Market Power and Price Informativeness*

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Abstract

The asset ownership structure in financial markets worldwide has changed significantly over the last few decades. Institutional investors own a larger fraction of assets, the distribution of their ownership is more concentrated, and the ownership by passive investors is getting increasingly more important. To study implications of these facts, we develop a general equilibrium portfolio-choice model with endogenous information acquisition and market power. We show that the size and concentration of institutional investors have opposite effects on price informativeness. Further, the introduction of passive investors has a negative effect on price informativeness, both through quantities and through changes in active investors' learning. Finally, we show that predictions of the model with endogenous information acquisition are significantly different from those implied by models with exogenous information, such as Kyle (1985).

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1 Introduction

The last few decades have witnessed a number of significant changes in the composition of U.S. equity ownership. First, institutional ownership of an average stock more than doubled, to around 60%. Similarly, the equity holdings by ten largest institutional investors more than doubled, to an average 35%. Finally, the holdings of passive funds have grown considerably relative to those of active funds. The percentage of equity mutual funds' total assets that are held by index funds has grown from 12% in 2007 to 25% in 2016.

These changes in ownership structure have drawn considerable attention from market participants, policy makers, and academics. One major consideration is the implication of changing market structure on asset prices, particularly the effect of *large* active and passive investors on price discovery and, more broadly, on the efficiency of capital allocation in the economy. On the one hand, larger active investors have greater capacity to conduct fundamental research, which would increase the amount of information revealed in their trading. On the other hand, they also recognize the price impact of their trades, which makes them trade less on any information they have (as in Kyle, 1985). In this paper, we provide a micro-founded theory of endogenous choices on both of these margins: which assets to learn about and which assets to optimally hold. We then study the implications of the theory for the interaction between market structure and *price informativeness*—a measure summarizing how well the variation in asset prices predicts the variation in fundamentals.[1]

Our theoretical framework incorporates three new elements crucial for capturing the impact of market structure on price informativeness: (i) investors have different sizes, and therefore different price impacts; (ii) investors have different abilities to acquire information, which yields signals of endogenous quality about future shocks; (iii)

[1]Formally, we define price informativeness as the covariance of the price of that asset with its future fundamental, normalized by the volatility of the price. This definition has been also used in Bai, Philippon, and Savov (2016). However, other standard measures correlate positively with ours and would not change our conclusions.

investors make learning and portfolio decisions across a range of risky assets. These elements allow us to study the interaction between investors' absolute and relative sizes and the incentives to acquire and reveal information about assets via trades. In particular, we identify *information pass-through*—how much an investor is willing to respond in quantity adjustment to the arrival of new private information— as the main (and novel) force in the model responsible for determining price informativeness.

The model features a financial market with heterogeneous assets and investors. A subset of investors are atomistic (*competitive fringe*), while the rest have market power (*oligopolists*) and internalize the effect of their trades on equilibrium prices. (*Active*) investors have differential ability to learn about the future paths of prices, and other (*passive*) investors have no ability to do so. Oligopolists play a Nash equilibrium in their acquisition of information, and then play a Cournot game in the asset market to optimize their portfolios. Inputs into the model are the size of the investors, interpreted as assets under management, and their learning capacities. The presence of large investors is a novel component of portfolio choice models with endogenous information acquisition. The combination of endogenous learning and trading in a multi-asset model is a significant extension of the theoretical literature on market power, whose focus is either on exogenous information or a single asset market.

We identify three key forces that affect price informativeness. The first one is the degree to which prices track fundamentals, which can be viewed as average ownership-weighted information. This effect increases with agents' ability to learn, and is positively related to price informativeness. The second one is the sensitivity of the oligopolists' quantities to private information, or what we call the information pass-through to quantities. Information pass-through is also positively related to price informativeness, as increases in pass-through reduce price volatility. We find that information pass-through is hump-shaped in the size of the oligopoly sector, which implies that size has a non-monotonic effect on price informativeness. On the one hand, the larger an oligopolist is, the larger will be her asset positions, and hence a change in asset position will reveal more in price. On the other hand, because of her price impact, she will actively reduce her reaction to signals in terms of quantities. The third factor impacting price informativeness is concentration of ownership, which in our model takes the form of the Herfindahl-Hirschman Index, weighted by agents' learning intensity—we term this measure the learning HHI or *LHHI*. This force arises due to the noise introduced by large players' trades and is independent of any learning by the fringe. It is, therefore, unambiguously negatively related to price informativeness. The overall effect of size on price informativeness is a result of a tension between these three forces. Quantitatively, in numerical examples parameterized to match the features of the U.S. market, we find that the information pass-through effect dominates the other two forces and gives rise to price informativeness that is hump-shaped in size.

We further study the effects of different market structures on price informativeness by changing the distribution of sizes (assets under management). Two features of the theory shed light on the response of the model. The first feature is the effect of changing size on a single investor. When an investor is atomistic she will specialize in her information collection, choosing only one asset to learn about. As the investor increases in size, she continues to specialize for a time, but also internalizes the increasing effect of her learning and trading on prices. When the magnitude of that effect gets large enough, the investor's information pass-through declines and it is no longer optimal to specialize and so she diversifies her learning. This is a novel finding, and one that is central to the results that follow. The second feature is that, from an aggregate price informativeness perspective, diversified learning is beneficial because price informativeness exhibits decreasing returns to learning on an asset-by-asset basis.

We present three sets of results from the model. First, we find that aggregate price informativeness has a non-monotonic, hump shape in the size of the oligopoly sector. On the one hand, as oligopolists increase in size, they diversify their learning, which helps average price informativeness. On the other hand, over a certain size, their informational pass-through starts decreasing, hurting price informativeness for all assets, and also the average. For small sizes, information pass-through is increasing and both effects increase price informativeness, but eventually information pass-through starts declining and dominates the diversification effect. Second, we show that increases in the concentration of ownership (or equivalently, size) unambiguously decrease price informativeness. A concentrated distribution of oligopolists means that comparatively larger parts of the market are controlled by just a few agents. The increased presence of oligopolists reduces price informativeness because their own private signal mistakes increase the noise introduced in the price, as well as because their information pass-through goes down.

Finally, we find that an increase in the size of passive investors—those who do not trade in an informed way (e.g., index funds)—at the expense of active investors, unambiguously reduces price informativeness. This result, while partly expected, sheds additional light on the information effect of passive investors' size on active investors' behavior. There are two aspects to this effect. First, assets in the market get diverted from active to passive investors, resulting in less informed funds for trade. Second, as the active investors lose market power, they specialize more in their learning. Specialization leads to a decrease in aggregate price informativeness, but because of agents' preferences for learning about high-volatility over low-volatility assets, the decrease is not uniform across all assets—some asset prices become more informative.

In order to further elucidate the key economic channels at play, we compare the results from the general model to those coming from two special cases. In the first case, the oligopoly sector is represented by one large, monopolistic investor. In the second case, all investors are atomistic. In these special cases, we obtain closed-form solutions for price informativeness and learning, which provide useful points of comparison. In additional tests, we derive closed-form solutions for the size thresholds at which investors find it privately optimal to specialize in their learning, and at which it would be optimal *from a price informativeness standpoint* for them to specialize in their learning.

Our work is reminiscent of the literature developed by Kyle (1985), and subsequent papers. Similar to that literature, we are interested in issues related to price informativeness when dealing with non-atomistic investors. However, unlike any of the papers in that literature, we introduce endogenous learning across many risky assets, which allows for the results on specialization and spreading of attention. Further, ours is one of the first papers to think about the impact of passive investing *on active investors' behavior* and the subsequent effects on asset pricing. We compare the results of our paper to a setup that is closer in spirit to Kyle's work, in which information quality is exogenously determined. We show that changing market structure in an exogenous information world has vastly different implications for price informativeness because of the inability of active investors to adjust their learning optimally. In particular, we show that the optimal level of institutional ownership coming from a model with exogenous information choice can be biased upwards or downwards relative to the fully endogenous model, depending on the exogenous information structure one assumes. We conclude that modeling endogenous information choices is crucial when making statements about the size and structure of the asset management sector.

In Section 2, we present a set of motivating facts from the U.S. data on institutional ownership and its concentration. Section 3 presents the theoretical framework, the equilibrium concept, and derives basic theoretical tradeoffs between the ownership structure and price informativeness. In Section 4, we derive numerical solutions for the more general settings and discuss various policy experiments. Section 6 concludes. Any omitted proofs and derivations are in the Appendix.

1.1 Related Literature

Our paper spans several research themes. The literature on informed trading with market power dates back to Kyle (1985) and Grinblatt and Ross (1985) whose setup is one strategic trader, and Holden and Subrahmanyam (1992), which extends the model of Kyle into an oligopolistic framework. Lambert, Ostrovsky, and Panov (2016) extend the Kyle's model to study the relation between the number of strategic traders and information content of prices.[2] In all these studies, information is an exogenous process, which is a key dimension along which our model works. Also, they do not examine the role of concentration and active/passive traders, both being the main focus of our study. Kyle, Ou-Yang, and Wei (2011) allow for endogenous information acquisition but their mechanism depends on differences in risk aversion. Also, they focus on the contracting features of delegation and only consider one risky asset. In turn, our framework utilizes heterogeneity in information capacity and multi-asset economy.

Our general equilibrium model is anchored in the literature on the endogenous information choice, in the spirit of Sims (1998, 2003). More closely related to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), and Kacperczyk, Nosal, and Stevens (2017). Ours is the first theoretical study to introduce market power into a model with endogenous information acquisition. This novel aspect allows us to study strategic responses of oligopolistic traders in terms of their demand and information choices.

We also contribute to the literature on information production and asset prices. Bond, Edmans, and Goldstein (2012) survey the literature on information production in financial markets, emphasizing the differences between new information produced in markets (revelatory price efficiency: RPE) and what is already known and merely

[2] Models in which traders condition on others' decisions also include Foster and Viswanathan (1996) and Back, Cao, and Willard (2000).

reflected in prices (forecasting price efficiency: FPE). Our focus is solely on RPE and is largely dictated by the modeling framework we use.[3] Stein (2009) develops a model of market efficiency and sophisticated (arbitrage) capital in the presence of capital constraints. Garleanu and Pedersen (2015) examine the role of search frictions in asset management for price efficiency. Breugem and Buss (2018) study the impact of benchmarking on price informativeness in a costly information acquisition competitive equilibrium model.

On an empirical front, Chen, Goldstein, and Jiang (2007) and Bakke and Whited (2010) find that the relation between stock prices and investment is stronger for firms with more informative stock prices, whereas Baker, Stein, and Wurgler (2003) find that it is stronger for firms that issue equity more often. None of the above studies investigates the role of market power and endogenous information acquisition. The exception is Bai, Philippon, and Savov (2016) who show empirically that price informativeness is greater for stocks with greater institutional ownership. We confirm their findings for the range of the ownership values. However, we show that beyond certain levels (not observed in their data) ownership may in fact reduce price informativeness. Separately, we also investigate the role of ownership concentration and provide a micro-founded general equilibrium model that allows us to study the underlying economic mechanism in more depth. In a contemporaneous work, Farboodi, Matrey, and Veldkamp (2017) examine differences in price informativeness between companies included and not included in the S&P 500 index. They show that the indexed companies exhibit larger efficiency, which they attribute to composition effect of these companies, being older and larger. Their focus, however, is not on market power and changes in market structure.

Finally, we add to a growing empirical literature that studies the impact of market structure in asset management on various economic outcomes. Following the disec-

[3]Theoretical work on asset prices and real efficiency also includes Dow and Gorton (1997), Subrahmanyam and Titman (1999), Kurlat and Veldkamp (2015), and Edmans, Goldstein, and Jiang (2015).

onomies of scale argument of Chen et al. (2004), Pástor, Stambaugh, and Taylor (2015) show significant diseconomies of scale at the industry level. Using a merger between BlackRock and BGI as a shock to market power, Massa, Schumacher, and Yan (2016) study the asset allocation responses of their competitors. They find that competitors scale down positions which overlap with those held by the merged entity. More broadly, He and Huang (2014) and Azar, Schmalz, and Tecu (2016) study consequences of common asset ownership by large blockholders for product market competition and prices. Our work complements these studies by studying, theoretically and empirically, the effect of ownership structure on price informativeness.

2 Motivating Facts

The growth in institutional ownership has been previously documented in several studies, starting with Gompers and Metrick (2001). The evidence on concentration is much more sparse. Similarly, evidence on passive investments has been largely explored, theoretically, from the agency perspective (e.g., Basak and Pavlova (2013)).

In this section, we show details of the three empirical facts that motivate our study. First, institutional stock ownership has increased over the last 35 years. Second, the ownership structure is skewed towards the largest owners. Third, the ownership mix has shifted from active towards passive investors. Except for the recent paper by Bai, Philippon, and Savov (2016) which emphasizes the first fact, no other study has exploited the implications of these facts for longer-horizon price informativeness.[4]

Our data on institutional stock ownership come from Thomson Reuters and span the period 1980–2015. Even though the formal requirements to report holdings allow for smaller companies not to report, the representation of institutions in the data is more than 98% in value-weighted terms. We calculate the stock-level institutional

[4] A parallel microstructure literature (Boehmer and Kelley (2009)) examines empirically the relation between institutional ownership and price efficiency due to trading intensity. Efficiency there is measured using variance ratios and pricing errors. The conclusions from this literature are akin to those reported in our paper.

ownership by taking the ratio of the number of stocks held by financial institutions at the end of a given year to the corresponding number of shares outstanding. Next, we aggregate the measures across stocks by taking a simple average across all stocks in our sample. Using equal weighting, rather than value weighting, gives a conservative metric of the trends in the data. Subsequently, we calculate a similar measure, but only taking into account the holdings of the top-10 largest holders for a given stock. We present the time-series dynamics of the two quantities in Figure 1.

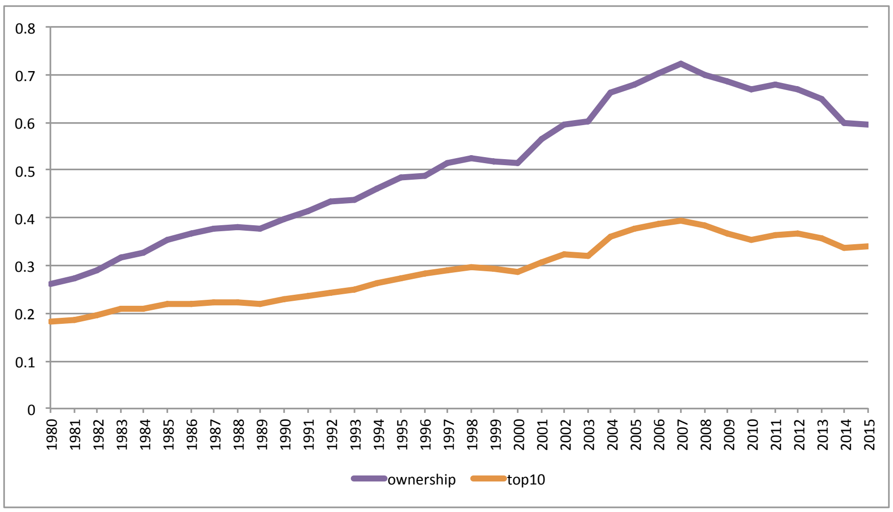


Figure 1: Institutional Ownership: Unconditional and Top-10 Holders

Both series indicate a clear pattern underlying the recent policy discussions: Institutional ownership has grown and the increase has been mostly fueled by the growing concentration of ownership. The magnitudes of the growth are economically large: Over the period of over 35 years, each ownership statistic has more than doubled. While we focus here on the average trends in the data, even stronger effects can be observed in the cross section of stocks with different characteristics.

In our model, a more natural way to measure concentration is the Herfindahl-Hirshman Index (HHI), defined as the sum of squared shares of all institutional owners of a given stock. However, the problem with using the index is its mechanical correlation with the number of investors. To the extent that the number of institutions has been growing steadily over the same period the unadjusted index would reflect two effects going in opposite direction. To filter out this mechanical sorting, we take out the predicted component in the HHI accounted by the number of investors.

The untabulated results indicate that the concentration levels have been generally going up over time. This pattern has been particularly visible since the early 1990s. The magnitude of the growth is economically large and the large values of concentration, especially in the last few years, reflect the concerns policy makers have voiced with regard to this phenomenon.

To illustrate the effects on ownership mix we define active investors as those engaged in information acquisition process and passive investors as those who strictly invest in pre-defined index portfolios. The latter group includes both index mutual funds and ETFs. Because identifying passive funds in the institutional investors data is not trivial we borrow the evidence from the Investment Company Institute (ICI) Fact Book. We show the time-series evolution of the percentage of passively managed equity mutual funds in the U.S. in Figure 2.

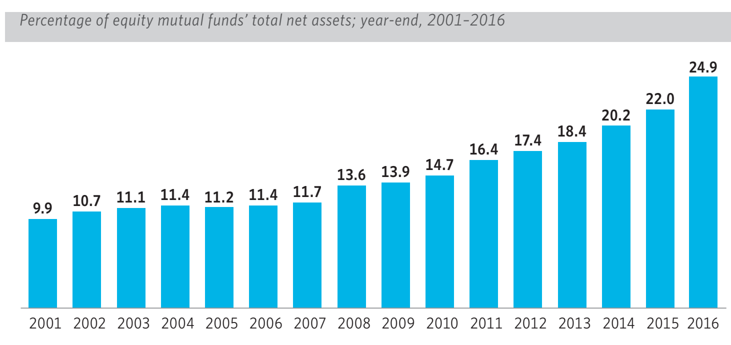


Figure 2: Passive Equity Fund Share

The results indicate a significant increase in passive ownership in the period 2001–2016. While passive funds accounted for less than 10% of total equity fund market in the U.S., this share has increased to almost 25% by 2016. In the paper, we take this trend as given and merely focus on its consequences for stock price informativeness.

To conclude, we note that while the motivating facts we present relate to institutional investors, the model we present next is a general theory of asset allocation and information acquisition by investors with market power. We believe institutions are natural candidates for this type of investors.

3 Model

This section presents a noisy rational expectations portfolio choice model in which investors are constrained in their capacity to process information about assets payoffs. The setup departs from standard information choice models (e.g., Van Nieuwerburgh and Veldkamp (2010) or Kacperczyk et al. (2017)) by introducing market power for some investors. In the model, we solve for price informativeness of the aggregate economy and individual assets differentiated by their volatility.

3.1 Setup

The model features a finite continuum of traders, divided into $L+1$ many segments, represented by λ_i, $i \in \{0, ..., L\}$. The traders in the first segment, λ_0, are atomistic—these traders act as a *competitive fringe*, in that they are able to pay attention to innovations in asset prices, but do not have any market power. They are indexed by h. Measures $\{\lambda_1, \lambda_2, ..., \lambda_L\}$ of investors act as *oligopolists*, indexed by j. Each measure collects information and trades, as the fringe does, but the oligopolists collect and trade as a unit, and therefore have market power in information, *and* market power in trading.

Every member of the fringe and every oligopolist observes signals about innovations in asset prices. The vector of signals for the oligopolists for asset j is $s_j = (s_{j1}...s_{jL})$. The vector of signals for the fringe for asset j is indexed by h. Investors of both types maximize mean-variance utility function, with common risk aversion ρ.

The market comprises one risk-free asset, with a price normalized to one and a net payout of r, and $n > 1$ risky assets, indexed by i, with prices p_i and independent payoffs $z_i = \bar{z} + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_i^2)$. The risk-free asset has unlimited supply, and each risky asset has a stochastic supply with mean $\bar{x}$ and variance $\{\sigma_{xi}\}$. We can think of these as noisy supply shocks.

Agents make portfolio decisions and can choose to obtain information about price innovations for some or all of the risky assets. The capacity to process information for the oligopolists is denoted $\{K_j\}$, while the capacity of each member of the fringe is constant at K_h. We place no restrictions on the values of K_j and K_h other than they must be finite and nonnegative. Investors do not learn from prices. Oligopolists and members of the fringe can use their capacities to receive informative signals about the payoff of the asset and reduce that variance accordingly. We model signal choice using entropy reduction, as in Sims (2003).

We denote an agent j's posterior variance on asset i as $\hat{\sigma}_{ji}^2 \leq \sigma_i^2$. For simplicity, we also define $\alpha_{ji} \equiv \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \geq 1$. We conjecture and later verify the following price structure:

$$p_i = a_i + b_i \varepsilon_i - c_i \nu_i - \sum_{j=1}^{L} d_{ji} \zeta_{ji} \tag{1}$$

where ε_i and ν_i are the innovations in the payoff and noisy supply shocks, respectively. The first term corresponds to the base price, and the second one to the innovation. The innovation is not typically revealed completely in prices, because agents cannot perfectly observe it. The third term corresponds to noise or liquidity shocks, while the fourth one is defined as follows: First, define δ_{ji} as the data loss of oligopolist j: $\delta_{ji} \equiv z_i - s_{ji}$, then define $\zeta_{ji} \equiv \delta_{ji} - \frac{1}{\alpha_{ji}} \varepsilon_i$ to be the portion of the data loss that is uncorrelated with the price innovation. Then $p_i \sim \mathcal{N}\left(a_i, \sigma_{pi}^2\right)$ where σ_{pi} can be expressed as:

$$\sigma_{pi}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^{L} d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}}\right) \widehat{\sigma}_{ji}^2 \tag{2}$$

See the Appendix for the derivations. Before solving the oligopolists' problem, we first analyze the problem faced by the competitive fringe.

3.1.1 Competitive fringe

Portfolio problem Given posterior beliefs and equilibrium prices, each competitive investor h solves the following problem:

$$U_h = \max_{\{q_{hi}\}_{i=1}^n} E_h(W_h) - \frac{\rho}{2} V_h(W_h) \quad s.t. \quad W_j = r\left(W_{0h} - \sum_{i=1}^n q_{hi} p_i\right) + \sum_{i=1}^n q_{hi} z_i \quad (3)$$

where E_h and V_h are the perceived mean and variance of investor h conditional on her information set, and W_{0h} is her initial wealth. Optimal portfolio holdings are given by:

$$q_{hi} = \frac{\hat{\mu}_{hi} - r p_i}{\rho \hat{\sigma}_{hi}^2} \quad (4)$$

where $\widehat{\mu}_{hi}$ and $\widehat{\sigma}_{hi}^2$ are the mean and variance of investor h's posterior beliefs about payoff z_i.

Given this second-stage solution, the fringe agents solve a first-stage information choice problem. Each member of the fringe can choose to receive signals s_{hi} about each asset payoff ϵ_i. The vector of signals is subject to an information capacity constraint, based off Shannon's (1948) mutual information measure: $I(z; s_h) \leq K_h$. Since K_h is finite, this expression constrains the ability of fringe members to reduce the uncertainty of signals.

Information problem Each member of the fringe faces the following information problem:

$$\max_{\{\widehat{\sigma}_{hi}^2\}_{i=1}^n} U_{0h} \equiv \frac{1}{2\rho} \sum_{i=1}^n \frac{E_{0h}\left(\widehat{\mu}_{hi} - r p_i\right)^2}{\widehat{\sigma}_{hi}^2} \quad (5)$$

subject to the relative entropy constraint

$$\prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} \leq e^{2K_h}. \tag{6}$$

The information problem can also be written as:

$$U_{0h} = \sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2}, \tag{7}$$

We obtain a corner solution: each investor h learns about one asset $l_h \in \arg\max\{G_i\}$. The gain to the competitive investors from learning about asset i is:

$$G_i \equiv \frac{(\overline{z} - ra_i)^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2} + r^2 \left(\sum_{j=1}^{L} d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}}\right) \widehat{\sigma}_{ji}^2 \right) - \frac{\widehat{\sigma}_{hi}^2}{\sigma_i^2}(1 - 2rb_i)$$

Derivation in Appendix. The gain from learning about a particular asset is the same across all competitive investors. However, this gain is a function of the learning by the oligopolist in that asset (namely, it is a function of the oligopolist's posterior variance, $\widehat{\sigma}_{ji}^2$). The gains to learning about an asset's payoff result from the fact that the fringe traders can take advantage of deviations in the price from their perception of the asset's value. The first term of G_i corresponds to the gains of trade from the fundamental; the second one to the gains of trade from deviations in the innovation; the third one to the gains of trade from noise traders; and the fourth one to alterations in price due to data loss by the oligopolists.

3.1.2 Oligopolists

Portfolio problem Oligopolists have a similar trading problem as the fringe, and the quantity demanded by each oligopolist is:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i\left(q_{ji}\right)}{\rho \widehat{\sigma}_{ji}^2 + r\frac{dp_i(q_{ji})}{dq_{ji}}}, \tag{8}$$

The derivative in the denominator reflects the fact that oligopolists have market power. Each oligopolist internalizes the fact that their asset purchase decisions affect the equilibrium price. Using market clearing, we can solve for this derivative to get:

$$\frac{dp_i(q_{ji})}{dq_{ji}} = \frac{\lambda_j \rho \sigma_i^2}{\lambda_0 r(1+\Phi_{hi})} > 0, \tag{9}$$

where

$$\Phi_{hi} \equiv m_{hi}\left(e^{2K_h} - 1\right), \tag{10}$$

and m_{hi} is the mass of competitive investors learning about asset i. Hence, how sensitive the price is to an oligopolist's demand depends (inversely) on what fraction of the competitive fringe is learning about that asset, and how much.

The oligopolist's demand becomes:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right)} \tag{11}$$

where $\hat{\lambda}_{ji} = \frac{\lambda_j}{\lambda_0(1+\Phi_{hi})}$ - essentially a ratio of the effective size shares of the oligopolists relative to the fringe. Given the expression for quantity demanded, we can calculate indirect utility as:

$$U_j = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\widehat{\mu}_{ji} - rp_i\right)^2\left[\frac{\widehat{\sigma}_{ji}^2 + 2\widehat{\lambda}_{ji}\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right)^2}\right]. \tag{12}$$

Derivation in Appendix. As with the fringe, oligopolists' expected utilities depend positively on the deviations of their personal estimates from the equilibrium price (larger deviations mean larger quantities demanded). Further, the smaller is the oligopolists' posterior variance the larger is their utility. The larger is the oligopolist's market power (or conversely the smaller is the fringe, or the less informed the fringe), the larger is the oligopolist's price impact, and therefore the smaller her utility.

Information problem The oligopolist's information problem is

$$\max_{\left\{\widehat{\sigma}_{ji}^2\right\}_{i=1}^n} U_{0j} \quad s.t. \quad \prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}. \tag{13}$$

We can also write the constraint as

$$\prod_{i=1}^{n} \alpha_{ji} \leq e^{2K_j} \Leftrightarrow \sum_{i=1}^{n} \ln\alpha_{ji} \leq 2K_j. \tag{14}$$

with

$$\ln\alpha_{ji} \geq 0. \tag{15}$$

The Lagrangean is [dropping the $1/2\rho$ term]

$$\mathcal{L} = \sum_{i=1}^{n} \left[u_i\left(\alpha_{ji}\right) - \mu\ln\alpha_{ji} + \eta_i\ln\alpha_{ji}\right] + n\gamma 2K_j, \tag{16}$$

The optimality conditions are

$$u_i'\left(\alpha_{ji}\right) - \frac{\mu}{\alpha_{ji}} + \frac{\eta_i}{\alpha_{ji}} = 0, \quad \forall i = 1, ..., n. \tag{17}$$

The capacity constraint is always binding, so $\sum_{i=1}^{n} \ln\alpha_{ji} = 2K_j$ and $\mu > 0$. Let L denote the set of assets that are learned about by the oligopolist. We have that

$$\alpha_{jl} > 1 \quad \text{and} \quad \eta_l = 0 \quad \text{and} \quad \mu = \alpha_{jl} u_l'\left(\alpha_{jl}\right) \quad \forall l \in L \tag{18}$$

and

$$\sum_{l=L} \ln\alpha_{jl} = 2K_j. \tag{19}$$

For assets $i \notin L$,

$$\alpha_{jl} = 1 \quad \text{and} \quad \eta_l = \mu - u_i'(1) \geq 0 \quad \Leftrightarrow \quad \alpha_{ji} u_i'\left(\alpha_{ji}\right) \geq u_i'(1) \quad \forall l \in L. \tag{20}$$

These conditions yield the oligopolist's allocation of attention across assets, $\{\alpha_{ji}\}$, as a function of the equilibrium price coefficients, a_i, b_i, c_i, d_i, and the share of competitive investors' learning about each asset, m_{hi}. Given the oligopolist's choice of the set $\{\alpha_{ji}\}$, variance of the posterior belief of the oligopolist is σ_i^2/α_{ji} and the corresponding mean is just the signal s_{ji}. The signal is distributed, conditional on the realizations $z_i = \bar{z} + \varepsilon_i$, as

$$E(s_{ji}|z_i) = \bar{z} + \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right)\varepsilon_i = \bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right)\varepsilon_i,$$

$$Var(s_{ji}|z_i) = \sigma_i^2\left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right)\frac{\hat{\sigma}_{ji}^2}{\sigma_i^2} = \left(1 - \frac{1}{\alpha_{ji}}\right)\frac{1}{\alpha_{ji}}\sigma_i^2.$$

3.2 Equilibrium

We solve for the coefficients of equation (1): a_i, b_i, c_i, d_{ki}, and d_{ji} (derivation in the Appendix):

$$a_i = \frac{\bar{z}}{r} - \frac{\bar{x}}{r}\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})} \tag{21}$$

$$b_i = N_i\left(\sum_{j=1}^{L}\frac{M_{ji}(\alpha_{ji}-1)}{r\alpha_{ji}} + \frac{\Phi_{hi}}{r(1+\Phi_{hi})}\right) \tag{22}$$

$$c_i = \frac{N_i\rho\sigma_i^2}{r\lambda_0(1+\Phi_{hi})} \tag{23}$$

$$d_{ji} = \frac{N_i M_{ji}}{r} \tag{24}$$

where $M_{ji} \equiv \frac{\hat{\lambda}_{ji}\sigma_i^2}{\left(\hat{\sigma}_{ji}^2+\hat{\lambda}_{ji}\sigma_i^2\right)}$ and $N_i \equiv \frac{1}{1+\sum_{j=1}^{L}M_{ji}}$. The fundamental component of the price, a_i, depends positively on $\bar{z}$. An increase in supply also decreases a_i, as do increased risk aversion and fundamental volatility. As the fringe's size or attentional capacity increase, their demand increases, and thus prices increase. As the oligopolists' size increases, or as their attention to asset i increases, demand goes up, M_{ji} increases, and N_i decreases, again driving up the price.

The coefficient b_i depends almost exclusively on the information choices of the fringe and oligopolists. If the fringe cannot pay attention, then Φ drops to zero, and so does the second term of the expression. If the oligopolists cannot pay attention, each α_{ji} goes to zero. b_i is increasing in Φ_{hi} and α_{ji}, because increased attention increases investors' ability to predict the innovation, and therefore their information will be better reflected in prices.

The same reasons for demand's fluctuation in a_i apply to c_i, as c_i corresponds to the random component, while a_i corresponds to the mean component. We next show the existence of an equilibrium.

Proposition 1. *An equilibrium in learning exists.*

All proofs are in the Appendix.

3.3 Price Informativeness

Without loss of generality, we analyze a form of the model in which the fringe cannot learn. Price informativeness in the model is given by the covariance of the price with the fundamental shock, normalized by the standard deviation of the price. Alternatively, this can be seen as the correlation of the price with the fundamental, multiplied by the asset's price variance. This definition is taken from Bai, Philippon, and Savov (2016).

$$PI = \frac{b_i \sigma_i}{\sqrt{b_i^2 + c_i^2 \sigma_{xi}^2 / \sigma_i^2 + \sum_j d_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}},$$

where a_i, b_i, c_i and d_{ji} are the coefficients of the equilibrium price function. In the expression for PI, b_i parameterizes the covariance of the price with the shock z_i; the second term in the denominator captures noise in the price coming from the noise trader demand shock, and the third term in the denominator captures the noise in the price coming from the noise in the oligopolists' private signals. Clearly, the lower are the noise terms relative to the signal term b_i, the higher is price informativeness. Plugging in terms, we get:

We can use the equilibrium expressions for the price coefficients to express PI as

$$PI = \frac{\sigma_i \sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\left(\sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right)^2 + \frac{1}{(\sum_j W_{ji})^2} \frac{\sigma_{xi}^2}{\sigma_i^2} + \sum_j \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{25}$$

where ω_{ji} is the average share of asset i held by oligopolist j, given by

$$\omega_{ji} \equiv \frac{Q_{ji}}{\sum_k Q_{ki}},$$

and Q_{ji} is the average quantity of asset i held by oligopolist j, and W_{ji} is the responsiveness of the quantity traded of asset i by oligopolist j to the private signal of oligopolist j. We call W the *information passthrough to quantities*, and calculate as,

$$W_{ji} = \frac{\partial \lambda_j q_{ji}}{\partial \hat{\mu}_{ji}} = \frac{\lambda_j \alpha_{ji}}{\rho \sigma_i^2 (1 + \hat{\lambda}_{ji} \alpha_{ji})}.$$

Intuitively, PI is increasing in the term $\sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}$, which is the ownership share-weighted average of the reduction in uncertainty[5] about asset i's payoffs due to learning by oligopolist j. PI is also increasing in W, the information passthrough to quantities. The more information affects trading, the more it shows up in prices.

Finally, PI is decreasing in the term $\sum_j \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$, which is given by the weighted sum of the noise in private signals, with weights given by the square ownership shares of each oligopolist. We define this term as the *Learning HHI* or $LHHI$. To see that this expression is related to ownership concentration, notice that, in a symmetric case of $\alpha_{ji} = \alpha_i$, it simplifies to $\frac{\alpha_i - 1}{\alpha_i^2} HHI_i$, where HHI_i is the Herfindahl index for asset i. Therefore, if the noise in oligopolists' signals is equally volatile, high concentration hurts PI through this channel.

[5]Note that $(\alpha_{ji} - 1)/\alpha_{ji} = (\sigma_i^2 - \hat{\sigma}_{ji}^2)/\sigma_i^2$

The expression in (25) highlights the importance of modeling the choice of information for price informativeness. For an exogenously fixed learning structure (that is, fixed $\{\alpha_{ji}\}_{j=1,..,l,i=1,...,n}$), putting high weight on the highest-α oligopolist is beneficial as it always increases the numerator of PI. However, working through the third term in the denominator, high concentration of ownership could be detrimental (e.g., for equal αs), or beneficial (e.g., for very unequal distribution of α). Hence, the information structure one assumes in an exogenous information model dictates the conclusion on the benefits of concentration.

3.4 Economic Forces

Given the derivations of the model, we can now set up our next two Propositions. These results describe the economic forces that drive all of the results of this section. The first one concerns the shape of the PI function:

Proposition 2. *Price Informativeness for asset j is concave in α_{ji}, $\forall i$. That is, $\frac{\partial^2 PI}{\partial \alpha_{ji}^2} < 0$.*

Intuitively, price informativeness exhibits decreasing returns to learning. This result is significant because if we care about higher levels of aggregate price informativeness, we would want agents to learn about many assets, as opposed to concentrating their learning in a few assets.

The second force concerns the shape of the utility function:

Proposition 3. *An oligopolist's utility function is concave in her own learning, and the degree of its concavity is increasing in the oligopolist's size. That is, $\frac{\partial^2 U_{ij}}{\partial \alpha_{ji}^2} < 0$ and $\frac{\partial^3 U_{ij}}{\partial \alpha_{ji}^2 \partial \lambda_j} < 0$.*

The result on concavity follows from the proof of the existence of an equilibrium. Further, we also know that as λ increases, agents reduce their quantities of trade due to their increased price impact. What this Proposition implies, is that as an investor gets bigger, the same level of informational investment is less attractive, as returns diminish more quickly. Therefore, an oligopolist would prefer to spread her attention as she grows rather than to specialize.

In sum, we can observe a quantity effect and a learning effect of size on price informativeness. The two effects are related, but distinct. When an oligopolist gets bigger, the quantity effect implies that she reduces trade size to mitigate price impact. A reduction in trade size would *reduce* price informativeness for all assets, regardless of whether the oligopolist learned about them or not. But as she gets bigger, she will spread her learning to more assets, due to the concavity of her utility function. By Proposition 2, this would *increase* aggregate price informativeness. Therefore, one can observe a clear tradeoff when an oligopolist grows in size.

3.5 Price Informativeness Effects

Three primary results related to price informativeness are born out of the above two forces.[6] First, PI is non-monotonic in the size of the oligopoly sector. We can prove this in the case with a single oligopolist (a monopolist):

Lemma 1. *Suppose $L = 1$ and $K_h = 0$. Then, $PI''(\lambda_1) < 0$.*

When λ_1 is very small, the monopolist will specialize in her learning, and will have limited capital at her disposal. As a result, her impact on the price will be very small. When λ_1 is arbitrarily close to 0, PI is also close to 0. As λ_1 increases, the increase in capital offsets the increased price impact. As λ_1 increases to the size of the entire market, the monopolist will barely trade at all, because such trade would be fully revealed in prices, thus pushing price informativeness to zero again.

Second, PI monotonically decreases as the concentration of the λs increases. This effect can be clearly seen in the decomposition of price informativeness above—the

[6]We cannot show all three analytically in full generality, but at least we can always show a version of the result analytically.

volatility of the price is impacted positively by the sum of squared ownership shares weighted by α.

The third result concerns the changes to PI when λ of passive investors is increased at the expense of active investors' λ. Because capital is now moving from being actively managed to being passively managed, there is an immediate quantity effect that decreases price informativeness. That is, holding α's constant, the marginal change of λ's results in non-learning terms getting weighed more, and learning terms getting weighed less. Theoretically, the learning effect could offset this force, but, by Proposition 3, we know that the opposite will happen. Therefore, a flow of assets from the active sector to the passive sector will decrease price informativeness through two channels: a quantity channel, and a learning channel.

4 Numerical Analysis

In this section, we provide a set of quantitative results from the solution to the equilibrium of the model.[7] We select parameter values for the return distribution $\bar{z}$ and $\{\sigma_i\}_{i=1}^n$, the liquidity distribution $\bar{x}$ and $\{\sigma_{xi}\}_{i=1}^n$, the risk-free return r, risk aversion ρ, fringe and oligopolists' learning capacities K_h and $\{K_j\}_{j=1}^l$, and their respective sizes λ_0 and $\{\lambda_j\}_{j=1}^l$. The simulation generates equilibrium levels of price informativeness, oligopoly holdings, and oligopoly concentration for each asset.

In our simulations, we choose the parameters with two goals in mind: they have to be in an empirically relevant region of the parameter space and the solution needs to involve some degree of learning. Specifically, we consider parameters such that the benchmark model exhibits: (i) learning about all assets, (ii) aggregate institutional holding share of between 60% and 70% (which corresponds to the information in Figure 1), (iii) market excess real return of around 7% (which corresponds to the average over 1980-2015). For the results reported below, we set the number of assets

[7] This involves solving a fixed point of the best responses of the oligopolists to each other's learning and trading policies.

to $n = 10$ and the number of oligopolists to $l = 6$.[8] We report parameter values in Table 1.

Table 1: Parameter values

Parameter	Symbol	Value
Mean payoff, supply	$\bar{z}_i$, $\bar{x}_i$	10, 5 for all i
Number of assets	n, l	$10, 6$
Risk-free rate	r	2.5%
Vol. of noise shocks	σ_{xi}	0.41 for all i
Vol. of asset payoffs	σ_i	$\in [1, 1.5]$, linear distribution
Risk aversion	ρ	1.3
Information capacities	K_h, $\{K_j\}$	$0, 4.5$, constant
Investor masses	$\lambda_0, \lambda_l/\lambda_1$	$0.45, 4$ λ_js linearly distributed

4.1 Cross-sectional Patterns

We begin by analyzing the cross section of equilibrium output variables across assets for the benchmark parameter values in Table 1. Figure 3 presents the relation between equilibrium price informativeness per asset (on the y-axis) and equilibrium oligopoly holdings per asset (on the x axis). We find that price informativeness is increasing in underlying volatility, and so are total oligopoly holdings.

Figure 4 presents the relation between equilibrium price informativeness and equilibrium oligopoly concentration. The larger an oligopolist's presence in a particular asset's market, the more likely she is to internalize the price effect of her trade. As such, she would like to be less informed than she would be if she had a small presence. As a result, concentration in a particular asset is associated with lower levels of price informativeness.

In Figure 5, we present the above cross-sectional relations for the part of price informativeness due to only the correlation of prices and shocks. That is the part

[8] The choice is largely dictated by the computational speed. Experiments with larger values of each parameter do not change the conclusions from this exercise.

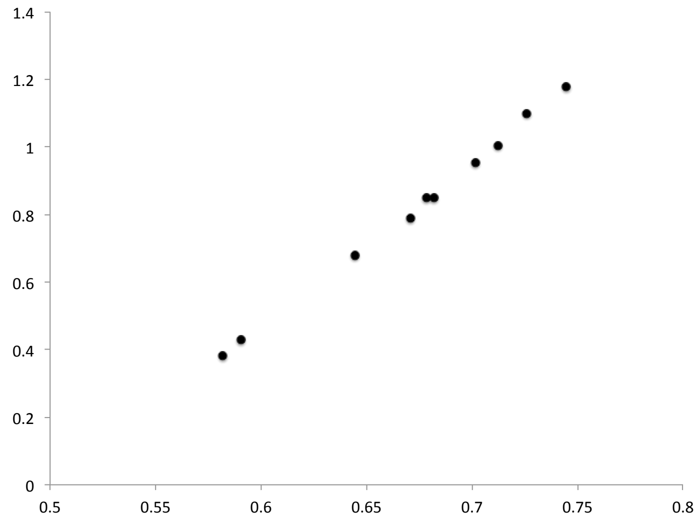


Figure 3: Price informativeness and institutional ownership

of the information measure that is endogenous to the information choices of agents, and does not come from pure cross-sectional dispersion of the exogenous shocks. As the figure indicates, the positive relation with institutional holdings and the negative relation with concentration hold for the correlation part of price informativeness, consistently with the empirical patterns documented before.

4.2 Policy Experiments

The different signs of the relation suggest an interesting interaction between ownership and concentration for the overall effect on price informativeness. We now move on to analyze the effect of policy on the aggregate price informativeness. While in Figures 3 and 4, each point corresponded to one asset, in the following exercise, each point corresponds to one iteration of a full financial market (with several assets). The experiments are useful as a way to isolate the relative effects of levels and concentration of institutional holdings on price informativeness.

The size of the oligopoly In our first experiment, we look at how average price informativeness across assets changes in response to different levels of λ_0. Holding the

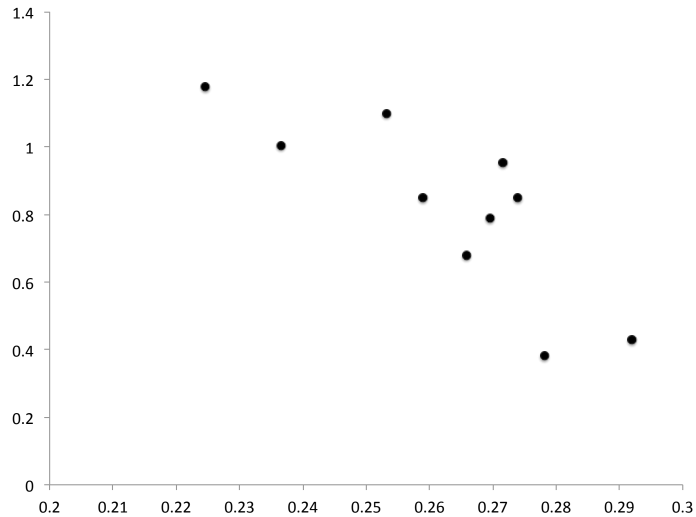
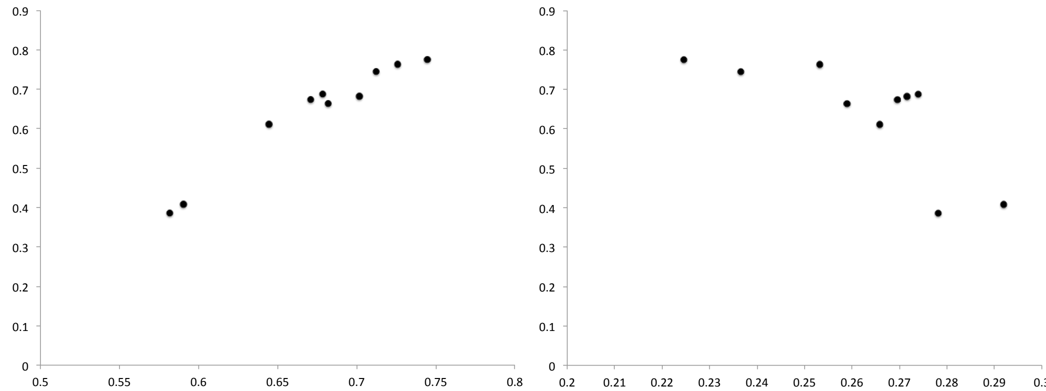


Figure 4: Price informativeness and concentration

distribution of λ_j fixed, we look at simulations of the model by varying λ_0 from 0.05 to 0.95. The type of policy we test here could be thought of as a limit on entry, or a limit on a per-agent size in a given market, which would then affect the *composition* of ownership in the market, keeping the total mass of investors constant.

Figure 6 shows the relation between the size of the institutional sector (parameterized by $1 - \lambda_0$) and endogenous variables of interest. The price informativeness in Panel (a) shows a hump-shaped relation, on average and also for each asset (as indicated by interquantile 10-90 range) with the parameterized size, and hence also with the actual realized ownership which is monotonically increasing (Panel (b)). The model's results point to an interior solution for optimal institutional sector size. This result can be explained by the tradeoff between more efficient (that is, diversified) learning due to larger size of the institutional sector, and an inefficiency due to the endogenous restriction of size of trades (quantities) due to the price impact considerations of the large investors.

When initially the size of the institutional sector is small, the oligopolists' price impact considerations are not very important in their quantity decisions, which means that increasing their size will mean more diversified learning without adverse impact

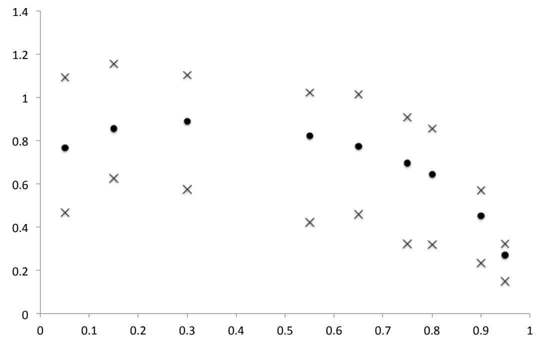


(a) Relative to institutional ownership. (b) Relative to concentration.

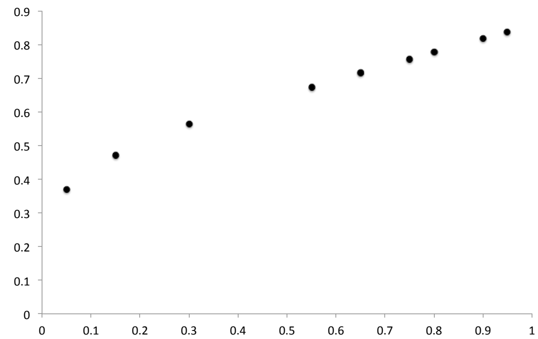
Figure 5: Price informativeness: correlation only

of how quantities react to individual signals (and hence how they show up in price). As the size of the institutional sector increases further, learning about each asset becomes more diversified: Additional oligopolists start learning (Panel (d)) and trading any given asset, which results in a large drop in concentration (Panel (c)), and increased ownership (Panel (b)). The increased diversification in learning means more efficient price informativeness while still relatively small size means that the negative quantity effects have not kicked in. Above a certain size of the institutional sector, the information choice is fully diversified and does not change by much further, but the size considerations are very significant and result in *decreasing* the size of trades as the size of the sector goes up—too much information is revealed in prices as quantity reacts to private signals. These effects give rise to a hump-shaped relation in the model between price informativeness and both institutional ownership and the concentration measure of that ownership. We present these results in Figure 7.

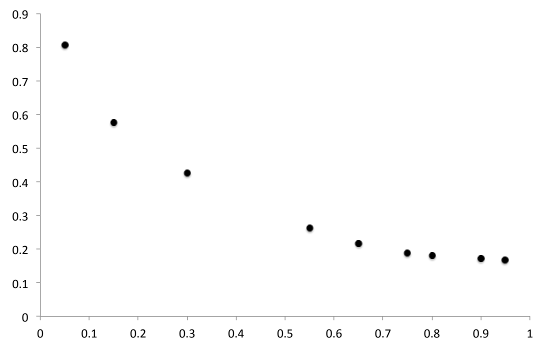
The concentration of the oligopoly In our second experiment, we consider the effects of a policy that affects the concentration of actively trading oligopolists. Holding λ_0 constant, we vary the size distribution of $\{\lambda_j\}$ in order to measure an impact on the concentration measure. Specifically, we vary λ_l/λ_1 from 1.05 to 10, with in-



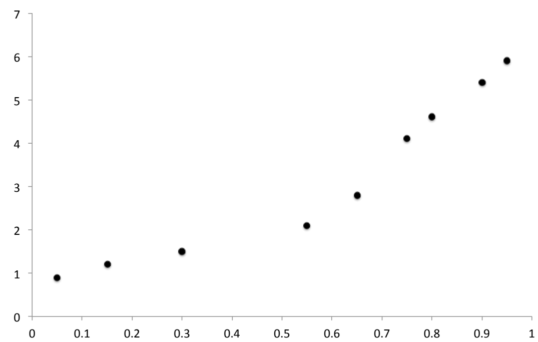
(a) Price Informativeness, average and 10, 90 percentiles



(b) Institutional Ownership



(c) Concentration

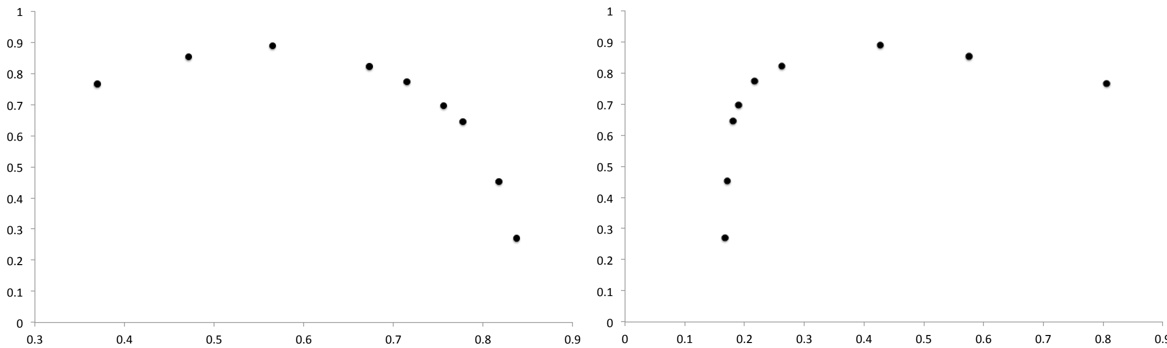


(d) Average Num. of Active Traders per Asset

Figure 6: Response of model to changing size of institutional sector $(1 - \lambda_0)$.

termediate λ_js growing linearly from λ_1 to λ_l. In doing so, we keep the sum of all λ_js equal to $1 - \lambda_0$ to isolate the effect of concentration on endogenous variables. Figure 8 presents the results for price informativeness and concentration.[9] The results with respect to concentration are roughly monotonic: Holding ownership relatively constant, a decrease in concentration increases the price informativeness in the aggregate. This is in line with the intuition from the previous exercise. If ownership is relatively stable, then there is no change in *average* market power across these markets. However, changing the size distribution of the oligopolists towards a more unequal one increases the concentration of ownership and hence increases market power of some of the oligopolists, distorting their quantity choices more. That leads

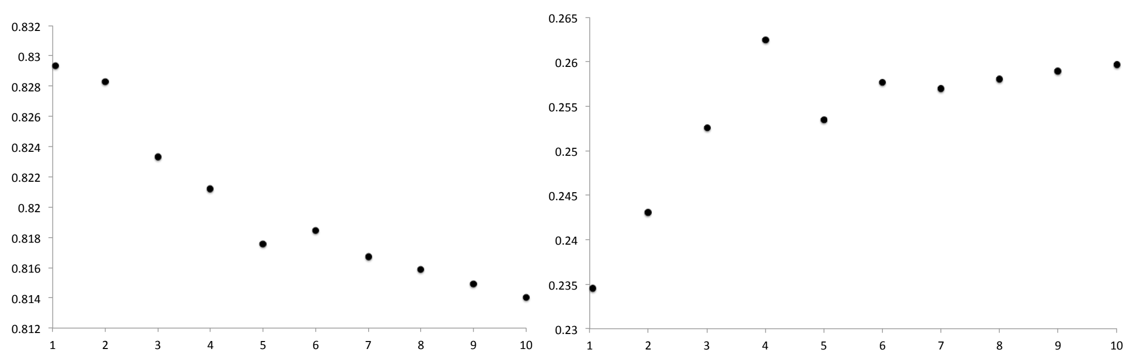
[9]Institutional ownership in this case varies only by 1.4% of the mean—by design—and hence we do not show its graph explicitly.



(a) Relative to institutional ownership (b) Relative to concentration.

Figure 7: Price informativeness in the model when changing $1 - \lambda_0$.

to a negative relation between concentration and price informativeness, keeping the ownership stable.



(a) Price Informativeness (b) Concentration

Figure 8: Average price informativeness and concentration relative to dispersion λ_l/λ_1

Decomposition To better characterize the channels through which size affects price informativeness, we conduct the following decomposition exercise. We take the experiment as before—that is, changing the size of the oligopoly sector while maintaining the same relative concentration of oligopolists within the sector. We calculate the actual values of price informativeness under that experiment, as well as counterfactuals under three other alternatives. Each alternative corresponds to a world in which we allow the terms of the model to adjust as we adjust the size of the oligopoly sector, but we keep one part of the expression for price informativeness fixed at the value obtained in the first (smallest size) experiment. We plot the different informativeness levels in Figure 9.

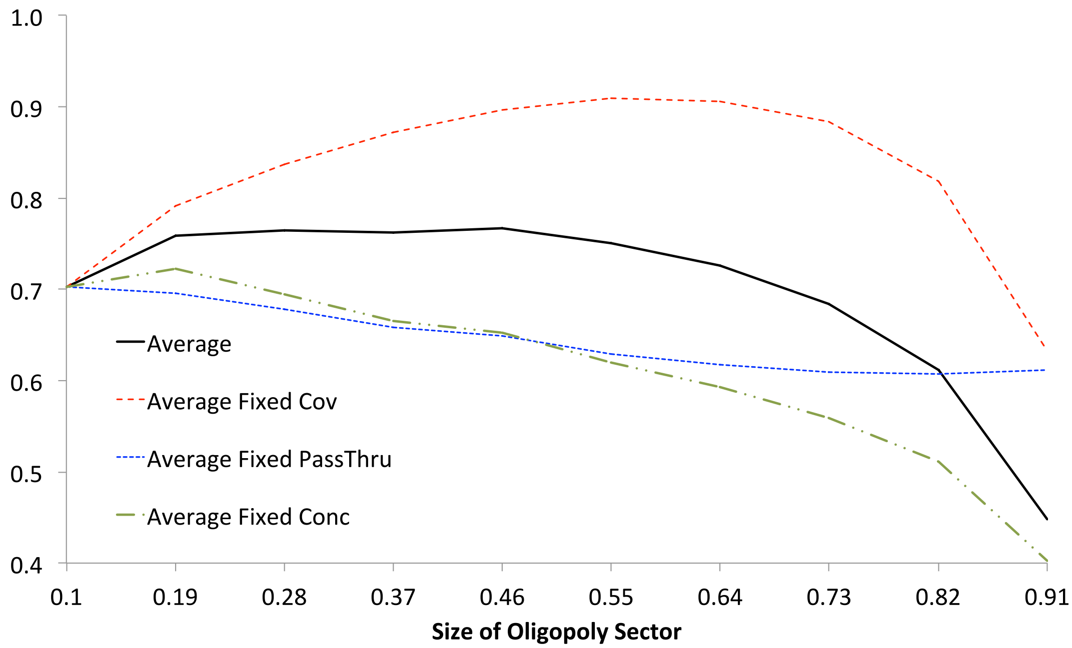


Figure 9: Price Informativeness decomposed into the relative contribution of the three component parts.

The red, dashed line shows the evolution of price informativeness across different oligopoly sector sizes, *holding the covariance term constant*. When the sector is small, oligopolists specialize in their learning, and their price impact is relatively small, which makes the covariance term relatively high. As the oligopolists get bigger, they spread their attention, and trade less on their information. Consequently, the holdings become relatively less concentrated, and learning also becomes relatively more dispersed, thus reducing the covariance term. Therefore, holding the covariance term fixed and increasing the size of the sector results in the overestimation of aggregate price informativeness.

For the same reason, *holding concentration fixed* (the dashed, green line) results in consistent underestimation of aggregate price informativeness. But holding either constant does not change the hump-shape of the experiment. That shape comes from the final component: pass-through. When *pass-through is held fixed*, the resulting shape is flat. This is because at low levels of oligopoly sector size, oligopolists will feel free to adjust their quantity decisions due to their small level of price impact. As their size increases, their quantities increase, but their sensitivity to their information declines as their price impact increases. As the two effects interact, holding pass-through constant first underestimates and then overestimates the level of aggregate price informativeness.

We conduct a similar decomposition for the concentration experiment. Here, we keep the overall size of the oligopoly sector constant at 45%. We also keep the relative size of the five smallest oligopolists the same, but vary the assets under management of the largest oligopolist, from 35% of the oligopoly sector to 98% of the oligopoly sector. We present the results in Figure 10. The decomposition effect looks

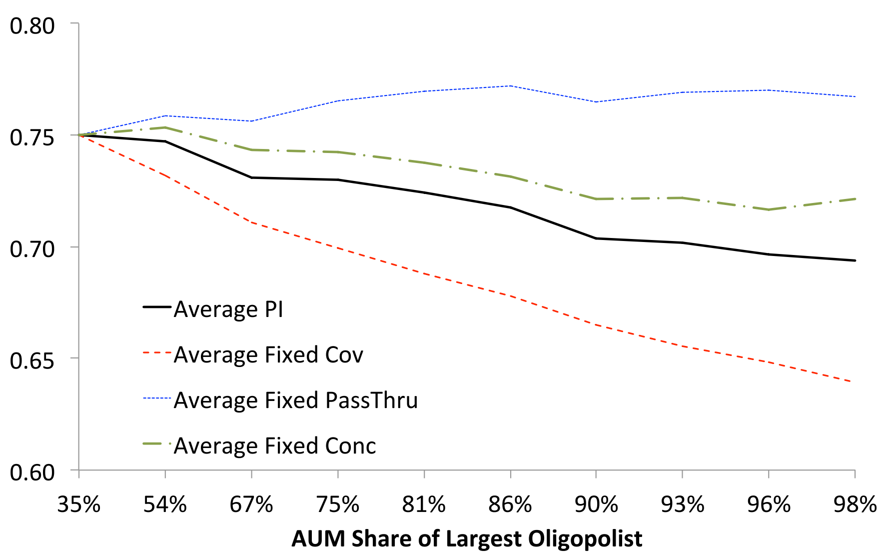


Figure 10: Price Informativeness decomposed into the relative contribution of the three component parts.

quite different, but some similarities still exist. As the concentration of the sector increases, the concentration in holdings is going to increase, but the concentration in learning is going to decrease, as the largest player starts to spread her attention across several assets. As a result, holding concentration fixed overestimates aggregate price informativeness, while holding covariance fixed underestimates it. However, as before, holding either of the margins constant still yields a downward-sloping relationship. This downward slope is again driven by the pass-through, which is our most important channel. If pass-through is held fixed, we miss the fact that the largest trader has more and more price impact, making her less and less sensitive to information. Thus, we overestimate not only the level of price informativeness, but also the trend.

The role of passive investors The importance of oligopoly traders results from two sources: their informational advantage and their size. While all oligopolists exert price impact not all of them need to be equally informed. In particular, passive investors do not directly participate in the market for information. In this section, we explore the predictions of the model with respect to the size of such passive investors. In Figure 11, we present results from the simulation in which the size of the passive sector increases from 25% to 52% of the market. As is evident, when the passive sector increases in size, there are two effects. The red line shows the effect of a growth in the passive sector on PI when only quantities are allowed to change and prices are not. The effect is uniformly negative, which is not surprising—assets under management are transferred from active to passive investors, so quantities will reflect less information. The blue line also allows for learning to adjust. The green bars plot the number of assets in total that are learned about by the active sector. As the active sector gets smaller in size, they will choose to specialize in their learning, resulting in fewer assets being learned about. Whenever an asset stops being learned about, the blue line drops faster than the red, indicating that the decline in price informativeness from the transfer of assets is amplified by the endogenous learning response of the active sector.

We can also show the breakdown of these effects using the decomposition method employed earlier. Consider Figure 12.

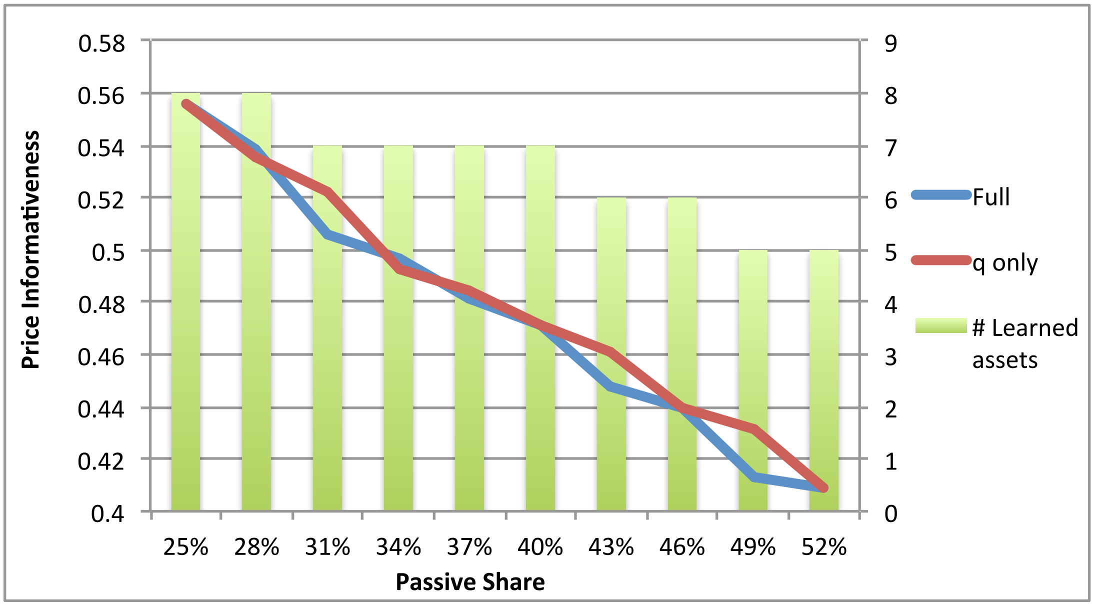


Figure 11: Price Informativeness as a function of passive share.

Here the experiment is as follows. There are three active investors, who take up 45% of assets under management. From left to right, the share of the institutional sector's AUM that is managed by passive investors is plotted on the x-axis (it increases from left to right). Thus the experiment shows the effects on PI of fringe members allowing their cash to be managed by a passive investor, while the size of the active sector remains unchanged. Because the fringe does not collect information, this has no effect on the overall capacity of information-collection in the market. The changes nonetheless have a deleterious effect on price informativeness. Unlike previous cases, where the main mechanism for the decline was passthrough, the main mechanism here is covariance. As the passive sector increases in size, they trade less and less, and the fringe's size decreases, making the price impact of all agents higher. Therefore, specializing becomes less and less attractive, indicating that fixing Covariance at a small passive sector would over-state the level of price informativeness.

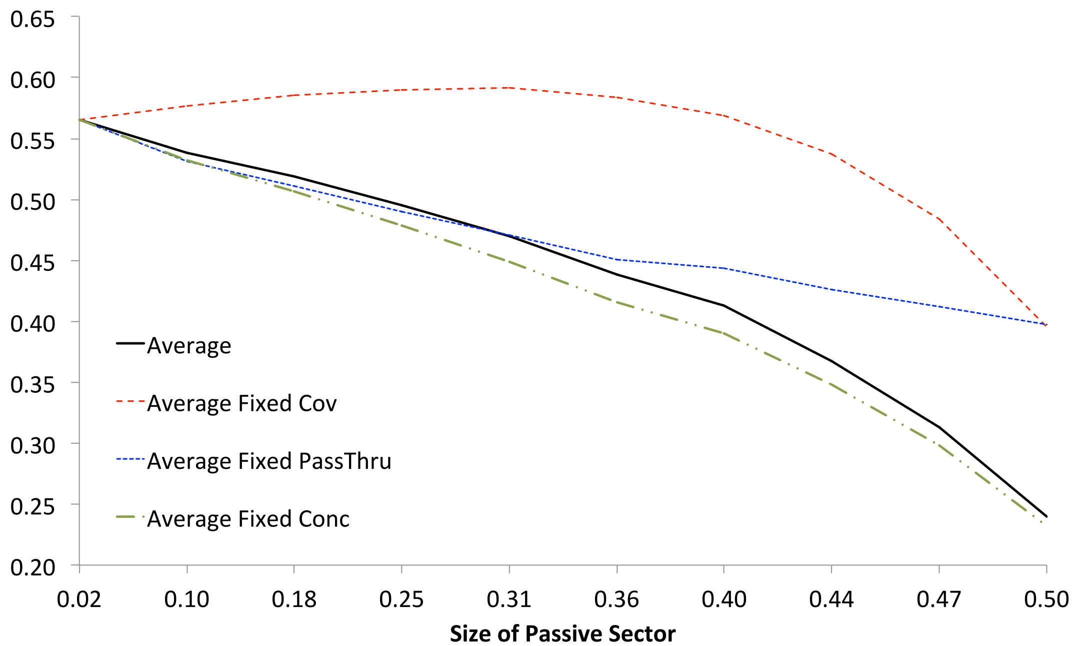


Figure 12: Price Informativeness decomposed into the relative contribution of the three component parts.

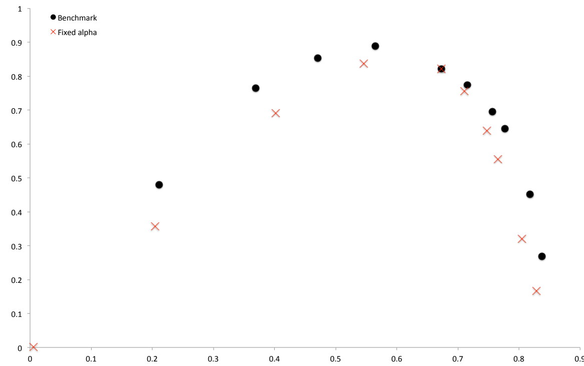
4.3 The Role of Endogenous Learning Choice

In this section, we present a comparison of the model with endogenous learning choice (our benchmark) to a model in which the information structure is given and set the same as in the benchmark model. The model with a fixed information structure is similar in spirit to that presented in Kyle (1985), in that the effect of market power in the absence of endogenous reoptimization of information choices depends entirely on how the quantities adjust.

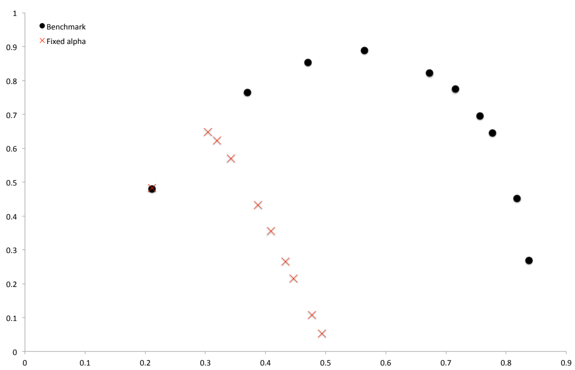
Figure 13 presents the interaction of institutional ownership and price informativeness, where the different points are generated by varying λ_0. The black dots represent the benchmark case in which we allow both quantities and information choices to adjust in response to changing λ_0. The red crosses correspond to a case with a fixed learning structure. For the fixed learning cases, the information choice is either fixed at the benchmark value (i.e., $\lambda_0 = 0.45$, Panel (a)), or at values such that information structures are optimal at $\lambda_0 = 0.999$ (small oligopolists, Panel (b)) and $\lambda_0 = 0.05$ (large oligopolists, Panel (c)). In all the fixed-information cases, the level of price informativeness is below that of the benchmark model for which capacity choice adjusts optimally. The gains in price informativeness from optimal learning can be quite large. For example, for the benchmark specification of fixed alphas, price informativeness is reduced by up to 40%. More important, fixing the learning choices leads to very different conclusions about the optimal size of the institutional sector. Depending on what values of learning one exactly fixes, the optimal size lies either below or above the actual optimum derived when all the choices are endogenous. This finding underscores the importance of modeling the information choice margin when making normative statements about the size of the institutional sector.

Next, we evaluate the 'concentration of oligopoly' exercise of Section 4.2, in which we hold λ_0 fixed but vary λ_l/λ_1. Figure 14 presents the relation between concentration of ownership and price informativeness for the benchmark model with endogenous information choice (black dots), as well as three cases of fixing the information choice at the benchmark values (i.e., for $\lambda_l/\lambda_1 = 4$, Panel (a)), as well as at values that are optimal at two extremes of the size distribution of the oligopolists, $\lambda_l/\lambda_1 = 9$ (Panel (b)) and $\lambda_l/\lambda_1 = 2$ (Panel (c)). For all the three cases, the exogenous and endogenous information models give remarkably different predictions in terms of the relation of concentration and price informativeness. In particular, for the benchmark model, lower concentration always increases price informativeness. In contrast, models with fixed information structure exhibit a hump-shaped relation between concentration and price informativeness. Similar to the previous exercise, the two models give very different recommendations regarding the level of concentration that maximizes price informativeness. The exogenous information models optimally imply an intermediate level of concentration, while at the same time the fully endogenous model prescribes a concentration level that is at the lower bound of the potential values.

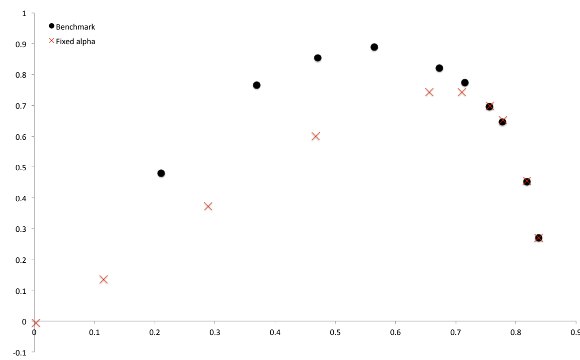
Overall, we conclude that the predictions resulting from a model with endogenous



(a) Fixing information at benchmark market structure



(b) Fixing information when $\lambda_0 = 0.999$



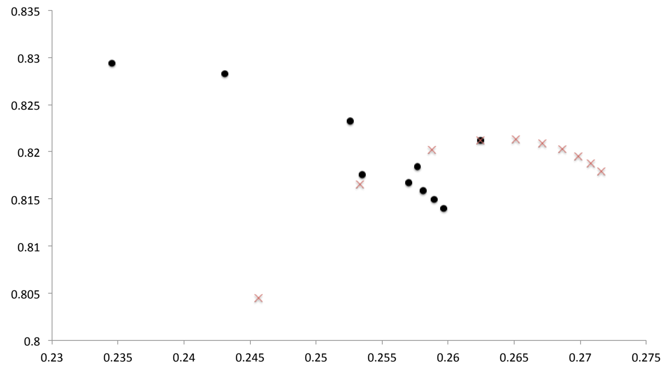
(c) Fixing information when $\lambda_0 = 0.05$

Figure 13: Aggregate price informativeness and institutional ownership with varying λ_0

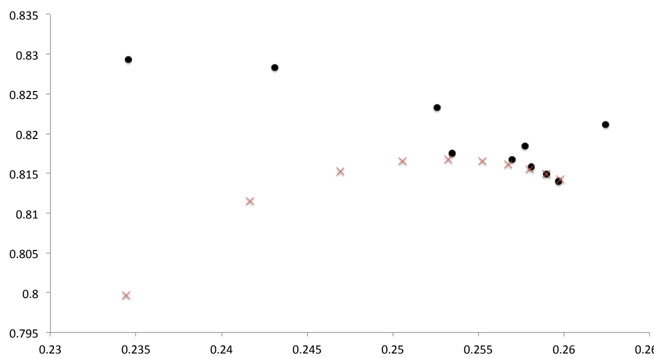
learning choices are not a simple extension of the model where information choices are fixed. The differences are not only quantitatively important but also qualitatively relevant from the perspective of policy making.

5 Other Results and Extensions

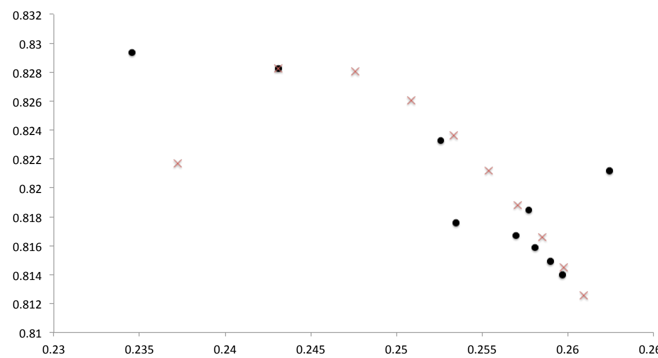
In this section, we discuss a number of additional results that shed more light on the importance of oligopolists' size and asset prices for learning process.



(a) Fixed alpha at benchmark value



(b) Fixed alpha for $\lambda_l/\lambda_1 = 9$



(c) Fixed alpha for $\lambda_l/\lambda_1 = 2$

Figure 14: Aggregate price informativeness and concentration of institutional ownership with varying λ_l/λ_1

5.1 Thresholds

Our model shows that oligopolists try to spread their learning across assets whenever possible to mitigate their price impact. In this section, we provide the characterization of the optimal size threshold at which such diversification effect takes place. In particular, the point at which the oligopolist stops specializing in her learning might be different from the point at which she would stop specializing if her objective was to maximize aggregate PI. For analytical tractability, we consider a special case of a monopolist, of size λ_1, who has a positive K and a fringe that is uninformed.

From our earlier discussion, we know that, for sufficiently small levels of λ_1, the monopolist will choose to specialize in her learning, and learn only about the most volatile asset. This specialization result arises from the fact that the monopolist's returns to learning are diminishing when she has positive size. We can characterize this threshold implicitly using the following expression:

$$\frac{e^{2K_j}}{(1+2\frac{T}{\lambda_0}e^{2K_j})^2}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_x^2)\sigma_1^2+1+2\frac{T}{\lambda_0}\right)$$
$$=\frac{1}{(1+2\frac{T}{\lambda_0})^2}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_x^2)\sigma_2^2+1+2\frac{T}{\lambda_0}\right)$$

where σ_1 is the volatility of the most volatile asset, and σ_2 is the volatility of the second-most volatile asset. If the monopolist specializes, she learns only about asset 1. When the marginal benefit of additional learning about asset 1 when the agent specializes is equal to the marginal benefit of starting to learn about asset 2, an increase in the agent's size will make her not specialize.

Next, we characterize the size threshold between specialization and diversification from the perspective of a monopolist maximizing aggregate price informativeness. This threshold is given by comparing the derivative of price informativeness for the most volatile asset with respect to the monopolist's learning (assuming specialization) to the derivative of price informativeness of the second-most volatile asset with respect to the monopolist's learning (assuming no learning):

$$\begin{aligned}
\frac{\partial PI}{\partial \alpha}=mP(\alpha,\lambda_1) &\equiv \frac{\sigma_i^2(((2\lambda_1^2+2\lambda_1\lambda_0)\rho^2\sigma_x^2+\lambda_0^2+\lambda_1^2)\alpha_i+(2\lambda_0\lambda_1+2\lambda_1^2)\rho^2\sigma_x^2-\lambda_0^2-\lambda_1^2)}{2(\lambda_0^2+\lambda_1^2)\left((\alpha_i-1)^2\sigma_i^2+(\alpha_i-1)\sigma_i^2+\frac{(\lambda_0+\lambda_1\alpha_i)^2\rho^2\sigma_i^4\sigma_x^2}{\lambda_0^2\lambda_1^2}\right)^{3/2}} \\
mP(e^{2K},T) &= mP(1,T)
\end{aligned}$$

Subsequently, we analyze the sensitivity of the threshold level with respect to the monopolist's capacity and differences in volatilities between asset 1 and 2. To provide a numerical solution to the above equations, for the analysis based on changes in capacity, we choose the following parameter values: $K_h=0$, $\sigma_1=2$, $\sigma_2=1$, $\bar{x}=5$,

$\rho = 1.3$, and $\sigma_x = 0.41$, consistent with our calibration exercise. We plot the results in Figure 15 below.

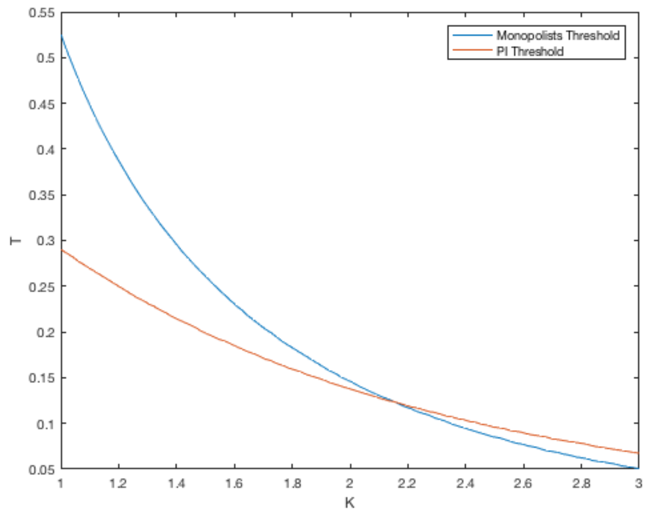


Figure 15: Optimal size thresholds for the monopolist as a function of her capacity.

The two size thresholds vary with the parameters as follows: First, they are both decreasing in K. As the monopolist has greater capacity to learn, specializing in learning means better ability to trade that asset, and higher price impact. Therefore, a monopolist wants to diversify her learning at smaller sizes. Similarly, the more information a monopolist can collect, the more quickly a planner might want her to spread her wealth (learning-wise) to other assets. For lower values of K, the monopolist wants to specialize later than the planner. For higher values of K, the monopolist wants to specialize sooner.

For the analysis based on differences in volatilities, we set $K_h = 0$, $K_j = 2$, $\bar{x} = 5$, $\rho = 1.3$, and $\sigma_x = 0.41$. We report the results from this analysis in Figure 16.

As is evident from the graph, the larger the gap in volatilities, the more a monopolist wants to specialize. However, the opposite is true for the planner. As the gap in volatilities grows, the monopolist would diversify sooner to increase PI. Notably, we only analyze the threshold conditions for two sets of parameter values, because in-

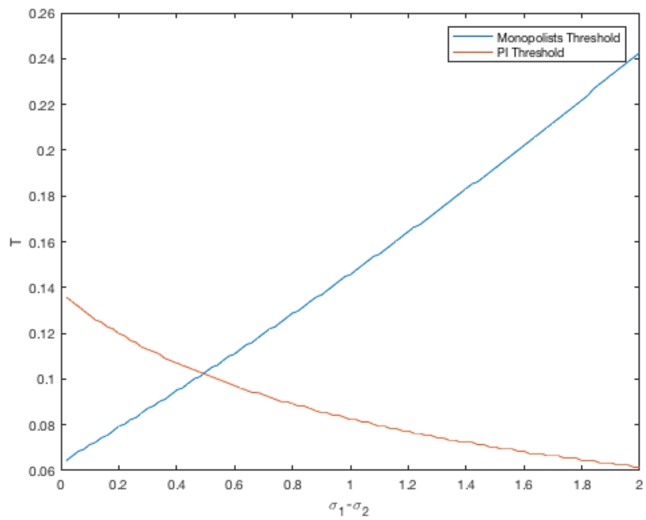


Figure 16: Optimal size thresholds for the monopolist as a function of differences in asset volatilities.

creases in ρ, $\bar{x}$, and σ_x all increase the size threshold for a monopolist by economically small margins.

In summary, we derive closed-form solutions for the size thresholds at which investors find it privately optimal to specialize in their learning, and at which it would be optimal for them to specialize in their learning from a price informativeness standpoint. We relate these thresholds to changes in the monopolist's capacity and differences in volatilities between most volatile assets. We find that the optimal thresholds vary depending on what objective function a monopolist maximizes and also depending on the assumptions about her information capacity and underlying asset volatilities. These results suggest that the objective function plays an important role to establish the optimality conditions in learning behavior.

6 Concluding Remarks

The last few decades have witnessed important changes in institutional equity ownership structure, with significant consequences for financial stability and social welfare. These trends have triggered an active discussion among financial regulators and finance industry. While several participants in the debate have raised important reasons for or against regulatory changes, the ultimate verdict is difficult to reach in the absence of a well-specified economic model. This paper aims to take one step in this direction by offering a general equilibrium model in which asymmetric information, market power, and heterogeneity of assets play an important role. We think this setting is a good way to characterize the world of equity ownership. Our goal is to rank various equilibria by comparing their implications for average price informativeness.

We show that for the level of ownership equal to the currently observed levels in the U.S. (roughly 60%), the average price efficiency is positively related to the levels of institutional ownership and negatively related to its concentration. This cross-sectional result is strongly supported by the data. Further, we show that the average price informativeness across assets is maximized for the values of ownership and concentration that are strictly within the range of admissible outcomes. This result suggests an interesting role for policy makers to enforce optimal structure of equity ownership.

Our model can be flexibly applied to settings with rich cross-section of assets, differences in information asymmetry across agents, and differences in market power. Hence, it can generate interesting policy implications at the aggregate and cross-sectional dimensions. It can also be a good tool to evaluate asset pricing implications in the presence of market power and information asymmetry. We leave these questions for future research. At the same time, while our research can inform the debate for the role of institutional owners for price informativeness and learning in the economy, it naturally abstracts from other important dimensions relevant for policy makers, such as investment costs or flows of funds in and out of the sector.

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7 Appendix: Proofs

7.0.1 Derivation of Equation 2

$$
\begin{aligned}
\sigma_{pi}^2 &= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n}\left(\widehat{\sigma}_{ji}^2 + \frac{\widehat{\sigma}_{ji}^4}{\sigma_i^4}\sigma_i^2 - \frac{2}{\alpha_{ji}}Cov(\epsilon_i,\delta_{ji}) - 2b_i d_{ji} Cov\left(\varepsilon_i,\zeta_{ji}\right) - 2c_i d_{ji} Cov\left(\nu_i,\delta_{ji}\right)\right) \\
&\quad + \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}Cov(\zeta_{ji},\zeta_{ki}) \\
&= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n}\left(d_{ji}^2\left(\widehat{\sigma}_{ji}^2 + \frac{\widehat{\sigma}_{ji}^4}{\sigma_i^2} - \frac{2\widehat{\sigma}_{ji}^2}{\sigma_i^2}\widehat{\sigma}_{ji}^2\right) - 2b_i d_{ji}\left(\widehat{\sigma}_{ji}^2 - \frac{\sigma_i^2}{\alpha_{ji}}\right)\right) \\
&\quad + \sum_{j=1}^{n}\sum_{j\neq k} 2d_{ji}d_{ki}Cov\left(\delta_{ji} - \frac{1}{\alpha_{ji}}\varepsilon_i, \delta_{ki} - \frac{1}{\alpha_{ki}}\varepsilon_i\right) \\
&= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \widehat{\sigma}_{ki}^2 - \widehat{\sigma}_{ji}^2 + \sigma_i^2 - \frac{1}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right) \\
&= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)
\end{aligned}
$$

7.0.2 Derivation of Equation 8

The objective is $U_{0h} = \frac{1}{2\rho}\sum_{i=1}^{n}\frac{E_{0h}(\widehat{\mu}_{hi} - rp_i)^2}{\widehat{\sigma}_{hi}^2} = \frac{1}{2\rho}\sum_{i=1}^{n}\frac{\widehat{R}_i^2 + \widehat{V}_{hi}}{\widehat{\sigma}_{hi}^2}$, where

$\widehat{R}_i \equiv E_{0h}\left(\widehat{\mu}_{hi} - rp_i\right) = \overline{z} - r\overline{p}_i = \overline{z} - ra_i,\ \widehat{V}_{hi} \equiv V_{0h}\left(\widehat{\mu}_{hi} - rp_i\right) = Var\left(\widehat{\mu}_{hi}\right) + r^2\sigma_{pi}^2 - 2rCov\left(\widehat{\mu}_{hi}, p_i\right).$

$Var\left(\widehat{\mu}_{hi}\right) = \sigma_i^2 - \widehat{\sigma}_{hi}^2$. $\sigma_{pi}^2 = b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right).$

Posterior beliefs and prices are conditionally independent given payoffs.

$$
\begin{aligned}
Cov\left(\widehat{\mu}_{hi}, p_i\right) &= \frac{1}{\sigma_i^2} Cov\left(\widehat{\mu}_{hi}, z_i\right) Cov\left(z_i, p_i\right) \\
&= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right)\left(Cov(\varepsilon_i, b_i \varepsilon_i) - \sum_{j=1}^n Cov(\varepsilon_i, d_{ji}\zeta_{ji})\right) \\
&= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right)\left(b_i \sigma_i^2 - \sum_{j=1}^n d_{ji} Cov\left(\varepsilon_i, \delta_{ji} - \frac{1}{\alpha_{ji}}\varepsilon_i\right)\right) \\
&= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right)\left(\left(b_i + \sum_{j=1}^n \frac{d_{ji}}{\alpha_{ji}}\right)\sigma_i^2 - \sum_{j=1}^n d_{ji}\widehat{\sigma}_{ji}^2\right) \\
&= \left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i
\end{aligned}
$$

Hence

$$
\begin{aligned}
\widehat{V}_{hi} = {} & \sigma_i^2 - \widehat{\sigma}_{hi}^2 + r^2\Bigg(b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^n d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \\
& \sum_{j=1}^n \sum_{k \neq j} 2 d_{ji} d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)\Bigg) - 2r\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i
\end{aligned}
$$

Expected utility becomes Hence $U_{0h} = \frac{1}{2\rho}\sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} - \frac{1}{2\rho}\sum_{i=1}^n (1 - 2rb_i)$, where

$$
G_i \equiv G^{KNS} + r^2\left(\sum_{j=1}^n d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \sum_{j=1}^n \sum_{k \neq j} 2 d_{ji} d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)\right).
$$

Note: $G_i^{KNS} \equiv \frac{\widehat{R}_i^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2}$.

7.0.3 Derivation of Equation 12

Market clearing for each asset i is

$$\begin{aligned}
x_i &= \sum_{j=1}^{n} \lambda_j q_{ji} + \int_{H_i} q_{hi} dh \\
&= \sum_{j=1}^{n} \lambda_j q_{ji} + \int_{H_i} \frac{\widehat{\mu}_{hi} - rp_i}{\rho \widehat{\sigma}_{hi}^2} dh \\
&= \sum_{j=1}^{n} \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} \left[e^{2K_h} \int_{H_i} \widehat{\mu}_{hi} dh - m_{hi} e^{2K_h} rp_i + (1 - m_{hi})(\overline{z} - rp_i) \right]
\end{aligned}$$

where H_i is the mass of competitive investors learning about asset i, of measure m_{hi}.

Using $E[s_{hi}|z_i] = \begin{cases} \overline{z} + \left(1 - e^{-2K_h}\right) \varepsilon_i & \text{if } i = l_h \\ \overline{z} & \text{if } i \neq l_h, \end{cases}$

$\int_{H_i} \widehat{\mu}_{hi} dh = m_{hi} \left[\overline{z} + \left(1 - e^{-2K_h}\right) \varepsilon_i \right].$

Then market clearing becomes

$$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} \left[\left(1 - m_{hi} + e^{2K_h} m_{hi}\right) \overline{z} + \left(e^{2K_h} - 1\right) \varepsilon_i m_{hi} - \left(1 - m_{hi} + e^{2K_h} m_{hi}\right) rp_i \right]$$

Defining $\Phi_{hi} \equiv m_{hi}(e^{2K_h} - 1)$,

$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + + \frac{\lambda_0}{\rho \sigma_i^2} [\bar{z}(1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - rp_i(1 + \Phi_i)]$
which becomes
$\frac{\rho \sigma_i^2}{\lambda_0} x_i = \frac{\rho \sigma_i^2}{\lambda_0} \sum_{j=1}^{n} \lambda_j q_{ji} + \bar{z}(1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - rp_i(1 + \Phi_i)$
and then
$rp_i = \frac{\rho \sigma_i^2}{\lambda_0(1+\Phi_{hi})} \sum_{j=1}^{n} \lambda_j q_{ji} + \bar{z} + \frac{\Phi_{hi}}{1+\Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1+\Phi_{hi})} x_i$
Hence,
$\frac{dp_i(q_{ji})}{dq_{ji}} = \frac{\lambda_j \rho \sigma_i^2}{\lambda_0 r (1+\Phi_{hi})} > 0$
Let $\hat{\lambda}_{ji} \equiv \frac{\lambda_j}{\lambda_0(1+\Phi_{hi})}$.
Then $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)}$, and similarly for k.
Plugging in the expression for q_{ji}:
$rp_i = \sum_{j=1}^{n} \hat{\lambda}_{ji} \rho \sigma_i^2 \frac{\widehat{\mu}_{ji} - rp_i}{\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)} + \bar{z} + \frac{\Phi_{hi}}{1+\Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1+\Phi_{hi})} x_i$
which becomes
$rp_i \left(1 + \sum_{j=1}^{n} \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)}\right) = \sum_{j=1}^{n} \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)} \widehat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1+\Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1+\Phi_{hi})} x_i$
dividing through gives

$rp_i = \frac{\sum_{j=1}^{n} \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\widehat{\mu}_{ji}+\bar{z}+\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i-\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i}{\left(1+\sum_{j=1}^{n}\frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\right)}$

The indirect utility function $U_j = \sum_{i=1}^{n} q_{ji}\left(\widehat{\mu}_{ji} - rp_i\right) - \frac{\rho}{2}\sum_{i=1}^{n} q_{ji}^2\widehat{\sigma}_{ji}^2$ becomes

$U_j = \sum_{i=1}^{n}\left[q_{ji}^2\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right) - \frac{\rho}{2}q_{ji}^2\widehat{\sigma}_{ji}^2\right]$

$U_j = \sum_{i=1}^{n}\left[\rho q_{ji}^2\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2 - \frac{1}{2}\widehat{\sigma}_{ji}^2\right)\right]$

$U_j = \sum_{i=1}^{n}\left\{\frac{(\widehat{\mu}_{ji}-rp_i)^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)^2}\left(\frac{1}{2}\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right)\right\}$

$U_j = \frac{1}{2\rho}\sum_{i=1}^{n}\left\{(\widehat{\mu}_{ji} - rp_i)^2\left[\frac{\widehat{\sigma}_{ji}^2+2\widehat{\lambda}_i\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_i\sigma_i^2\right)^2}\right]\right\}.$

More detailed expression for U: We can rewrite $E_{0j}(\widehat{\mu}_{ji} - rp_i)^2$ as $\widehat{R}_i^2 + \widehat{V}_{ji}$, where $\widehat{R}_i$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, which means: $\widehat{R}_i \equiv E_{0j}\left(\widehat{\mu}_{ji} - rp_i\right) = \frac{\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}}{\left(1+\sum_{j=1}^{n}\frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\right)}\bar{x}$ Define $M_{ji} \equiv \frac{\hat{\lambda}_{ji}\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}$ $N_i \equiv \frac{1}{1+\sum_{j=1}^{n} M_{ji}}$

$\widehat{V}_{ji} \equiv V_{0j}\left(\widehat{\mu}_{ji} - rp_i\right)$
$= V_{0j}\left(\widehat{\mu}_{ji} - N_i\sum_{k=1}^{n} M_{ki}\widehat{\mu}_{ki} - N_i\bar{z} - N_i\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i + N_i\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right)$
$= V_{0j}\left(N_i\left(\widehat{\mu}_{ji} + \sum_{k=1}^{n} M_{ki}(\widehat{\mu}_{ji} - \widehat{\mu}_{ki})\right) - N_i\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i + N_i\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right)$
$= N_i^2 V_{0j}\left(\widehat{\mu}_{ji} + \sum_{k\neq j}^{n} M_{ki}\widehat{\mu}_{ji} - \frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i + \frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right)$
$= N_i^2\left(1 + \sum_{k\neq j}^{n} M_{ki}\right)^2(\sigma_i^2 - \widehat{\sigma}_{ji}^2) + \left(\frac{N_i\Phi_{hi}}{1+\Phi_{hi}}\right)^2\sigma_i^2 + \left(\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2\sigma_{xi}^2 - 2\left(\frac{N_i^2\Phi_{hi}\left(1+\sum_{k\neq j}^{n} M_{ki}\right)}{1+\Phi_{hi}}\right)(\sigma_i^2 - \widehat{\sigma}_{ji}^2)$

$$\begin{aligned} U_{0j} \;\; &= \;\; \frac{1}{2\rho}\sum_i \frac{N_i^2(\widehat{\sigma}_{ji}^2 + 2\widehat{\lambda}_i\sigma_i^2)}{\left(\hat{\sigma}_{ji}^2 + \hat{\lambda}\sigma_i^2\right)^2}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2 + \sigma_{ix}^2) + \left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\sigma_i^2 \right. \\ & \left. \quad + \left(\frac{(1+\Phi_{hi})\left(1+\sum_{k\neq j}^{n} M_{ki}\right) - 2\Phi_{hi}}{1+\Phi_{hi}}\right)(\sigma_i^2 - \hat{\sigma}_i^2)\left(1 + \sum_{k\neq j}^{n} M_{ki}\right)\right] \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2\rho}\sum_i \frac{N_i^2(1 + 2\hat{\lambda}\alpha_{ji})}{(1+\hat{\lambda}\alpha_{ji})^2}\left[\left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\alpha_{ji} + \left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2 + \sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2}\right. \\ &\left. + \left(\frac{(1+\Phi_{hi})\left(1+\sum_{k\neq j}^{n} M_{ki}\right) - 2\Phi_{hi}}{1+\Phi_{hi}}\right)\left(1 + \sum_{k\neq j}^{n} M_{ki}\right)(\alpha_{ji} - 1)\right] \end{aligned}$$

7.0.4 Derivation of Equations 21

The market clearing condition is

$$rp_i = \frac{\sum_{j=1}^{n} \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\widehat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i - \frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i}{\left(1+\sum_{j=1}^{n}\frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\right)} \tag{26}$$

From here we identify the price coefficients as a function of the monopolist learning and the competitive fringe learning. Now, conditionally on z_i, we have

$$\hat{\mu}_{ji} = s_{ji}$$

and s_{ji} is normally distributed with mean $\bar{z}+(1-\frac{1}{\alpha_{ji}})\varepsilon_i$ and variance $(1-\frac{1}{\alpha_{ji}})\frac{1}{\alpha_{ji}}\sigma_i^2$. What we want is to express the posterior mean in terms of delta as in $z_i = s_i + \delta_i$. Given that,

$$\delta_{ji} = z_i - s_{ji} = -\frac{1}{\alpha_{ji}}\varepsilon_i + noise$$

$$rp_i = N_i\sum_{j=1}^{n} M_{ji}\left(\bar{z}+\left(1-\frac{1}{\alpha_{ji}}\right)\epsilon_i - \zeta_{ji}\right) + N_i\left[\bar{z}+\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i - \frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right] \tag{27}$$

$$\begin{aligned} rp_i \quad = \quad & \bar{z} - \bar{x}\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})} + \epsilon_i N_i\left(\sum_{j=1}^{n}\frac{M_{ji}(\alpha_{ji}-1)}{\alpha_{ji}} + \frac{\Phi_{hi}}{1+\Phi_{hi}}\right) \\ & -\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\nu_i - N_i\sum_{j=1}^{n} M_{ji}\zeta_{ji} \end{aligned}$$

7.0.5 Derivation of Proposition 1

Proof. In order to apply Kakutani's Fixed Point Theorem, we need to define a few terms. Agents select α_i First, define $A_i(\{\alpha_{-j}\})$ to be the best response correspondence for oligopolist j. Next define $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_L\}$, and let $\aleph$ define the set of all possible α. Then the best response correspondence can be defined as $A : \aleph \Rightarrow \aleph$ such that for all $\alpha \in \aleph$, we have that $A(\alpha) = [A_j(\alpha_{-j})]_{j\in L}$. This best response function takes into account the associated demand schedule for every oligopolist, as well as the learning and demand decisions for the fringe. Now we need to check whether there is a fixed point to A.

- $\aleph$ is compact and convex. Each α_j must satisfy the capacity constraint. Therefore each α_j is convex, closed, and bounded, and therefore compact. Therefore $\aleph$ is as well.

- A is non-empty. This is trivially true if an interior solution exists. If an interior solution does not exist, then the solutions are corners, so A is always non-empty.

- A is convex-valued. A is convex iff A_i are all convex. The oligopolist's objective function is weakly more concave than the fringe's due to size. We show here that the second derivative is negative.

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^4}{\alpha_{ji}N_i^2} = X_i(\lambda_0{+}2\lambda\alpha)\Big(\lambda_0^2{-}\lambda_j^2\alpha_{ji}^2{-}2N\lambda_0\lambda_j\alpha_{ji}{+}\alpha_{ji}\Big){+}Y_i\Big[2\lambda_j{-}2\lambda_j(N_i\lambda_0{+}\lambda_0{+}\lambda_j\alpha_{ji})(\lambda_0{+}2\lambda_j\alpha_{ji})\Big]$$

$$\underbrace{u'-P(x)u^2-Q(x)u-R(x)}_{\text{=0, since } u \text{ is a particular solution.}}$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^2}{\alpha N_i} = \left[2N_i'(\lambda_0+2\lambda_j\alpha_{ji})-\frac{2\lambda_jN_i\lambda_j\alpha_{ji}}{(\lambda_0+\lambda_j\alpha_{ji})}\right]$$

$$\left[\frac{\alpha_{ji}\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0(\alpha_{ji}-1)\right]$$

$$+\left[N_i(\lambda_0+2\lambda_j\alpha_{ji})\right]\left[\frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\right]$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^2}{\alpha N_i} = \frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)\left[(N_i+2\alpha_{ji}N_i')(\lambda_0+2\lambda_j\alpha_{ji})-\frac{2\lambda_jN_i\lambda_j\alpha_{ji}^2}{(\lambda_0+\lambda_j\alpha_{ji})}\right]$$

$$+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\left[(N_i+2(\alpha_{ji}-1)N_i')(\lambda_0+2\lambda_j\alpha_{ji})-\frac{2\lambda_jN_i\lambda_j\alpha_{ji}(\alpha_{ji}-1)}{(\lambda_0+\lambda_j\alpha_{ji})}\right]$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3}{\alpha N_i^2} = \frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2{+}\sigma_{ix}^2)\left[((\lambda_0+\lambda_j\alpha_{ji})-2\alpha_{ji}N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})-2\lambda_j^2\alpha_{ji}^2\right]$$

$$+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\left[((\lambda_0+\lambda_j\alpha_{ji})-2(\alpha_{ji}-1)N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})-2\lambda_j^2\alpha_{ji}(\alpha_{ji}-1)\right]$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3}{\alpha N_i^2} = \left(\frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\right)$$

$$\left[((\lambda_0+\lambda_j\alpha_{ji})-2\alpha_{ji}N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})-2\lambda_j^2\alpha_{ji}^2\right]$$

$$+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\left[(2N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_j^2\alpha_{ji}\right]$$

$$\frac{\partial^2U}{\partial\alpha_{ji}^2}\propto-\left[\frac{(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right](4X_iN_iN_i'\lambda_j\alpha_{ji}{+}2X_iN_i^2\lambda_j{-}4Y_iN_iN_i'\lambda_j)$$

$$-\frac{(2X_iN_i^2\lambda_j\alpha_{ji}-2Y_iN_i^2\lambda_j)}{(\lambda_0+\lambda_j\alpha_{ji})^8}\Bigg[(\lambda_0+\lambda_j\alpha_{ji})^4\left((N_i'\lambda_0{+}\lambda_j)(\lambda_0{+}2\lambda_j\alpha_{ji}){+}2\lambda_j(N_i\lambda_0{+}\lambda_0{+}\lambda_j\alpha_{ji})\right)$$

$$-2\lambda_j(\lambda_0+\lambda_j\alpha_{ji}))-4\lambda_j\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3\Big((N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2\Big)\Bigg]$$

$$+\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]X_i$$

$$\frac{\partial^2U}{\partial\alpha_{ji}^2}\propto-\underbrace{\left[\frac{(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]}_{\frac{A-1}{(\lambda_0+\lambda_j\alpha_{ji})^2}>0}\underbrace{2N_i^2\lambda_j\left(-2N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}(\alpha_{ji}X_i-Y_i)+X_i\right)}_{>0}$$

$$-\underbrace{\frac{(2X_iN_i^2\lambda_j\alpha_{ji}-2Y_iN_i^2\lambda_j)}{(\lambda_0+\lambda_j\alpha_{ji})^8}}_{>0}\Bigg[(\lambda_0+\lambda_j\alpha_{ji})^4\left((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\right)$$

$$-\underbrace{4\lambda_j\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3\Big((N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2\Big)}_{>0}\Bigg]$$

$$+\underbrace{\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]X_i}_{<0}$$

$$\frac{\partial^2U}{\partial\alpha_{ji}^2}\propto-\left[\frac{A-1}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]2N_i^2\lambda_j\left(-2N_i\lambda_j\lambda_0(X_i\alpha_{ji}-Y_i)+X_i(\lambda_j\alpha_{ji}+\lambda_0)^2\right)$$

$$-\frac{2N_i^2\lambda_j(X_i\alpha_{ji}-Y_i)}{(\lambda_0+\lambda_j\alpha_{ji})^4}\left[\left((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\right)-4\lambda_j\Big(\left(A-1\right)\left(\lambda_0+\lambda_j\alpha_{ji}\right)\Big)\right]$$

$$+\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]X_i$$

$$\frac{\partial^2U}{\partial\alpha_{ji}^2}\propto-(A-1)2N_i^2\lambda_j\left(-2N_i\lambda_j\lambda_0(X_i\alpha_{ji}-Y_i)+X_i(\lambda_j\alpha_{ji}+\lambda_0)^2\right)$$

$$-(2N_i^2\lambda_j(X_i\alpha_{ji}-Y_i))\left[\left((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\right)-4\lambda_j\Big(\left(A-1\right)\left(\lambda_0+\lambda_j\alpha_{ji}\right)\Big)\right]$$

$$+\left[(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})\right]X_i$$

$$\frac{\partial^2U}{\partial\alpha_{ji}^2}\propto-2N_i^2\lambda_j\Big(\left((A-1)\left(-2N_i\lambda_j\lambda_0-4\lambda_j(\lambda_0+\lambda_j\alpha_{ji})\right)+\left((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\right)\right)$$

$$\times(X_i\alpha_{ji}-Y_i)+X_i(\lambda_j\alpha_{ji}+\lambda_0)^2(A-1)\Big)$$

$$+\Big[(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})\Big]X_i$$

$$\frac{\partial^2 U}{\partial\alpha_{ji}^2}\propto\left((2AN_i\lambda_j\lambda_0+(A-1)4\lambda_j(\lambda_0+\lambda_j\alpha_{ji}))+\left(\left(\frac{2N_i^2\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0-\lambda_j\right)(\lambda_0+2\lambda_j\alpha_{ji})\right)\right)(X_i\alpha_{ji}-Y_i)$$

$$-X_i(\lambda_j\alpha_{ji}+\lambda_0)^2(A-1)-\left[2N_i\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2+(\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\right]X_i$$

$$\frac{\partial^2 U}{\partial\alpha_{ji}^2}\propto-\left((2AN_i+4A-5)\lambda_j\lambda_0+(4A-6)\lambda_j^2\alpha_{ji}+\frac{2N_i\lambda_j\alpha_{ji}\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})\right)Y_i$$

$$-X_i\Bigg[(\lambda_j\alpha_{ji}+\lambda_0)^2(A-1)+\left[2N_i\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2+(\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\right]$$

$$-\left((2AN_i+4A-5)\lambda_j\alpha_{ji}\lambda_0+(4A-6)\lambda_j^2\alpha_{ji}^2+\frac{2N_i\lambda_j\alpha_{ji}\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})\right)\Bigg]$$

$$\frac{\partial^2 U}{\partial\alpha_{ji}^2}\propto-\left((2AN_i+4A-5)\lambda_j\lambda_0+(4A-6)\lambda_j^2\alpha_{ji}+\frac{2N_i\lambda_j\alpha_{ji}\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})\right)Y_i$$

$$-\frac{X_i}{\lambda_0^2+\lambda_j^2\alpha_{ji}^2+2\lambda_0\lambda_j\alpha_{ji}}\Bigg[(\lambda_j\alpha_{ji}+\lambda_0)^2((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3)-(\lambda_j\alpha_{ji}+\lambda_0)^2)$$

$$+2N_i\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_j\alpha_{ji}+\lambda_0)^2-(\lambda_j\alpha_{ji}+\lambda_0)^4+(\lambda_0+\lambda_j\alpha_{ji})^3(\lambda_0+2\lambda_j\alpha_{ji})-2N_i\lambda_j\alpha_{ji}\lambda_0\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})$$

$$-\Big((2N_i+4)((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3))-5(\lambda_j\alpha_{ji}+\lambda_0)^2\Big)\lambda_j\lambda_0\alpha_{ji}$$

$$-(4((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3))-6(\lambda_j\alpha_{ji}+\lambda_0)^2)\lambda_j^2\alpha_{ji}^2\Bigg]$$

$$\frac{\partial^2 U}{\partial\alpha_{ji}^2}\propto-\left(\Big((2AN_i+4A-5)\lambda_j\lambda_0+(4A-6)\lambda_j^2\alpha_{ji}\Big)(\lambda_j^2\alpha_{ji}^2+\lambda_0^2+2\lambda_0\lambda_j\alpha_{ji})+2N_i\lambda_j\alpha_{ji}\lambda_0^2(\lambda_0+2\lambda_j\alpha_{ji})\right)Y_i$$

$$-X_i\left[3N_i\lambda_0^4+\Big((3+2N_i-2N_i^2)\lambda_0^2+(9-2N_i)\lambda_j^2\alpha_{ji}^2+(5N_i+15)\lambda_0\lambda_j\alpha_{ji}\Big)\lambda_j\lambda_0\alpha_{ji}\right]$$

$$2AN_i+4A-5=(2N_i+4)((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3))-5(\lambda_0^2+\lambda_j^2\alpha_{ji}^2+2\lambda_0\lambda_j\alpha_{ji})$$

$$=\lambda_0^2(2N_i^2+6N_i-1)+\lambda_j^2\alpha_{ji}^2(4N_i+3)+\lambda_0\lambda_j\alpha_{ji}(4N_i^2+14N_i+2)$$

$$\begin{aligned}4A-6 &= 4((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3))-6(\lambda_0^2+\lambda_j^2\alpha_{ji}^2+2\lambda_0\lambda_j\alpha_{ji})\\ &= \lambda_0^2(4N_i-2)+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(8N_i)\end{aligned}$$

Therefore $\frac{\partial^2 U}{\partial \alpha_{ji}^2} < 0$.

- A has a closed graph. The first order conditions of the oligopolist are continuous, so this is trivial. (see above).

□

7.0.6 Derivation of Proposition 2

Proof.

$$PI' = \frac{b_i'\sigma}{\sqrt{b_i^2+c_i^2\frac{\sigma_x^2}{\sigma_i^2}+\sum d_j^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}$$

$$+\frac{b_i\sigma_i}{\left(\sqrt{b_i^2+c_i^2\frac{\sigma_x^2}{\sigma_i^2}+\sum d_j^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}}\right)^3}\left(2b_ib_i'+\frac{2c_ic_i'\sigma_x^2}{\sigma_i^2}-\frac{d_{ji}^2(\alpha_{ji}-2)}{\alpha_{ji}^3}+\frac{2d_{ji}d_{ji}'(\alpha_{ji}-1)}{\alpha_{ji}^2}\right)$$

$$PI'' \propto \frac{\underbrace{b_i''}_{<0}\underbrace{\left(b_i^2+c_i^2\frac{\sigma_x^2}{\sigma_i^2}+\sum d_j^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}\right)}_{>0}}{\left(\underbrace{2b_ib_i'+\frac{2c_ic_i'\sigma_x^2}{\sigma_i^2}-\frac{d_{ji}^2(\alpha_{ji}-2)}{\alpha_{ji}^3}+\frac{2d_{ji}d_{ji}'(\alpha_{ji}-1)}{\alpha_{ji}^2}}_{>0}\right)}\underbrace{-2b_i'}_{<0}-\underbrace{\frac{3b_i}{\left(b_i^2+c_i^2\frac{\sigma_x^2}{\sigma_i^2}+\sum d_j^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}\right)}}_{<0}$$

$$+\underbrace{\frac{b_i\left(2b_ib_i'+\frac{2c_ic_i'\sigma_x^2}{\sigma_i^2}-\frac{d_{ji}^2(\alpha_{ji}-2)}{\alpha_{ji}^3}+\frac{2d_{ji}d_{ji}'(\alpha_{ji}-1)}{\alpha_{ji}^2}\right)'}{\left(2b_ib_i'+\frac{2c_ic_i'\sigma_x^2}{\sigma_i^2}-\frac{d_{ji}^2(\alpha_{ji}-2)}{\alpha_{ji}^3}+\frac{2d_{ji}d_{ji}'(\alpha_{ji}-1)}{\alpha_{ji}^2}\right)}}_{<0}$$

$$PI'' < 0$$

□

7.0.7 Derivation of Proposition 3

Proof. The sign of $\frac{\partial^2 U}{\partial \alpha_{ji}^2}$ was shown in the proof of proposition 1. Because $\alpha_{ji}\lambda_j$ and A are both increasing in λ_j, the triple-derivative is immediate. □

7.0.8 Derivation of Lemma 1

Proof. If only the monopolist can learn, then $\Phi_{hi} = 0$. First we can write: $M_{ji} = \frac{\hat{\lambda}_{ji}\alpha_{ji}}{1+\hat{\lambda}_{ji}\alpha_{ji}}$, $N_i = \frac{1+\hat{\lambda}_{ji}\alpha_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}$. Then we need to solve the monopolist's information problem:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial\alpha_{ji}}\frac{1}{2\rho}\sum_i \frac{1}{1+2\hat{\lambda}\alpha_{ji}}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2}+(\alpha_{ji}-1)\right] \\
0 &= \frac{1}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+1\right] - \\
&\quad \frac{2\hat{\lambda}_{ji}}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2}+(\alpha_{ji}-1)\right]+2\rho\frac{\eta_i-\mu}{\alpha_{ji}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{2\rho\mu(1+2\hat{\lambda}_{ji}\alpha_{ji})}{\alpha_{ji}} &= \left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+1\right] - \frac{2\hat{\lambda}_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2}+(\alpha_{ji}-1)\right] \\
&= \left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+1\right]\left(1-\frac{2\hat{\lambda}_{ji}\alpha_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\right)+\frac{2\hat{\lambda}_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}} \\
\frac{2\rho\mu(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}{\alpha_{ji}} &= \left[\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1\right]+2\hat{\lambda}_{ji}
\end{aligned}
$$

$$
\begin{aligned}
\mu &= \frac{\alpha_{ji}}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2} X_i \\
X_i &= \frac{1}{2\rho}\left(\left[\left(\frac{\rho}{\lambda_0}\right)^2 (\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1\right]+2\hat{\lambda}_{ji}\right) \\
\frac{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}{(1+2\hat{\lambda}_{jk}\alpha_{jk})^2} &= \frac{\alpha_{ji}X_i}{\alpha_{jk}X_k} \\
\prod_{k=1}^{n}\frac{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}{(1+2\hat{\lambda}_{jk}\alpha_{jk})^2} &= \prod_{k=1}^{n}\frac{\alpha_{ji}X_i}{\alpha_{jk}X_k} \\
\frac{(1+2\hat{\lambda}_{ji}\alpha_{ji})^{2n}}{\prod_{k=1}^{n}(1+2\hat{\lambda}_{jk}\alpha_{jk})^2} &= \frac{(\alpha_{ji}X_i)^n}{e^{2K_h}\prod_{k=1}^{n}X_k} \\
M_{ji} &= \frac{\lambda_1\alpha_{ji}}{\lambda_0+\lambda_1\alpha_{ji}} \\
N_i &= \frac{\lambda_0+\lambda_1\alpha_{ji}}{\lambda_0+2\lambda_1\alpha_{ji}} \\
a_i &= \frac{\bar{z}}{r}-\frac{\bar{x}}{r}\frac{(\lambda_0+\lambda_1\alpha_{ji})\rho\sigma_i^2}{\lambda_0(\lambda_0+2\lambda_1\alpha_{ji})} \\
b_i &= \frac{\lambda_1(\alpha_{ji}-1)}{r(\lambda_0+2\lambda_1\alpha_{ji})} \\
c_i &= \frac{(\lambda_0+\lambda_1\alpha_{ji})\rho\sigma_i^2}{r\lambda_0(\lambda_0+2\lambda_1\alpha_{ji})} \\
d_{ji} &= \frac{\lambda_1\alpha_{ji}}{r(\lambda_0+2\lambda_1\alpha_{ji})} \\
\sigma_{pi}^2 &= b_i^2\sigma_i^2+c_i^2\sigma_{xi}^2+d_{ji}^2\left(1-\frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 \\
&= \frac{1}{r^2(\lambda_0+2\lambda_1\alpha_{ji})^2}\left[\lambda_1^2(\alpha_{ji}-1)^2\sigma_i^2+(\lambda_0+\lambda_1\alpha_{ji})^2\rho^2\sigma_i^4\sigma_{xi}^2\lambda_0^{-2}+\lambda_1^2\sigma_i^2\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{Cov(p_i,z_i)}{\sigma_{pi}} &= \frac{Cov(a_i+b_i\epsilon_i-c_i\nu_i-d_i\zeta_{ji},z_i)}{\sigma_{pi}} \\
&= \frac{\lambda_1(\alpha_{ji}-1)\sigma_i^2}{\sqrt{\left[\lambda_1^2(\alpha_{ji}-1)^2\sigma_i^2+(\lambda_0+\lambda_1\alpha_{ji})^2\rho^2\sigma_i^4\sigma_{xi}^2\lambda_0^{-2}+\lambda_1^2\sigma_i^2\right]}} \\
&= \frac{\lambda_1(\alpha_{ji}-1)\sigma_i}{\sqrt{\left[\lambda_1^2(\alpha_{ji}-1)^2+(1+\hat{\lambda}_{ji}\alpha_{ji})^2\rho^2\sigma_i^2\sigma_{xi}^2+\lambda_1^2\right]}}
\end{aligned}
$$

It is straightforward to see that $PI=0$ when $\alpha=0$ and when $\alpha=1$. Because $PI\geq 0$, it is nonmonotonic, and by Proposition 2. □