrum, with each new asset being 1% more volatile than the previous highest-volatility asset. The emergence of these new assets actually *reduces* the growth in capital income inequality, as shown in the bottom panel of Table 2. High-volatility assets make the information processing more difficult, making effective capacity lower. In response, the ownership shares of sophisticated investors grow less rapidly and the price impact is reduced, resulting in higher market returns. This general equilibrium effect amplifies the direct effect of lower effective capacity, leading to more moderate capital income inequality growth. Because the volatility of the asset market is growing in this exercise, excess returns are higher for a given rate of capacity growth. We consider a reparameterization of the model that increases the rate of aggregate capacity growth to 7.4%, to match the decline in the market return seen in the data. In that case, the growth in capital income inequality is 85%—much higher than in the benchmark model—as sophisticated investors take advantage of the now more volatile asset set. The sophisticated investors hold 34% more of high-volatility assets and 30% more of low-volatility assets, relative to their population weights, at the end of the simulation. This result reinforces the strong link between structural change in the financial market and inequality in capital income.

Asymmetric Growth As an additional specification, we also investigate the additional impact of a heterogeneous growth in capacity for the evolution of inequality. Specifically, we assume that the growth in capacity of each investor type is proportional to that investor group’s own returns, rather than to the market return on equity.²⁵ This specification amplifies the feedback loop from high capacity to high returns, and hence increases the growth in inequality over time. The asymmetric growth model gives 54% growth in capital income inequality, which is an additional 30% effect.

Behavior in the Long Run Does the model predict inequality growth forever, as technology continues to improve? As described above, growth in aggregate information capacity, while generating an increase in capital income inequality, also decreases average market return. That force by itself acts as a mitigating factor in the growth of capital income inequality. Intuitively, if market excess return is very small, that is, market returns are close to the risk free rate, then there is less scope in the economy for extracting informational rents. In Figure 2, we present the simulation of the model for 50 years, in order to explore the long-run implications of capacity growth. Both in the benchmark specification, and in the specification where we introduce new assets, capital income inequality initially rises, accelerates and peaks as rates of return reach the risk free rate, and subsequently starts to decline. Eventually, it stabilizes at a level implied by the differences in risk-free return income earned on on previously accumulated wealth. In the limit, all information is revealed in the model and capital income inequality becomes flat. In the model with innovation in the asset space, this process is slower and inequality peaks and plateaus at higher levels.

²⁵We also scale the constant of proportionality to be 0.93 in order for this exercise to exhibit the same average growth of aggregate capacity as does the benchmark model, equal to 5.1% annually.

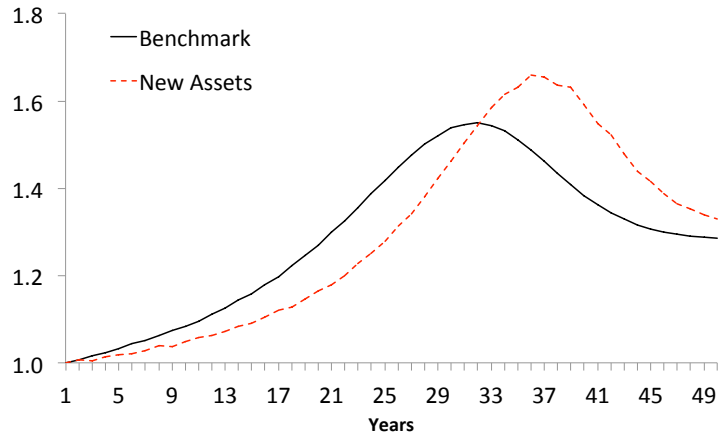


Figure 2: Model: capital income inequality in the long run.

4.3 Skill versus Risk

How much of the growth in inequality comes from differences in exposure to risk versus differences in skill? Fagereng et al. (2016b) document that risk taking is only partially responsible for the difference in returns among Norwegian households, with approximately half of the return difference being attributed to unobservable heterogeneity. Our model is one in which both risk-taking differences and pure compensation for skill generate return heterogeneity. Sophisticated investors are more exposed to risk because they hold a larger share of risky assets (compensation for risk); and they have informational advantage (compensation for skill). To shed more light on the relative importance of these two effects, we decompose the returns of each investor type by computing the unconditional expectation of the return on the portfolio held by investor type $j \in \{S, U\}$:

$$R_j = E \sum_i \omega_{jit} (r_{it} - r) = \sum_i Cov(\omega_{jit}, r_{it}) + \sum_i E \omega_{jit} E[r_{it} - r], \quad (24)$$

where $r_{it} = z_{it}/p_{it}$ is the time t return on asset i and ω_{jit} is the portfolio weight of asset i for investor j at time t , defined as $\omega_{jit} = q_{jit}p_{it}/\sum_l q_{jlt}p_{lt}$. The first term of the decomposition captures the covariance conditional on investor j information set,

that is, the investor’s reaction to information flow via portfolio weight adjustment (*skill effect*); the second term captures the *average effect*, unrelated to active trading.

Quantitatively, the skill effect accounts for the majority of the return differential in the model. To show that, we compute the counterfactual return of sophisticated investors if their skill matched that of unsophisticated (plus noise) investors, but their average holdings stayed the same

$$\hat{R}_I = \sum_i Cov(\omega_{Rit}, r_{it}) + \sum_i E\omega_{Iit}E[r_{it} - r]. \quad (25)$$

Such a portfolio would have generated an annualized return of 10.3%, which implies that the compensation for skill accounts for more than 75% of the return differential between the sophisticated and unsophisticated investors.

4.4 Robustness

In this section we discuss alternative specifications for preferences and the information technology function.

Heterogeneity in Risk Aversion The overall growth in inequality can be increased by augmenting the model with differences in risk attitudes that, like information capacity, are correlated with wealth. The less risk averse investors would hold a greater share of risky assets, and hence they would have higher expected capital income.²⁶ Within our mean-variance specification, a growing difference in risk aversion produces growing *aggregate* ownership in risky assets of less risk averse investors, and a uniform, proportional retrenchment from all risky assets of more risk averse investors. However, heterogeneity in risk aversion alone cannot generate the empirical investor-specific rates of return on equity, differences in portfolio weights within

²⁶Such setting would also encompass situations in which investors are exposed to different levels of volatility in areas outside capital markets, like labor income.

a class of risky assets or differential growth in ownership by asset volatility.²⁷ Hence, the information asymmetry would have to be retained.

Alternative Preferences In the Online Appendix, we analyze the model with CRRA utility. Since a closed-form solution to the full model is not feasible, we focus on a local approximation of the utility function. We show that the model solution under no capacity differences predicts the same portfolio shares for risky assets, *independent of wealth*. Intuitively, if agents have common information, then wealth differences affect the composition of their allocations between the risk-free asset and the risky portfolio, but not the composition of the risky portfolio, which is determined optimally by the (common) belief structure. As a result, differences in capacity are a necessary component for the model to generate any risky return differences across agents.

Endogenous Capacity Choice In the benchmark model, we assume an exogenous relationship between initial capacity and an investor’s wealth. In the Online Appendix, we show how such relation could arise endogenously. Intuitively, if investors endogenously choose different portfolio sizes, then their net benefit of investing in information will increase with portfolio size. We apply this idea in a model in which investors have identical CRRA preferences and make endogenous capacity choice decisions. In the context of the information choice model, CRRA utility specification is not tractable; hence, we map a common relative risk aversion together with wealth differences locally into different absolute risk aversion coefficients. In a numerical example, we show how initial wealth differences observed in the 1989 SCF map into endogenous capacity differences, for different values of the cost of capacity and different relative risk aversion coefficients. We show that for a wide range of the risk aversion specifications and for capacity cost away from zero, the implied differences in

²⁷On the other hand, Gomez (2016) shows that when macro asset pricing models with heterogeneous risk aversion are parameterized to match the volatility of asset prices, they require a degree of heterogeneity in preferences that leads to counterfactual predictions about wealth inequality.

capacity are equal or actually larger than the ones specified in the benchmark model. Hence, we view our parameterization as conservative in that it implies more modest initial capacity differences.

5 Auxiliary Evidence

In this section, we discuss three important questions regarding our framework vis-a-vis the empirical evidence. First, a large part of investing in risky assets such as stocks is done through intermediaries such as mutual funds. Couldn't unsophisticated households overcome their information disadvantage by simply delegating their money management to more informed asset management companies? We argue below that there are both institutional and informational barriers that prevent unsophisticated households from gaining access to high quality investment services.

Second, if less sophisticated investors retreat from risky assets in response to overall progress in the information processing capacity of the economy, should we not observe this retrenchment in data on flows to different types of mutual funds? Using data on flows into mutual funds from Morningstar, we find that since 2000, unsophisticated investors have been shifting their funds out of equity mutual funds and into less risky non-equity funds.

Third, how important is this intensive portfolio choice margin compared with the extensive participation margin, for generating wealth inequality? Using data from the SCF, we find that much of the recent growth in financial wealth inequality has occurred among household who participate in financial markets, and that trends in capital income growth mirror trends in total financial wealth inequality. Capital income inequality is large and growing fast, accounting for a significant portion of total income inequality.

5.1 Mutual Funds and Delegation

Barriers to High-Return Institutional Funds We compare returns from different types of mutual funds, using data from Morningstar, which contains information for two types of funds: those with a minimum investment of \$100,000 (institutional funds) and those without such restrictions (retail funds). Our fund data span the period 1989 through 2012.

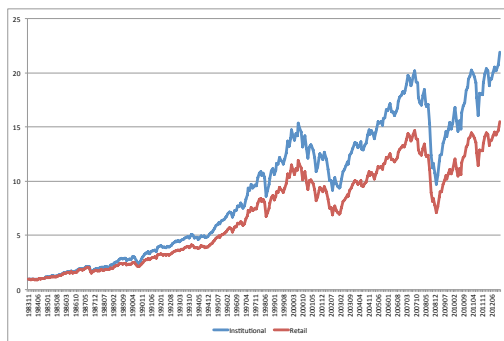


Figure 3: Cumulative investment returns in equity mutual funds by investor type.

Figure 3 plots the cumulative return series for institutional versus retail mutual funds. To construct the figure, we compute the value of one dollar invested in each fund type in January of 1989 and assume that the monthly after-fee return is subsequently reinvested until December 2012. A cumulated value of the dollar in 1989 grows to \$22 for institutional funds and to \$16 for retail funds. This difference amounts to about 3% return difference per year between the two types of funds. Since the institutional funds have a minimum investment threshold, less sophisticated, less wealthy investors do not have access to the higher returns earned by institutional funds, even for “plain vanilla” assets like equities.

Dispersion in the Quality of Asset Management Companies We document a large heterogeneity in mutual fund returns in the data depending on the investment size (typically related to wealth). Additionally, below we show that the average fund does not outperform the passive benchmark and that the performance of a typical

mutual fund is not persistent over time. Taken together, these findings suggest that selecting a mutual fund in any particular period is an informationally intensive task, similar to trading individual stocks.²⁸

Average Mutual Fund Does Not Outperform Passive Benchmark. We construct a sample of risk-adjusted after-fee fund returns by regressing monthly excess fund returns, net of the risk-free rate, on four risk factors: market, size, value, and momentum as in Carhart (1997). The abnormal return from this regression is our definition of a risk-adjusted return. We present in Figure 4 a histogram of monthly returns pooled across all funds and all months in our sample. The mean and median value of the distribution are not statistically different from zero.

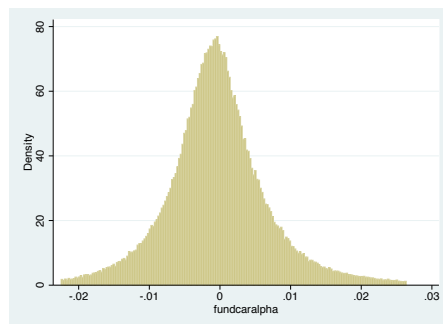


Figure 4: Distribution of equity funds' returns.

Mutual Fund Performance Is Not Persistent Over Time. While an average fund does not beat a passive benchmark, we observe a large cross-sectional dispersion in returns with both small and large values of alpha. It is thus possible that investors could focus their attention only on funds with positive returns thus beating the market portfolio. The issue with such approach is whether funds with positive returns tend to outperform the benchmark on a consistent basis. If not, the strategy of focusing on current winners may not be profitable. To test for such predictability, each month we sort funds into five equal-sized portfolios according to their current risk-adjusted

²⁸We are not the first ones to point out these regularities. Extant literature in finance, such as Kacperczyk, Sialm, and Zheng (2005) or Pástor, Stambaugh, and Taylor (2015) finds that while the average abnormal gross returns of mutual funds are positive, the distribution of returns is highly dispersed and the returns are not predictable.

returns and test whether the ranking of funds into such portfolios is preserved one month and one year into the future. We show the result using a transition matrix of being in a particular quintile portfolio conditional on starting in a given portfolio at time t . Each of the 25 cells of the transition matrix illustrates the probability of being in quintile $j = 1 - 5$ at time $t + k$ conditional on being in quintile $i = 1 - 5$ at time t . We set k to be equal to 1 and to 12 months. The results are in Table 3.

Table 3: Transition Probabilities of Fund Performance

		Performance quintiles				
		at $t + k$				
at t		1	2	3	4	5
		k=1 month				
1		77.8	16.6	3.4	1.3	0.7
2		16.5	56.2	20.7	5.2	1.4
3		3.6	20.5	52.1	20.3	3.4
4		1.2	5.4	20.3	56.6	16.4
5		0.8	1.5	3.6	16.6	77.4
		k=12 months				
1		29.3	19.6	16.6	16.6	17.8
2		20.0	22.2	21.6	20.3	15.9
3		16.6	22.0	23.7	21.9	15.7
4		16.0	21.0	21.9	22.1	19.1
5		18.5	16.7	17.0	19.8	27.9

We observe that fund performance is not very persistent over time. For example, a fund that starts in the top-performing quintile at time t has a 77% chance of ending up in the same quintile one month later. The same probability for one-year ahead transition drops to 28%. Similar patterns emerge for other quintiles in the matrix. We conclude that an uninformed household would face a difficult task to invest in a successful fund by simply following past winners.

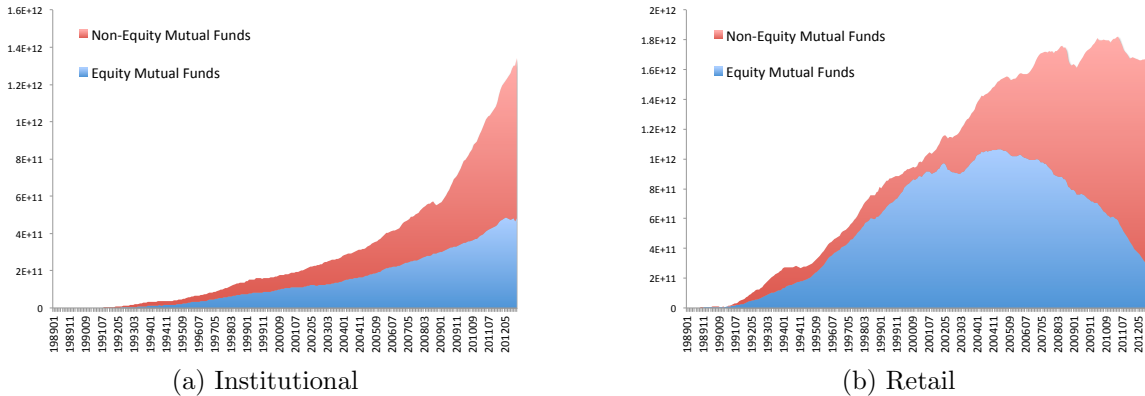


Figure 5: Cumulative Flows to Mutual Funds: Institutional vs. Retail

5.2 Expansion of Ownership

As aggregate capacity grows, sophisticated investors expand their ownership of risky assets by order of volatility: starting from the highest volatility assets and then moving down.

To test this prediction, we consider flows into mutual funds by investor type. Given that equity funds are generally more risky than non-equity funds one would expect unsophisticated investors be less likely to invest in the equity funds, especially if aggregate information capacity grows.

We use data on flows into equity and non-equity mutual funds from Morningstar by sophisticated (institutional) and unsophisticated (retail) investors. As shown in Figure 5, the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the entire sample period. In contrast, the flows from unsophisticated investors display a markedly different pattern. The flows into equity funds grow until 2000 but subsequently decrease at a significant rate to drop by a factor of 3 by 2012. Moreover, this decrease coincides with a significant increase in cumulative flows to non-equity funds. Notably, the increase in equity fund flows by unsophisticated investors observed in the early sample period is consistent with the steady decrease in holdings of individual equity in the U.S. data. To the extent

that direct equity holdings are more risky than diversified equity portfolios, such as mutual funds, this implies that unsophisticated investors have been systematically reallocating their wealth from riskier to safer asset classes.

Overall, these findings qualitatively support our model’s predictions: Sophisticated households have a large exposure to risky assets and subsequently add exposure to less risky assets, and as unsophisticated households face greater information disadvantage they increasingly move their money into safer assets.

5.3 Capital Income Inequality in the SCF

Our evidence on capital income inequality reinforces existing results using more detailed U.S. and European data, e.g. Saez and Zucman (2016), Fagereng, Guiso, Malacrino, and Pistaferri (2016b) and Bach, Calvet, and Sodini (2015).

First, we document that inequality in total financial wealth has grown within the group of households who participate in financial markets, but it has remained essentially unchanged along the extensive margin (defined as the ratio of average financial wealth of the bottom 10% of participating households to that of the non-participating households). Thus the dynamics of financial wealth inequality do not appear to be driven by the extensive (participation) margin. These trends are shown in panel (a) of Figure 6.²⁹

Second, among participants, we find that the increase in inequality in financial wealth tracks the accumulation of capital income from the risky assets (namely, income from dividends, interest income, and realized capital gains). To see this, we consider the counterfactual financial wealth obtained from accruing capital income only.³⁰ Panel (b) of Figure 6 suggests that past capital income realizations may be

²⁹Financial wealth in the SCF contains holdings of risky assets (stocks, bonds, mutual funds), passive assets (life insurance, retirement accounts, royalties, annuities, trusts), and liquid assets (cash, checking and savings accounts, money market accounts).

³⁰For example, the counterfactual financial wealth level in 1995 is equal to the actual financial wealth in 1989 plus 3 times the capital income reported in the prior survey years (in this case, 1989 and 1992).

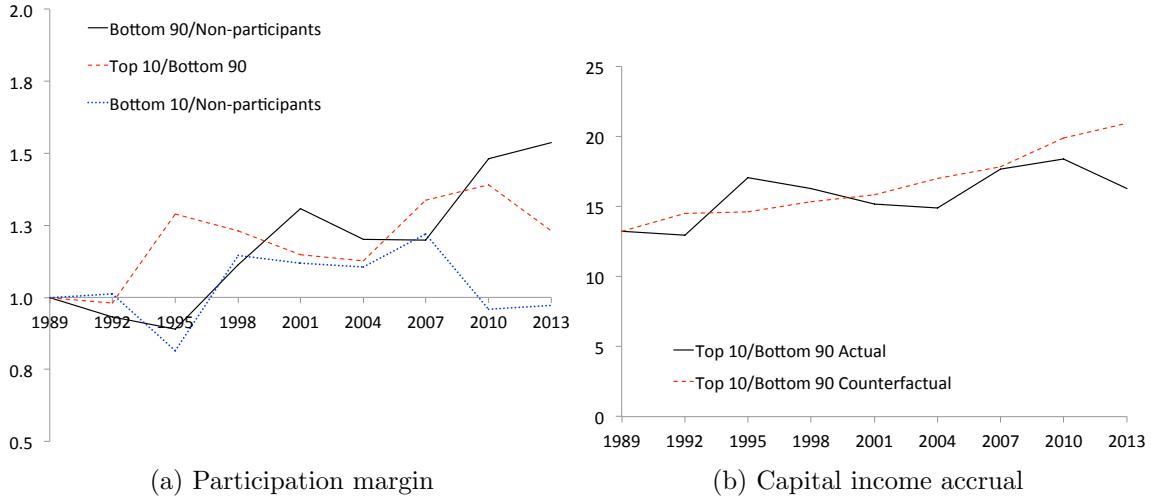


Figure 6: Financial wealth inequality in the SCF. Market participants are sorted in terms of their total wealth. (a) Inequality within the group of households who participate in financial markets, defined as the ratio of financial wealth of the top wealth decile to that of the bottom 90% of participants, versus inequality at the participating margin, measured as the ratio of financial wealth of the bottom 10% of participating households to that of the non-participating households. (b) Actual and counterfactual financial wealth inequality due to accrual of capital income only.

sufficient to explain the evolution of financial wealth inequality, without resorting to mechanisms that involve savings rates from other income sources.³¹

Third, among participating households, capital income inequality is large and growing fast. Panel (a) of Figure 7 shows that in the cross-section, capital income is an order of magnitude more unequal than either labor or total income. For example, in 1989, the average capital income of the top 10% of participants was 21 times larger than that of the bottom 90% of participants. This ratio increased to nearly 40 in 2013. By comparison, the corresponding ratio for wage income was 2.4 in 1989 and 3.9 in 2013. To compare the dynamics of inequality across income sources, we normalize the inequality of each income measure to 1 in 1989, and plot growth rates for capital, labor, and total income inequality in panel (b) of the figure. As is well

³¹By construction, the two wealth levels are identical in 1989, so the figure also implies that the counterfactual levels of financial wealth for each group are very close to those in the data. Still, we treat this evidence as suggestive, since our exercise imposes a panel interpretation on a repeated cross-section.

known, labor income inequality has grown significantly during this period, and so has capital income inequality, which nearly doubled.

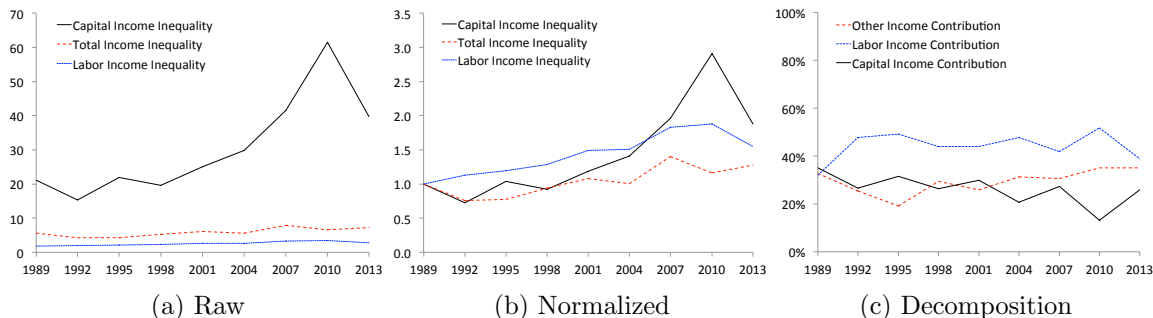


Figure 7: Income inequality growth in the SCF. Inequality is the ratio of the top 10% to the bottom 90% (in terms of total wealth) of participants in financial markets. (a) Inequality for capital income, labor income and other income in levels. (b) Same series, normalized to 1 in 1989. (c) Decomposition of total income inequality into its three components.

Since capital income is so unequal, it is an important contributor to total income inequality, even though its share in total income is not that large (14% on average). To see this, consider the decomposition of total income inequality in period t (denoted T_{10t}/T_{90t}) into shares coming from capital income (denoted by K), labor income (denoted by W), and other (residual) income (denoted by R).³² This process integrates two empirical drivers of inequality: the evolution of shares and the evolution of inequality within each income source.

$$\frac{T_{10t}}{T_{90t}} = \frac{K_{10t}}{K_{90t}} \frac{K_{90t}}{T_{90t}} + \frac{W_{10t}}{W_{90t}} \frac{W_{90t}}{T_{90t}} + \frac{R_{10t}}{R_{90t}} \frac{R_{90t}}{T_{90t}}$$

Panel (c) of Figure 7 plots the contribution at time t of each of the components of total income to the inequality in total income. On average, 26% of the total income

³²Labor income (income from wages and salaries) represents 56% of total income, while other income makes up the remaining 30%. Other income includes social security and other pension income, income from professional practice, business or limited partnerships, income from net rent, royalties, trusts and investment in business, unemployment benefits, child support, alimony and income from welfare assistance programs. In the literature on labor income inequality, business income is sometimes included in labor income. The split between labor and other income does not impact our calculations regarding the relative importance of capital income.

inequality in each year is attributable to capital income.³³

6 Concluding Remarks

What contributes to the growing income inequality across households? This question has been of great economic and policy relevance for at least several decades starting with the seminal work by Kuznets (1953). We approach this question from the perspective of capital income that is known to be highly unequally distributed across individuals. We propose a theoretical information-based framework that links capital income derived from financial markets to a level of investor sophistication. Our model implies the presence of income inequality between sophisticated and unsophisticated investors that is growing in the extent of total information in the market, and could be the result of aggregate technological progress. Predictions on asset ownership, market returns, and turnover provide additional support for the economic mechanism we propose.

One could argue that the overall growth of investment resources and competition across investors with different skill levels are generally considered as a positive aspect of a well-functioning financial market. However, our work suggests that one should assess any policy targeting overall information environment in financial markets as potentially exerting an offsetting and negative effect on socially relevant issues, such as distribution of income. Our work also sheds light on the overall benefits and redistribution aspects of progress in financial markets in terms of creating new financial instruments. Depending on where the new assets land on the volatility (or more generally, opaqueness) spectrum, the benefits will accrue to the relatively less (low-volatility assets) or more (high-volatility assets) sophisticated investors.

³³Furthermore, Benhabib, Bisin and Zhu (2011) show that it is capital income risk, not labor income risk, that is critical to generating the skewness in the wealth distribution seen in the data.

References

- Aiyagari, Rao S., 1994, Uninsured idiosyncratic risk and aggregate saving, *Quarterly Journal of Economics* 109(3), 659–684.
- Arrow, Kenneth J, 1987, The demand for information and the distribution of income, *Probability in the Engineering and Informational Sciences* 1, 3–13.
- Atkinson, Anthony B, Thomas Piketty, and Emmanuel Saez, 2011, Top incomes over a century or more, *Journal of Economic Literature* 49, 3–71.
- Bach, Laurent, Laurent Calvet, and Paolo Sodini, 2015, Rich pickings? Risk, return, and skill in the portfolios of the wealthy, Working Paper Stockholm School of Economics.
- Benhabib, Jess, and Alberto Bisin, 2017, Skewed wealth distributions: Theory and empirics, *Journal of Economic Literature* forthcoming.
- Benhabib, Jess, Alberto Bisin, and Shenghao Zhu, 2011, The distribution of wealth and fiscal policy in economies with finitely lived agents, *Econometrica* 79, 123–157.
- Cagetti, Marco, and Mariacristina De Nardi, 2006, Entrepreneurship, frictions, and wealth, *Journal of Political Economy* 114, 835–870.
- Calvet, Laurent E, John Y Campbell, and Paolo Sodini, 2009, Measuring the financial sophistication of households, *American Economic Review* 99, 393–398.
- Cao, Dan, and Wenlan Luo, 2017, Persistent heterogeneous returns and top end wealth inequality, *Review of Economic Dynamics* 26, 301–326.
- Chien, YiLi, Harold Cole, and Hanno Lustig, 2011, A multiplier approach to understanding the macro implications of household finance, *Review of Economic Studies* 78 (1), 199–234.
- De Nardi, Mariacristina, and Giulio Fella, 2017, Saving and wealth inequality, CEPR Working Paper 11746.
- Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri, 2016a, Heterogeneity and persistence in returns to wealth, NBER Working Paper No 22822.
- Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri, 2016b, Heterogeneity in returns to wealth and the measurement of wealth inequality, *The American Economic Review* 106, 651–655.
- Favilukis, Jack, 2013, Inequality, stock market participation, and the equity premium, *Journal of Financial Economics* 107 (3), 740–759.

- Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, 2016, The dynamics of inequality, *Econometrica* 84, 2071–2111.
- Gomez, Matthieu, 2016, Asset prices and wealth inequality, Working Paper Princeton University.
- Greenwood, Daphne, 1983, An estimation of US family wealth and its distribution from microdata, *Review of Income and Wealth* 29 (1), 23–44.
- Guvenen, Fatih, 2007, Do stockholders share risk more effectively than non-stockholders?, *Review of Economics and Statistics* 89(2), 275–288.
- Guvenen, Fatih, 2009, A parsimonious macroeconomic model for asset pricing, *Econometrica* 77(6), 1711–1750.
- Kacperczyk, Marcin, Clemens Sialm, and Lu Zheng, 2005, On the industry concentration of actively managed equity mutual funds, *Journal of Finance* 60, 1983–2012.
- Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp, 2016, A rational theory of mutual funds’ attention allocation, *Econometrica* 84(2), 571–626.
- Kessler, Dennis, and Edward N. Wolff, 1991, A comparative analysis of household wealth patterns in France and the United States, *Review of Income and Wealth* 37 (1), 249–266.
- Krusell, Per, and Anthony A. Smith, 1998, Income and wealth heterogeneity in the macroeconomy, *Journal of Political Economy* 106(5), 867–896.
- Kuznets, Simon, 1953, Shares of Upper Income Groups in Savings, in *Shares of Upper Income Groups in Income and Savings* (NBER, New York).
- Lusardi, Annamaria, Pierre-Carl Michaud, and Olivia S. Mitchell, 2017, Optimal financial knowledge and wealth inequality, *Journal of Political Economy* forthcoming.
- Maćkowiak, Bartosz, and Mirko Wiederholt, 2009, Optimal sticky prices under rational inattention, *American Economic Review* 99 (3), 769–803.
- Maćkowiak, Bartosz, and Mirko Wiederholt, 2015, Business cycle dynamics under rational inattention, *The Review of Economic Studies* 82, 1502–1532.
- Mankiw, N Gregory, and Stephen P Zeldes, 1991, The consumption of stockholders and nonstockholders, *Journal of Financial Economics* 29, 97–112.
- Matějka, Filip, 2015, Rationally inattentive seller: Sales and discrete pricing, *The Review of Economic Studies* 83, 1125–1155.

- Mondria, Jordi, 2010, Portfolio choice, attention allocation, and price comovement, *Journal of Economic Theory* 145, 1837–1864.
- Pástor, Luboš, Robert F. Stambaugh, and Luke Taylor, 2015, Scale and skill in active management, *Journal of Financial Economics* 116, 23–45.
- Pástor, Luboš, and Pietro Veronesi, 2016, Income inequality and asset prices under redistributive taxation, *Journal of Monetary Economics* 81, 1–20.
- Peng, Lin, 2005, Learning with information capacity constraints, *Journal of Financial and Quantitative Analysis* 40, 307–329.
- Peng, Lin, and Wei Xiong, 2006, Investor attention, overconfidence and category learning, *Journal of Financial Economics* 80, 563–602.
- Peress, Joel, 2004, Wealth, information acquisition and portfolio choice, *The Review of Financial Studies* 17(3), 879–914.
- Piketty, Thomas, 2014, *Capital in the Twenty-First Century*. (Harvard University Press).
- Piketty, Thomas, and Emmanuel Saez, 2003, Income inequality in the United States, 1913–1998, *The Quarterly Journal of Economics* 118, 1–41.
- Quadrini, Vincenzo, 1999, The importance of entrepreneurship for wealth concentration and mobility, *Review of Income and Wealth* 45(1), 1–19.
- Saez, Emmanuel, and Gabriel Zucman, 2016, Wealth inequality in the United States since 1913: Evidence from capitalized income tax data, *The Quarterly Journal of Economics* 131, 519–578.
- Shannon, Claude E, 1948, A mathematical theory of communication, *Bell System Technical Journal* 27, 379–423 and 623–656.
- Sims, Christopher A, 2003, Implications of rational inattention, *Journal of Monetary Economics* 50(3), 665–690.
- Stevens, Luminita, 2018, Coarse pricing policies, Working Paper, University of Maryland.
- Van Nieuwerburgh, Stijn, and Laura Veldkamp, 2009, Information immobility and the home bias puzzle, *Journal of Finance* 64(3), 1187–1215.
- Van Nieuwerburgh, Stijn, and Laura Veldkamp, 2010, Information acquisition and under-diversification, *Review of Economic Studies* 77(2), 779–805.

Vissing-Jorgensen, Annette, 2004, Perspectives on behavioral finance: Does “irrationality” disappear with wealth? Evidence from expectations and actions, in *NBER Macroeconomics Annual 2003, Vol. 18* (The MIT Press, Cambridge).

Appendix: Proofs

Model

Portfolio Choice. In the second stage, each investor chooses portfolio holdings q_{ji} to solve

$$\max_{\{q_{ji}\}_{i=1}^n} U_j = E_j(W_j) - \frac{\rho}{2} V_j(W_j) \quad s.t. \quad W_j = r(W_{0j} - \sum_{i=1}^n q_{ji} p_i) + \sum_{i=1}^n q_{ji} z_i,$$

where E_j and V_j denote the mean and variance conditional on investor j 's information set:

$$E_j(W_j) = E_j[rW_{0j} + \sum_{i=1}^n q_{ji}(z_i - rp_i)] = rW_{0j} + \sum_{i=1}^n q_{ji}[E_j(z_i) - rp_i],$$

$$V_j(W_j) = V_j[rW_{0j} + \sum_{i=1}^n q_{ji}(z_i - rp_i)] = \sum_{i=1}^n q_{ji}^2 V_j(z_i).$$

Let $\hat{\mu}_{ji} \equiv E_j[z_i]$ and $\hat{\sigma}_{ji}^2 \equiv V_j[z_i]$. The investor's portfolio problem is to maximize

$$U_j = rW_{0j} + \sum_{i=1}^n q_{ji}(\hat{\mu}_{ji} - rp_i) - \frac{\rho}{2} \sum_{i=1}^n q_{ji}^2 \hat{\sigma}_{ji}^2.$$

The first order conditions with respect to q_{ji} yield $q_{ji} = \frac{\hat{\mu}_{ji} - rp_i}{\rho \hat{\sigma}_{ji}^2}$. Since W_{0j} does not affect the optimization, we normalize it to zero. The indirect utility function becomes

$$U_j = \frac{1}{2\rho} \sum_{i=1}^n \frac{(\hat{\mu}_{ji} - rp_i)^2}{\hat{\sigma}_{ji}^2}. \quad \square$$

Posterior Beliefs. The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$\hat{\mu}_{ji} = \bar{z} + \frac{Cov(s_{ji}, z_i)}{\sigma_{s_{ji}}^2} (s_{ji} - \bar{s}_{ji}) = s_{ji},$$

$$\hat{\sigma}_{ji}^2 = \sigma_i^2 \left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{s_{ji}}^2 \sigma_i^2} \right) = \sigma_{\delta_{ji}}^2. \quad \square$$

Information Constraint. Let $H(z)$ denote the entropy of z , and let $H(z|s_j)$ denote the conditional entropy of z given the vector of signals s_j . Then

$$I(z; s_j) \equiv H(z) - H(z|s_j) \stackrel{(1)}{=} \sum_{i=1}^n H(z_i) - H(z|s_j) \stackrel{(2)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|z^{i-1}, s_j)$$

$$\stackrel{(1)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|s_j) \stackrel{(3)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|s_{ji}) = \sum_{i=1}^n I(z_i; s_{ji})$$

where (1) follows from the independence of the payoffs z_i ; (2) follows from the chain rule for entropy, where $z^{i-1} = \{z_1, \dots, z_{i-1}\}$; (3) follows from the independence of the signals s_{ji} .

For each asset i , the entropy of $z_i \sim \mathcal{N}(\bar{z}, \sigma_i^2)$ is $H(z_i) = \frac{1}{2} \ln(2\pi e \sigma_i^2)$.

The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$I(z_i; s_{ji}) = H(z_i) + H(s_{ji}) - H(z_i, s_{ji}) = \frac{1}{2} \log \left(\frac{\sigma_i^2 \sigma_{s_{ji}}^2}{|\Sigma_{z_i s_{ji}}|} \right) = \frac{1}{2} \log \left(\frac{\sigma_i^2}{\sigma_{\delta_{ji}}^2} \right),$$

where $|\Sigma_{z_i s_{ji}}| = \sigma_{s_{ji}}^2 \sigma_{\delta_{ji}}^2$ is the determinant of the variance-covariance matrix of z_i and s_{ji} .

Hence $I(z_i; s_{ji}) = 0$ if $\sigma_{\delta_{ji}}^2 = \sigma_i^2$.

$$\text{Across assets, } I(z; s_j) = \sum_{i=1}^n I(z_i; s_{ji}) = \frac{1}{2} \sum_{i=1}^n \log \left(\frac{\sigma_i^2}{\sigma_{\delta_{ji}}^2} \right) = \frac{1}{2} \log \left(\prod_{i=1}^n \frac{\sigma_i^2}{\sigma_{\delta_{ji}}^2} \right) \leq K_j. \quad \square$$

Information Objective. Expected utility is given by

$$E_{0j}[U_j] = \frac{1}{2\rho} E_{0j} \left[\sum_{i=1}^n \frac{(\hat{\mu}_{ji} - rp_i)^2}{\hat{\sigma}_{ji}^2} \right] = \frac{1}{2\rho} \sum_{i=1}^n \frac{E_{0j}[(\hat{\mu}_{ji} - rp_i)^2]}{\hat{\sigma}_{ji}^2} = \frac{1}{2\rho} \sum_{i=1}^n \left(\frac{\hat{R}_{ji} + \hat{V}_{ji}}{\hat{\sigma}_{ji}^2} \right),$$

where \hat{R}_{ji} and \hat{V}_{ji} denote the ex-ante mean and variance of expected excess returns, $\hat{\mu}_{ji} - rp_i$. Conjecture (and later verify) that prices are normally distributed, $p_i \sim \mathcal{N}(\bar{p}_i, \sigma_{pi}^2)$.

$$\hat{R}_{ji} \equiv E_{0j}(\hat{\mu}_{ji} - rp_i) = \bar{z} - r\bar{p}_i,$$

$$\hat{V}_{ji} \equiv V_{0j}(\hat{\mu}_{ji} - rp_i) = \text{Var}(\hat{\mu}_{ji}) + r^2 \sigma_{pi}^2 - 2r \text{Cov}(\hat{\mu}_{ji}, p_i).$$

The signal structure implies that $\text{Var}(\hat{\mu}_{ji}) = \sigma_{s_{ji}}^2$.

Following Admati (1985), we conjecture (and later verify) that prices are $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$, for some coefficients $a_i, b_i, c_i \geq 0$. We compute $\text{Cov}(\hat{\mu}_{ji}, p_i)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs. We obtain

$$\hat{V}_{ji} = \sigma_{s_{ji}}^2 + r^2 \sigma_{pi}^2 - 2rb_i \sigma_{s_{ji}}^2 = (1 - rb_i)^2 \sigma_i^2 + r^2 c_i^2 \sigma_x^2 - (1 - 2rb_i) \hat{\sigma}_{ji}^2.$$

Hence the distribution of expected excess returns is normal with mean and variance:

$$\hat{R}_{ji} = \bar{z} - ra_i \quad \text{and} \quad \hat{V}_{ji} = (1 - rb_i)^2 \sigma_i^2 + r^2 c_i^2 \sigma_x^2 - (1 - 2rb_i) \hat{\sigma}_{ji}^2.$$

Expected utility becomes

$$E_{0j}[U_j] = \frac{1}{2\rho} \sum_{i=1}^n G_i \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} - \frac{1}{2\rho} \sum_{i=1}^n (1 - 2rb_i),$$

where $G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\bar{z} - ra_i)^2}{\sigma_i^2}$, and where the second summation is independent of the investor's choices. \square

Proof of Lemma 1 (Information Choice). The linear objective function and the convex constraint imply that each investor allocates all capacity to learning about a single asset. For all other assets, the posterior variance is equal to the prior variance. Let l_j index the asset about which investor j learns. The information constraint becomes $\prod_{i=1}^n \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} = \frac{\sigma_{l_j}^2}{\hat{\sigma}_{j l_j}^2} = e^{2K_j}$, and hence the variance of the investor's beliefs is given by

$$\hat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j} \sigma_i^2 & \text{if } i = l_j, \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases}$$

The investor's problem becomes picking the asset l_j to maximize $\sum_{i=1}^n G_i \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} = (e^{2K_j} - 1) G_{l_j} + \sum_{i=1}^n G_i$. Since $e^{2K_j} > 1$, the objective is maximized by allocating all capacity to the asset with the largest utility gain: $l_j \in \arg \max_i G_i$. The distribution of posterior beliefs follows. \square

Conditional Distribution of Signals. Conditional on the realized payoff, the signal is a normally distributed random variable, with mean and variance given by

$$E(s_{ji}|z_i) = \bar{s}_{ji} + \frac{\text{Cov}(s_{ji}, z_i)}{\sigma_i^2} (z_i - \bar{z}) = \begin{cases} \bar{z} + (1 - e^{-2K_j}) \varepsilon_i & \text{if } i = l_j \\ \bar{z} & \text{if } i \neq l_j, \end{cases}$$

$$V(s_{ji}|z_i) = \sigma_{s_{ji}}^2 \left(1 - \frac{\text{Cov}^2(s_{ji}, z_i)}{\sigma_{s_{ji}}^2 \sigma_i^2} \right) = \begin{cases} (1 - e^{-2K_j}) e^{-2K_j} \sigma_i^2 & \text{if } i = l_j \\ 0 & \text{if } i \neq l_j. \end{cases} \quad \square$$

Proof of Lemma 2 (Equilibrium Prices). The market clearing condition for each asset in state (z_i, x_i) is

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1} \rho \sigma_i^2} \right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2} \rho \sigma_i^2} \right) dj + (1 - m_{1i} - m_{2i}) \left(\frac{\bar{z} - rp_i}{\rho \sigma_i^2} \right) = x_i,$$

where M_{1i} denotes the set of measure $m_{1i} \in [0, \lambda]$ of sophisticated investors who choose to learn about asset i , and M_{2i} denotes the set of measure $m_{2i} \in [0, 1 - \lambda]$, of unsophisticated investors who choose to learn about asset i .

Using the conditional distribution of the signals, $\int_{M_{1i}} s_{ji} dj = m_{1i} [\bar{z} + (1 - e^{-2K_1}) \varepsilon_i]$ for the type-1 investors, and analogously for the type-2 investors. Then, the market clearing condition can be written as $\alpha_1 \bar{z} + \alpha_2 \varepsilon_i - x_i = \alpha_1 rp_i$, where

$$\alpha_1 \equiv \frac{1 + m_{1i}(e^{2K_1} - 1) + m_{2i}(e^{2K_2} - 1)}{\rho \sigma_i^2} \quad \text{and} \quad \alpha_2 \equiv \frac{m_{1i}(e^{2K_1} - 1) + m_{2i}(e^{2K_2} - 1)}{\rho \sigma_i^2}.$$

We obtain identification of the coefficients in $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$ as

$$a_i = \frac{1}{r} \left[\bar{z} - \frac{\bar{x}}{\alpha_1} \right], \quad b_i = \frac{\alpha_2}{r \alpha_1}, \quad \text{and} \quad c_i = \frac{1}{r \alpha_1}.$$

Let $\Phi_i \equiv m_{1i}(e^{2K_1} - 1) + m_{2i}(e^{2K_2} - 1)$ be a measure of the information capacity allocated to learning about asset i in equilibrium. Further substitution yields

$$a_i = \frac{1}{r} \left(\bar{z} - \frac{\rho \sigma_i^2 \bar{x}}{1 + \Phi_i} \right), \quad b_i = \frac{1}{r} \left(\frac{\Phi_i}{1 + \Phi_i} \right), \quad c_i = \frac{1}{r} \left(\frac{\rho \sigma_i^2}{1 + \Phi_i} \right). \quad \square$$

Proof of Lemma 3 (Equilibrium Learning). Substituting a_i , b_i , and c_i in $G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\bar{z} - ra_i)^2}{\sigma_i^2}$ and defining $\xi_i \equiv \sigma_i^2 (\sigma_x^2 + \bar{x}^2)$ gives $G_i = \frac{1 + \rho^2 \xi_i}{(1 + \Phi_i)^2}$.

By Lemma 1, each investor learns about a single asset among the assets with the highest gain. WLOG, assets are ordered such that $\sigma_i > \sigma_{i+1}$, for all $i \in \{1, \dots, n-1\}$. First suppose that all investors learn about the same asset. Since G_i is increasing in σ_i , this asset is asset 1. All investors learn about asset 1 as long as $\phi \leq \phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1$. At this threshold, some investors switch and learn about the second asset.

For $\phi > \phi_1$, equilibrium gains must be equated among all assets with positive learning mass. Otherwise, investors have an incentive to switch to learning about the asset with the higher gain. Moreover, the gains of all assets with zero learning mass must be strictly lower. Otherwise, an investor would once again have the incentive to deviate and learn about one of these assets.

To derive expressions for the mass of investors learning about each asset, we assume that the participation of sophisticated and unsophisticated investors in learning about a particular asset is proportional to their mass in the population: $m_{1i} = \lambda m_i$ and $m_{2i} = (1 - \lambda) m_i$, where m_i is the total mass of investors learning about asset i . The necessary and sufficient conditions for determining $\{m_i\}_{i=1}^n$ are $\sum_{i=1}^k m_i = 1$; $\frac{1 + \phi m_i}{1 + \phi m_1} = c_{i1}$, for any $i \in \{2, \dots, k\}$, where $c_{i1} \equiv \sqrt{\frac{1 + \rho^2 \xi_i}{1 + \rho^2 \xi_1}} \leq 1$, with equality iff $i = 1$; and $m_i = 0$ for any $i \in \{k+1, \dots, n\}$. Recursively, $m_i = c_{i1} m_1 - \frac{1}{\phi} (1 - c_{i1})$, $\forall i \in \{2, \dots, k\}$. Using $\sum_{i=1}^k m_i = 1$, and defining $C_k \equiv \sum_{i=1}^k c_{i1}$, we obtain the solution for m_1 given by $m_1 = \frac{1}{C_k} + \frac{1}{\phi} \left(\frac{k}{C_k} - 1 \right)$. Using this expression, we obtain the solution for all m_i , $i \in \{1, \dots, k\}$, $m_i = \frac{c_{i1}}{C_k} + \frac{1}{\phi} \left(\frac{k c_{i1}}{C_k} - 1 \right)$. \square

Analytic Results

Proof of Proposition 1 (Learning Dynamics). (i) First, consider a local increase in ϕ to some $\phi' \leq \phi_k$, such that no new assets are learned about in equilibrium (k and C_k are unchanged). For $i \in \{1, \dots, k\}$,

$$\frac{dm_i}{d\phi} = -\frac{1}{\phi^2} \left(\frac{k c_{i1}}{C_k} - 1 \right), \text{ where } c_{i1} \equiv \sqrt{\frac{1 + \rho^2 \xi_i}{1 + \rho^2 \xi_1}} \leq 1 \text{ and } C_k \equiv \sum_{i=1}^k c_{i1}.$$

Hence m_i is strictly decreasing in ϕ if $c_{i1} > \frac{C_k}{k}$ (namely, if the asset is above average in terms of relative volatility), and m_i is increasing in ϕ otherwise. Since c_{i1} is decreasing in i , the condition $c_{\bar{1}1} = C_k/k$ defines the cutoff asset \bar{i} . Moreover, note that for $i \in \{1, \dots, \bar{i}\}$, the absolute value of $\frac{dm_i}{d\phi}$ is decreasing in i , such that the masses of the more volatile assets fall by more than those of the less volatile assets. Likewise, for $i \in \{\bar{i} + 1, \dots, k\}$, the value of $\frac{dm_i}{d\phi}$ is increasing in i , such that the masses of the less volatile assets increase by more than those of the more volatile assets. This results in a flattening of the distribution of investors across assets.

Next, suppose that $k < n$, and consider an increase in ϕ to some $\phi' > \phi_k$, such that $k' > k$ assets are learned about (with $k' \leq n$). Let the new equilibrium masses be denoted by m'_i for $i \in \{1, \dots, k'\}$. Hence, $\sum_{i=1}^{k'} m'_i < 1$. Using the recursive expression for m_i in terms of m_1 , for $i \in \{2, \dots, k\}$

$$m_i - m'_i = c_{i1} (m_1 - m'_1) - (1 - c_{i1}) \left(\frac{1}{\phi} - \frac{1}{\phi'} \right).$$

Suppose that $m_1 \leq m'_1$. Then $\sum_{i=1}^k m_i - \sum_{i=1}^k m'_i = 1 - \sum_{i=1}^k m'_i < 0$, which is a contradiction. Hence $m_1 > m'_1$. Moreover, since c_{i1} is decreasing in i , the condition $m_{\bar{i}} = m'_{\bar{i}}$ defines the threshold value for c_{i1} that defines the cutoff asset \bar{i} .

(ii) First, consider a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about (k and C_k are unchanged). For $i \in \{1, \dots, k\}$,

$$\frac{d(\phi m_i)}{d\phi} = \frac{c_{i1}}{C_k} > 0.$$

Next, suppose that $k < n$, and consider an increase in ϕ to some $\phi' > \phi_k$, such that $k' > k$ assets are learned about in equilibrium (with $k' \leq n$). First, for the new assets that are actively traded, $i \in \{k+1, \dots, k'\}$, $m'_i > m_i = 0$, hence, $\phi' m'_i > \phi m_i$. Second, consider an asset $i \in \{1, \dots, k\}$ and an asset $h \in \{k+1, \dots, k'\}$. Let the new equilibrium gains be denoted by G'_i and G'_h . Then $G_i > G_h$, which implies that $1 + \phi m_i < c_{ih}$, and $G'_i = G'_h$, which implies that $1 + \phi' m'_i = (1 + \phi' m'_h) c_{ih} > (1 + \phi' m'_h) (1 + \phi m_i) \Leftrightarrow \phi' m'_i > \phi m_i + \phi' m'_h (1 + \phi m_i) > \phi m_i$.

(iii) Let $K_1 = K$ and $K_2 = \delta K$, for some $\delta \in (0, 1)$, and consider a symmetric increase in K_j to $(1 + \gamma)K_j$ such that the first $k' \geq k$ assets are learned about.

Let \bar{i} denote the cutoff asset determined in part (i). For $i \in \{\bar{i}, \dots, k'\}$, both $m_i (e^{2K_1} - 1)$ and $m_i (e^{2K_2} - 1)$ increase, since $m'_i \geq m_i$, but $m_i (e^{2K_1} - 1)$ grows by more since e^x is convex.

For $i \in \{1, \dots, \bar{i} - 1\}$, m_i is decreasing in ϕ .

Let $m_{i\phi} \equiv \frac{dm_i}{d\phi}$. The derivatives of interest are

$$D_1 \equiv \frac{d[m_i(e^{2K_1}-1)]}{dK} = m_{i\phi} (e^{2K} - 1) \frac{d\phi}{dK} + 2e^{2K} m_i$$

$$D_2 \equiv \frac{d[m_i(e^{2K_2}-1)]}{dK} = m_{i\phi} (e^{2K\delta} - 1) \frac{d\phi}{dK} + 2\delta e^{2K\delta} m_i$$

where $\frac{d\phi}{dK} = 2\lambda e^{2K} + 2\delta(1 - \lambda) e^{2K\delta} > 0$.

Factoring out $2e^{2K}$ yields

$$\begin{aligned} D_1 &= 2e^{2K} \{ m_i + m_{i\phi} (e^{2K} - 1) [\lambda + (1 - \lambda) \delta e^{2K(\delta-1)}] \} \\ &= 2e^{2K} \{ m_i + m_{i\phi} [\lambda (e^{2K} - 1) + (1 - \lambda) \delta (e^{2K\delta} - e^{2K(\delta-1)})] \} \\ &\stackrel{(1)}{>} 2e^{2K} \{ m_i + m_{i\phi} [\lambda (e^{2K} - 1) + (1 - \lambda) (e^{2K\delta} - 1)] \} \end{aligned}$$

$$= 2e^{2K} \{m_i + m_{i\phi}\} = 2e^{2K} \left[\frac{d(\phi m_i)}{d\phi} \right] \stackrel{(2)}{>} 0,$$

where (2) follows from part (ii) above and (1) follows from the evaluation of the function

$F(\delta) = e^{2K\delta} - 1 - \delta(e^{2K\delta} - e^{2K(\delta-1)})$ for which $F(0) = F(1) = 0$, $F'(\delta) = 0$ has a unique solution, $F'(\delta) > 0$ and $F'(\delta) < 0$, which imply that $F(\delta) > 0$.

Next, note that $\lambda D_1 + (1 - \lambda) D_2 = \left[\frac{d(\phi m_i)}{d\phi} \right] \frac{d\phi}{dK} = 2 \left[\frac{d(\phi m_i)}{d\phi} \right] [\lambda e^{2K} + \delta(1 - \lambda) e^{2K\delta}]$. We have just shown that $D_1 > 2e^{2K} \left[\frac{d(\phi m_i)}{d\phi} \right]$, so for the equality to hold it must be the case that $D_2 < 2\delta e^{2K\delta} \left[\frac{d(\phi m_i)}{d\phi} \right]$. Hence $D_1 > 0$ and $D_1 > D_2$. It remains to be determined if $D_2 > 0$ as well.

$$\begin{aligned} D_2 &= 2e^{2K\delta} \left\{ \delta m_i + m_{i\phi} (1 - e^{-2K\delta}) [\lambda e^{2K} + \delta(1 - \lambda) e^{2K\delta}] \right\} \\ &= 2e^{2K\delta} \left\{ \delta m_i + m_{i\phi} [\lambda e^{2K} + \delta(1 - \lambda) e^{2K\delta} - \lambda e^{2K-2K\delta} - \delta(1 - \lambda)] \right\} \\ &= 2e^{2K\delta} \left\{ \delta m_i + m_{i\phi} [\lambda (e^{2K} - 1) - \lambda (e^{2K-2K\delta} - 1) + \delta(1 - \lambda) (e^{2K\delta} - 1)] \right\} \\ &> 2e^{2K\delta} \left\{ \delta m_i + m_{i\phi} [\lambda (e^{2K} - 1) + \delta(1 - \lambda) (e^{2K\delta} - 1)] \right\} \\ &> 2e^{2K\delta} \left\{ \delta m_i + m_{i\phi} [\lambda (e^{2K} - 1) + (1 - \lambda) (e^{2K\delta} - 1)] \right\} \\ &= 2e^{2K\delta} \left\{ \delta m_i + m_{i\phi} \phi \right\} = 2e^{2K\delta} \left[\frac{d(\phi m_i)}{d\phi} \right] - (1 - \delta) m_i. \end{aligned}$$

Hence, if δ is not too small (i.e. capacity dispersion is not too large), then $D_2 > 0$ for $i \in \{1, \dots, \bar{i} - 1\}$ as well.

Hence, for assets $i \in \{1, \dots, \bar{i} - 1\}$, for which the mass of investors falls in response to the capacity growth, $m_i (e^{2K_1} - 1)$ grows and $m_i (e^{2K_2} - 1)$ grows by less, or even falls, if capacity dispersion is large enough. \square

Proof of Proposition 2. Results follow from equations (17-19). \square

Proof of Proposition 3. (i) Follows from the definition of capital income per capita and equation (18). (ii) Since for all $i \in \{1, \dots, k\}$, the gains G_i are equated in equilibrium, then $E[\pi_{1i} - \pi_{2i}]$ is increasing in m_i , which in turn is increasing in σ_i^2 . \square

Proof of Proposition 4. (i) The increase in dispersion keeps ϕ unchanged. Therefore, using equation (14), the masses m_i are unchanged. With both ϕ and m_i unchanged, prices are unchanged. (ii) The result follows from equation (18): masses and prices do not change, and dispersion, $(e^{2K_1} - e^{2K_2})$ increases. (iii) Relative capital income is

$$\frac{\pi_{1i}}{\pi_{2i}} = \frac{(\bar{z}_i - rp_i)(z_i - rp_i) + (e^{2K_1} - 1) m_i (z_i - rp_i)^2}{(\bar{z}_i - rp_i)(z_i - rp_i) + (e^{2K_2} - 1) m_i (z_i - rp_i)^2} > 1.$$

Since prices are unchanged, $(\bar{z}_i - rp_i)(z_i - rp_i)$ and $m_i (z_i - rp_i)^2$ are unchanged. Since $K'_1 > K_1$ and $K'_2 < K_2$, the second term in π_{1i} increases and the second term in π_{2i} decreases. \square

Proof of Proposition 5. (i) Using equilibrium prices, $\bar{p}_i = \frac{1}{r} \left(\bar{z} - \frac{\rho\sigma_i^2\bar{x}}{1+\phi m_i} \right)$. Per Lemma 1, ϕm_i is increasing in ϕ . Hence, for $i \in \{1, \dots, k\}$, \bar{p}_i is increasing in ϕ . The result for equilibrium expected excess returns $E[z_i - r\bar{p}_i]$ follows.

(ii) Since $\lambda E[q_{1i}] + (1 - \lambda) E[q_{2i}] = \bar{x}$, it is sufficient to show that for $i \in \{1, \dots, k'\}$, $E[q_{1i}]$ increases in response to symmetric capacity growth. Let $K \equiv K_1$, and $K_2 = \delta K$, with $\delta \in (0, 1)$. Since

$$E[q_{1i}] = \frac{1+m_i(e^{2K}-1)}{(1+\phi m_i)} \bar{x}, \text{ then } \frac{dE[q_{1i}]}{dK} = \frac{\bar{x}}{(1+\phi m_i)^2} \left[\frac{d[m_i(e^{2K}-1)]}{dK} (1 + \phi m_i) - \frac{d(\phi m_i)}{d\phi} \frac{d\phi}{dK} m_i (e^{2K} - 1) \right].$$

$$\text{Hence } \text{sign} \left(\frac{dE[q_{1i}]}{dK} \right) = \text{sign} \left(\frac{d[m_i(e^{2K}-1)]}{dK} - \frac{d(\phi m_i)}{d\phi} \frac{d\phi}{dK} \frac{m_i(e^{2K}-1)}{1+\phi m_i} \right).$$

In the proof of Lemma 1, we show that $\frac{d[m_i(e^{2K}-1)]}{dK} > 2e^{2K} \frac{d(\phi m_i)}{d\phi} > 0$. Hence,

$$\begin{aligned} \text{sign} \left(\frac{dE[q_{1i}]}{dK} \right) &= \text{sign} \left(2e^{2K} - \frac{d\phi}{dK} \frac{m_i(e^{2K}-1)}{1+\phi m_i} \right) \\ &= \text{sign} \left(2e^{2K} - \frac{2m_i[\lambda e^{2K} + (1-\lambda)\delta e^{2K\delta}](e^{2K}-1)}{1+m_i[\lambda(e^{2K}-1) + (1-\lambda)(e^{2K\delta}-1)]} \right) \\ &= \text{sign} \left(e^{2K} - (e^{2K}-1) \frac{m_i[\lambda e^{2K} + (1-\lambda)\delta e^{2K\delta}]}{1+m_i[\lambda e^{2K} + (1-\lambda)e^{2K\delta}] - m_i} \right) \\ &\stackrel{(1)}{=} \text{sign} \left(e^{2K} - (e^{2K}-1) \left[\frac{m_i[\lambda e^{2K} + (1-\lambda)e^{2K\delta}]}{1+m_i[\lambda e^{2K} + (1-\lambda)e^{2K\delta}] - m_i} \right] \right) \\ &\stackrel{(2)}{=} \text{sign} (e^{2K} - (e^{2K}-1)) > 0 \end{aligned}$$

where (1) follows from $\delta \in (0, 1)$, and (2) follows from the fact that the term in square brackets is less than 1.

(iii) Let the per capita capital income be decomposed into a component C_i that is common across investor groups, and a component that is group-specific:

$\pi_{1i} = c_i + \frac{1}{\rho\sigma_i^2} m_i (e^{2K} - 1) (z_i - rp_i)^2$, where $c_i \equiv \frac{1}{\rho\sigma_i^2} (\bar{z} - rp_i) (z_i - rp_i)$, with expected value C_i . Then $E[\pi_{1i}] = C_i + \frac{1}{\rho\sigma_i^2} m_i (e^{2K} - 1) E[(z_i - rp_i)^2] = C_i + \frac{1}{\rho} m_i (e^{2K} - 1) G_i$, where G_i is the gain from learning about asset i , equated across all $i \in \{1, \dots, k\}$.

We then obtain that $\frac{E[\pi_{1i}]}{E[\pi_{2i}]} = \frac{C_i + \frac{1}{\rho} m_i (e^{2K}-1) G_i}{C_i + \frac{1}{\rho} m_i (e^{2K\delta}-1) G_i}$.

In response to an increase in K , C_i and G_i decrease, but they affect both sophisticated and unsophisticated profits in the same way. From Lemma 1, $m_i (e^{2K} - 1)$ increases by more than $m_i (e^{2K\delta} - 1)$ in response to a change in K . Hence overall, $\frac{E[\pi_{1i}]}{E[\pi_{2i}]}$ increases. \square

Derivation of volume per capita. We define the volume of trade in asset i between two periods, across all optimizing investors j in group g as $V_i^g \equiv \int |q'_{ji} - q_{ji}| dj$. Integrating over all possible realizations of q'_{ji} and q_{ji} , we obtain average volume across many periods, \bar{V}_i^g . We assume that investors do not change groups over time. To ease notation, most of the derivation omits group and asset superscripts.

Volume between two periods for a generic group First, we calculate the expected volume of trade for each asset by agents in each group from period t to $t + 1$. Let f and F denote the pdf and cdf of current holdings, with mean μ_q and standard deviation σ_q . Let f' and F' denote the pdf and cdf of future holdings, with mean μ'_q and standard deviation σ'_q .

STEP 1. Consider a particular investor with holdings q in the current period. The investor's expected volume of trade between the current and the next period is

$$v(q) \equiv \int_{-\infty}^{+\infty} |q' - q| f'(q') dq' = 2qF(q) - q - 2F'(q) E_{f'}[q' | q' < q] + \mu'_q.$$

Using the formula for the expected value of a normal truncated from above,

$$v(q) = 2qF(q) - q - 2\mu'_q F'(q) + 2\sigma_q'^2 f'(q) + \mu'_q.$$

STEP 2. Integrating over the (normal) distribution of holdings q in the group,

$$V^g = \int_{-\infty}^{+\infty} v(q) f(q) dq = 2 \int_{-\infty}^{+\infty} qF(q) f(q) dq - \mu_q - 2\mu'_q \int_{-\infty}^{+\infty} F'(q) f(q) dq + 2\sigma_q'^2 \int_{-\infty}^{+\infty} f'(q) f(q) dq + \mu'_q.$$

Using the formulas $\int_{-\infty}^{+\infty} \exp\{-ax^2 + bx + c\} dx = \sqrt{\frac{\pi}{a}} \exp\left\{\frac{b^2}{4a} + c\right\}$,

$$\int_{-\infty}^{+\infty} \Phi(a + bx) \phi(x) dx = \Phi\left(\frac{a}{\sqrt{1+b^2}}\right) \text{ and } \int_{-\infty}^{+\infty} x\Phi(bx) \phi(x) dx = \frac{b}{\sqrt{2\pi(1+b^2)}},$$

we compute $J_1 \equiv \int_{-\infty}^{+\infty} qF(q) f(q) dq = \frac{\mu_q}{2} + \frac{\sigma_q}{2\sqrt{\pi}}$,

$$J_2 \equiv \int_{-\infty}^{+\infty} F'(q) f(q) dq = \Phi\left(\frac{\mu_q - \mu'_q}{\sqrt{\sigma_q^2 + \sigma_q'^2}}\right),$$

$$J_3 \equiv \int_{-\infty}^{+\infty} f'(q) f(q) dq = \frac{1}{\sqrt{2\pi(\sigma_q^2 + \sigma_q'^2)}} \exp\left\{-\frac{(\mu_q - \mu'_q)^2}{2(\sigma_q^2 + \sigma_q'^2)}\right\}.$$

Hence $V^g = \frac{\sigma_q}{\sqrt{\pi}} - 2\mu'_q \Phi\left(\frac{\mu_q - \mu'_q}{\sqrt{\sigma_q^2 + \sigma_q'^2}}\right) + \frac{2\sigma_q'^2}{\sqrt{2\pi(\sigma_q^2 + \sigma_q'^2)}} \exp\left\{-\frac{(\mu_q - \mu'_q)^2}{2(\sigma_q^2 + \sigma_q'^2)}\right\} + \mu'_q$, where the means and standard deviations are group and asset specific.

Since the shocks are i.i.d., holdings have the same cross-sectional variance in all periods, $\sigma'_q = \sigma_q$, though they will have different means, depending on shock realizations. Hence

$$V^g = \frac{\sigma_q}{\sqrt{\pi}} \left[1 + \exp \left\{ -\frac{(\mu_q - \mu'_q)^2}{4\sigma_q^2} \right\} \right] + \mu'_q \left[1 - 2\Phi \left(\frac{\mu_q - \mu'_q}{\sqrt{2\sigma_q^2}} \right) \right].$$

Average volume across many periods for a generic group We assume no change in the environment, including no change in capacities and hence learning. Let the distribution of mean holdings μ_q be denoted by g , with mean and variance μ_μ and σ_μ^2 . Since the shocks are normal i.i.d., this distribution is stationary and also normal. The average volume across all possible realizations of μ_q and μ'_q is

$$\bar{V}^g = \int_{-\infty}^{+\infty} v^g(\mu_q) g(\mu_q) d\mu_q, \text{ with } v^g(\mu_q) = \int_{-\infty}^{+\infty} V^g(\mu_q, \mu'_q) g(\mu'_q) d\mu'_q.$$

STEP 3. Using the expression for V^g ,

$$v^g(\mu_q) = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp \left\{ -\frac{(\mu_q - \mu'_q)^2}{4\sigma_q^2} \right\} g(\mu'_q) d\mu'_q + \mu_\mu - 2 \int_{-\infty}^{+\infty} \mu'_q \Phi \left(\frac{\mu_q - \mu'_q}{\sqrt{2\sigma_q^2}} \right) g(\mu'_q) d\mu'_q$$

Using the formulas for integrals of normal distributions, we compute

$$J_1 \equiv \int_{-\infty}^{+\infty} \exp \left\{ -\frac{(\mu_q - \mu'_q)^2}{4\sigma_q^2} \right\} g(\mu'_q) d\mu'_q = \sqrt{\frac{2\sigma_q^2}{\sigma_\mu^2 + 2\sigma_q^2}} \exp \left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\},$$

$$J_2 \equiv \int_{-\infty}^{+\infty} \mu'_q \Phi \left(\frac{\mu_q - \mu'_q}{\sqrt{2\sigma_q^2}} \right) g(\mu'_q) d\mu'_q = \mu_\mu \Phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) - \frac{\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right).$$

Then $v^g(\mu_q) = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q}{\sqrt{\pi}} J_1 + \mu_\mu - 2J_2$ becomes

$$\frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q^2 \sqrt{2}}{\sqrt{\pi}(\sigma_\mu^2 + 2\sigma_q^2)} \exp \left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\} + \mu_\mu - 2\mu_\mu \Phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) + \frac{2\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right).$$

STEP 4. Finally, integrating $v^g(\mu_q)$ over all possible realizations of μ_q , we obtain

$$\begin{aligned} \bar{V}^g &= \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q^2 \sqrt{2}}{\sqrt{\pi}(\sigma_\mu^2 + 2\sigma_q^2)} \int_{-\infty}^{+\infty} \exp \left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\} g(\mu_q) d\mu_q + \mu_\mu \\ &\quad - 2\mu_\mu \int_{-\infty}^{+\infty} \Phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) g(\mu_q) d\mu_q + \frac{2\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \int_{-\infty}^{+\infty} \phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) g(\mu_q) d\mu_q. \end{aligned}$$

We compute

$$J_1 \equiv \int_{-\infty}^{+\infty} \exp \left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\} g(\mu_q) d\mu_q = \sqrt{\frac{\sigma_\mu^2 + 2\sigma_q^2}{2(\sigma_\mu^2 + 2\sigma_q^2)}},$$

$$J_2 \equiv \int_{-\infty}^{+\infty} \Phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) g(\mu_q) d\mu_q = \frac{1}{2},$$

$$J_3 \equiv \int_{-\infty}^{+\infty} \phi \left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) g(\mu_q) d\mu_q = \frac{J_1}{\sqrt{2\pi}}.$$

Then

$$\bar{V}^g = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q^2 \sqrt{2}}{\sqrt{\pi(\sigma_\mu^2 + 2\sigma_q^2)}} J_1 + \mu_\mu - 2\mu_\mu J_2 + \frac{2\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} J_3 = \frac{1}{\sqrt{\pi}} \left(\sigma_q + \sqrt{\sigma_q^2 + \sigma_\mu^2} \right).$$

Variations by Investor Group

Consider the groups $g = SL, UL$ of sophisticated and unsophisticated investors who learn about asset i . These groups differ in their capacities only. A particular investor j in group g holds $q_{ji} = e^{2K_g}(s_{ji} - rp_i)/\rho\sigma_i^2$. The cross-sectional variance of holdings for this group, conditional on the realized shocks, is

$$\left(\sigma_{qi}^g \right)^2 = \left(\frac{e^{4K_g}}{\rho^2 \sigma_i^4} \right) \text{Var}(s_{ji} - rp_i) = \frac{e^{2K_g} - 1}{\rho^2 \sigma_i^2}.$$

The cross-sectional mean is

$$\mu_{qi}^g = \left(\frac{e^{2K_g}}{\rho \sigma_i^2} \right) E(s_{ji} - rp_i) = \frac{e^{2K_g}}{1 + \phi m_i} (\bar{x} + \nu_i) + \frac{e^{2K_g} - 1 - \phi m_i}{\rho \sigma_i^2 (1 + \phi m_i)} \varepsilon_i.$$

The expected value of mean of holdings is $\mu_{\mu i}^g = \frac{e^{2K_g}}{1 + \phi m_i} \bar{x}$ and the variance of mean holdings is

$$\left(\sigma_{\mu i}^g \right)^2 = \left(\frac{e^{2K_g}}{1 + \phi m_i} \right)^2 \sigma_x^2 + \left(\frac{e^{2K_g} - 1 - \phi m_i}{1 + \phi m_i} \right)^2 \frac{1}{\rho^2 \sigma_i^2}.$$

Consider the group NL of investors who are not learning about asset i . All investors in this group hold the same quantity $q_{ji} = \mu_{qi} = (\bar{z} - rp_i)/\rho\sigma_i^2$. Hence

$$\left(\sigma_{qi}^{NL} \right)^2 = 0 \text{ and } \mu_{qi}^{NL} = \frac{1}{\rho \sigma_i^2} (\bar{z} - rp_i).$$

The mean and variance of mean holdings are

$$\mu_{\mu i}^{NL} = \frac{\bar{x}}{1 + \phi m_i} \text{ and } \left(\sigma_{\mu i}^{NL} \right)^2 = \left(\frac{1}{1 + \phi m_i} \right)^2 \sigma_x^2 + \left(\frac{\phi m_i}{1 + \phi m_i} \right)^2 \frac{1}{\rho^2 \sigma_i^2}.$$

Consider the assets with zero learning, ZL . For assets that are not learned about by anyone ($m_i = 0, \phi m_i = 0$), all investors hold $q_{ji} = \mu_{qi} = (\bar{z} - rp_i)/\rho\sigma_i^2$. Hence

$$\left(\sigma_{qi}^{ZL} \right)^2 = 0 \text{ and } \mu_{qi}^{ZL} = \frac{1}{\rho \sigma_i^2} (\bar{z} - rp_i).$$

The mean and variance of mean holdings are

$$\mu_{\mu i}^{ZL} = \bar{x} \text{ and } \left(\sigma_{\mu i}^{ZL} \right)^2 = \sigma_x^2.$$

□

Proof of Proposition 6. First, average volume of active investors, $g = SL, UL$, is

$$\bar{V}^g = \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}} \left[\sqrt{e^{2K_g} - 1} + \sqrt{e^{2K_g} - 1 + \left(\frac{e^{2K_g}}{1+\phi m_i}\right)^2 \rho^2 \sigma_i^2 \sigma_x^2 + \left(\frac{e^{2K_g} - 1 - \phi m_i}{1+\phi m_i}\right)^2} \right]$$

\bar{V}^g is increasing in K_g hence $\bar{V}^{SL} > \bar{V}^{UL}$.

Next, average volume of passive investors in actively traded assets is

$$\bar{V}_i^{NL} = \frac{\sigma_\mu^{NL}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}} \sqrt{\left(\frac{\rho\sigma_i\sigma_x}{1+\phi m_i}\right)^2 + \left(\frac{\phi m_i}{1+\phi m_i}\right)^2}.$$

Using $\sqrt{a} + \sqrt{b} > \sqrt{a+b}$,

$$\begin{aligned} \bar{V}^{UL} &> \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}} \left[\sqrt{2(e^{2K_2} - 1) + \left(\frac{e^{2K_2}}{1+\phi m_i}\right)^2 \rho^2 \sigma_i^2 \sigma_x^2 + \left(\frac{e^{2K_2} - 1 - \phi m_i}{1+\phi m_i}\right)^2} \right] \\ &= \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}} \sqrt{\frac{(\phi m_i)^2(2e^{2K_2} - 1) + 2\phi m_i(e^{2K_2} - 1) + (e^{4K_2} - 1)}{(1+\phi m_i)^2} + \left(\frac{e^{2K_2}}{1+\phi m_i}\right)^2 \rho^2 \sigma_i^2 \sigma_x^2} \\ &> \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}} \sqrt{\frac{(\phi m_i)^2}{(1+\phi m_i)^2} + \left(\frac{1}{1+\phi m_i}\right)^2 \rho^2 \sigma_i^2 \sigma_x^2} = \bar{V}_i^{NL}. \end{aligned}$$

Hence for $i \in \{1, \dots, k\}$, $\bar{V}_i^{SL} > \bar{V}_i^{UL} > \bar{V}_i^{NL}$.

□