

# Pricing-to-Market in Business Cycle Models \*

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## ABSTRACT

We evaluate the performance of leading micro-founded pricing-to-market models vis-a-vis a set of robust stylized facts about international prices. We embed each model into a unified IRBC framework parameterized in a uniform way to provide a meaningful evaluation of their applicability for policy-oriented DSGE modeling, and identify their individual strengths and weaknesses. We also attempt to devise empirical strategies that could be employed to discriminate between these models. We conclude by making three observations: (i) the mechanisms generating pricing to market are not always neutral to business cycle dynamics of quantities; (ii) some mechanisms require producer markups of at least 50% or more to account for a reasonable full range of estimates of the empirical exchange rate pass-through to export prices of 35%-50%, imposing precondition on pricing-to-market to arise such as fixed or sunk costs (or high profitability); (iii) some frictions crucially depend on a particular driver of uncertainty in the underlying model, and do not work with all types of shocks.

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# 1 Introduction

It is a well known fact that frictionless international macro models, while successful in accounting for business cycle dynamics of quantities, fail to account for international prices. In particular, they are inconsistent with a vast empirical literature documenting international deviations from the law of one price and incompleteness of exchange rate pass-through. Not surprisingly, reconciling the predictions of business cycle models for international prices with the data has been on the forefront of research in international economics, as it is critical to determine the expenditure switching role of exchange rates and thus the transmission mechanism of shocks across countries. Independently, it can be informative about the market structure and technological constraints that underly international trade in goods.

Thus far, the literature has stressed the importance of two potential sources of international deviations from the law of one price and incompleteness of exchange rate pass-through: sticky prices and broadly defined source of price discrimination (real rigidities). These two avenues should not be regarded as competing in light of recent empirical evidence. This is because this micro evidence strongly suggests a significant role for real rigidities in accounting for the behavior of prices, independently from nominal price rigidities<sup>1</sup>. Starting from this premise, however, the key question is whether the models are up to the task of potentially remedying shortcomings of models based on sticky prices alone.

At this point, the pricing to market literature offers a wealth of micro-founded theories of real rigidities generating pricing to market behavior (PTM hereafter). In various contexts, these models have been demonstrated as successful in improving upon their respective frictionless benchmarks. However, since the definition of the frictionless benchmark as well as the set of analyzed data moments typically vary across papers, it is hard to compare these mechanism with each other, and it is not clear how useful these mechanism are for policy oriented modeling in general. It is also not clear on what grounds one could discriminate between them using empirical evidence.

In an attempt to fill this void, our paper provides a consistent comparison of several

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<sup>1</sup>Johri & Lahiri (2008) argue that sticky prices alone cannot account for the persistence of the deviations from the law of one price. The empirical findings in Gopinath & Itshkhoki (2010a), Gopinath & Itshkhoki (2010b), and Fitzgerald & Haller (2012) on price responses to an exchange rate change, *conditional on a price change taking place*, point in the direction of strong and persistent pricing to market in exporting. Given that exchange rate is close to random walk, these findings are inconsistent with standard sticky price models. While pricing to market and variable markups are not the only source of deviation from law of one price under flexible prices, the consensus view is that these alternative sources are too weak to serve as a leading explanation.

leading models in the literature that can generate PTM — analyzed both in a flexible price environment and also in presence of nominal rigidities. To this end, as a point of our departure and benchmark, we consider here the same frictionless business cycle model that is relevant from an applied perspective, and embed all possible sources of real rigidities generating PTM into this common framework — and enhance the model with nominal rigidities as well as sources of volatile exchange rate fluctuations. We then focus on factors which in our view are likely to determine the broad-based applicability of these models to policy-oriented business-cycle work and helpful in identifying pre-conditions for PTM to arise: (i) dependence of the mechanism generating PTM on the source of economic fluctuations in the model, (ii) implications of the mechanism generating PTM for the dynamic behavior of quantities, and finally, (iii) the degree of deviations from the law of one price generated by the models. By introducing such systematic approach we aim at bringing more structure into this fairly dispersed literature.

The above criteria set a minimum set of requirements for a successful and flexible model from an applied point of view. For example, given fair degree of uncertainty about the actual shocks that drive the enormous exchange rate volatility in the data, the first criterion determines to what extent a given friction can be successfully employed in large scale models featuring various types of shocks. The second criterion is important because accounting for dynamics of prices should not be achieved by sacrificing the quantitative fit of the model. Finally, the third aspect determines how well a given friction fares with the primary goal of bringing prices closer to the data.

In particular, our analysis focuses on four state-of-the-art models that lead to imperfect competition and pricing to market: (i) *Sectoral Aggregation* model, based on a quantitative trade/macro model due to Atkeson & Burstein (2008)<sup>2</sup>; (ii) *Costly Search* model developed by Alessandria (2009); (iii) *Deep Habits*, developed in Ravn, Schmitt-Grohe & Uribe (2007); and (iv) *Costly Distribution* model, proposed by Corsetti & Dedola (2005). It is important to stress that all selected frictions are tractable and thus potentially amenable to quantitative analysis within large-scale DSGE models; they are widely cited in the literature, and all are micro-founded to certain degree<sup>3</sup>. Nevertheless, while guided by these criteria, our selection remains fairly arbitrary and incomplete.

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<sup>2</sup>First proposed by Dornbusch (1987).

<sup>3</sup>That is, the effects arise from a meaningful micro-level friction which allow for enhanced quantitative discipline potentially using micro-level evidence.

The specifics of our exercise are as follows. We embed each friction into a standard business cycle model a la Backus, Kehoe & Kydland (1995), and parameterize each model using a common set of data targets that are also standard in the literature. Whenever this is not possible, our preferred approach is to use the methodology or parameters from the original papers that introduced these frictions. To focus on the ability of each friction to generate deviations from the law of one price, in our qualitative analysis we focus on the predicted theoretical pass-through of exchange rates to export prices and contrast it with the available evidence. In our quantitative analysis, we study a set of moments pertaining to *real export prices*, i.e., prices deflated by the price of comparable domestic goods (or baskets). In particular, we compare the model vis-a-vis the consensus range of pass-through estimates to export prices<sup>4</sup> of 35%-50%. By focusing on the pass-through coefficient implied by the theory, our analysis is largely independent from the exact mechanism generating real exchange rate movements – which in the model will be counterfactual. Finally, we evaluate each PTM mechanism comprehensively by analyzing a set of price and quantity statistics, and assess the importance of the exact specification of the forcing process.

Even though we often refer to each friction by citing the paper that introduced it the first time, it is important to stress that our exercise actually modifies the original models, and takes a rather selective look at the implications of each theory. Thus, our results should not be interpreted as a criticism of the original contributions, but rather an extension. Our goal is to inform applied researchers about the differences between these mechanisms when modeled within a unified environment, and potential empirical strategies that can be used to discriminate among them.

In our quantitative exercises, we provide a comprehensive analysis of the pricing and quantity predictions of the theories. Our three key take away are summarized below:

**(1) Not all frictions generating PTM are neutral to business cycle dynamics of quantities. Moreover, some of the counterfactual implications are directly implied by the very mechanism generating PTM.** The *Costly Distribution* and *Deep Habits* model are fully neutral to the predictions for quantities. However, the remaining two: *Sectoral Aggregation* model due to Atkeson & Burstein (2008) and *Consumer Search* model due to Alessandria (2009) significantly affect the fit of the model for quantities.

First, the *Industry Aggregation* model, which was designed to work in a low-frequency-

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<sup>4</sup>Pass-through to import prices is approximately 100% *minus* the pass-through to export prices.

oriented trade model, requires a high elasticity between home and imported goods for the PTM mechanism to work (consistent with the trade literature), which hurts the implications of the model for quantities on business-cycle frequencies. In particular, the high elasticity resurrects some of the problems of the early Backus, Kehoe & Kydland (1992) setup with one homogeneous good, like excess comovement of TFP over output or excess comovement of consumption over output. Furthermore, the *Industry Aggregation* model delivers PTM through movements of importer’s market shares that affect the measured elasticity of demand. Consequently, in order to deliver deviations from LOP that are in line with the data, the model requires that the aggregate import shares be volatile. Our back-of-the-envelope calculation suggests that, for a plausible range of parameter values, they will be more volatile in the model than in the data and also too tightly correlated<sup>5</sup>.

Second, the *Costly Search* model alters the predictions for quantities when modeled as in Alessandria (2009) (we denote this formulation as *Price Dispersion\**). This original specification, featuring a linear utility function in labor featuring an exogenous labor-wedge shock, results in excess volatility of employment and output. With CRRA utility and only productivity shocks, similar problems arise<sup>6</sup>. We fix these problems in our baseline specification of the friction by modifying the original model and making search a market activity of intermediate good producers – rather than consumers. This restores a reasonable fit for quantities, while still delivering statistics for prices; however, it alters the economic interpretation of the friction in a crucial way.

**(2) The equilibrium level of markups affects the performance of most models.**

In particular, *Costly Search* and *Costly Distribution* model require high equilibrium markups in order to be consistent with the theoretical target for the pass-through coefficient of exchange rates to export prices. For example, in our benchmark comparison, we set markups equal to 30%, which turns out to be grossly insufficient to deliver any action. Specifically, the *Costly Search* friction, and especially *Costly Distribution* friction<sup>7</sup> deliver a subset of moments for prices at our targeted level of 30% markups, but in order to be consistent with a full range of empirical estimates of 35-50% of exchange rate pass-through to export prices, the

<sup>5</sup>This is less of a problem on lower frequencies—as suggested by much higher estimates of long-run price elasticities of trade.

<sup>6</sup>Employment is negatively correlated internationally, and there too little comovement of output.

<sup>7</sup>In *Costly Distribution* friction only part of the markup is in the foreign unit, whereas in *Costly Search* model the whole markup is. Thus, it is easier to get more PTM in *Costly Search* model than in *Costly Distribution* model for the same level of markups.

model requires markups of at least 50%. On the other hand, the *Sectoral Aggregation* model performs fairly well for the benchmark setting of markups, but quantity statistics deteriorate rapidly when markups are lowered significantly below this level (while still matching statistics on prices). In particular, when markups are set to 15%, as suggested by Basu & Fernald (1997), the country’s import shares need to be much more volatile in the model to deliver meaningful amounts of PTM. We conclude that markups are a critical aspect and challenge that pricing to market literature needs to deal with.

What is the appropriate target for markups? There is a lot of conflicting evidence about the level of markups in manufacturing. Industry studies point to numbers around 20-30% (as used by Atkeson and Burstein (2008) and Alessandria (2009)) or sometimes even higher, while on the other end, there are aggregate estimates by Basu & Fernald (1997) pointing to markups as low as 10-15%. While any level of markups can, in principle, be rationalized in the context of PTM models by sizable fixed or sunk costs of production, such features may also critically affect business cycle implications of the models, especially in the case of quantities. We leave these considerations for future research, and conclude that more evidence on exact production structure may be needed to support the assumption that PTM models need.

**(3) Not all models are independent from the specifics of the forcing process.**

We find that the *Deep Habits* model with productivity shocks or standard demand shocks delivers deviations from LOP in the opposite direction (export markups fall relative to home markups when the real exchange rate depreciates, and theoretical exchange rate pass-through to export prices is negative). This makes the model perform worse vis-a-vis our frictionless benchmark model in which LOP holds.<sup>8</sup>

The rest of the paper is organized as follows. Section 2 presents the frictionless framework in which we embed the frictions. Section 3 introduces the frictions. Section 5 provides analytical results and qualitative analysis of the sources of PTM in each friction. Sections 6 and 7 present the parameterization and quantitative results. Section 8 concludes.

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<sup>8</sup>It should be noted, however, that when demand shocks are modeled as stochastic government purchases that are additionally subject to deep habit formation, they can give rise to PTM in the right direction—as demonstrated by Ravn, Schmitt-Grohe & Uribe (2007). For such shocks, the model implies that following a government spending shock, the real exchange rate appreciates, rather than depreciates as in the standard models. According to some VAR-evidence, such correlation of the real exchange rate may be consistent with the data for some countries, including US and Japan (see Corsetti, Dedola & Leduc (2006)). We do not consider shocks of this kind here.

## 2 Unifying Model

In the next five sections, we set up the models for later quantitative comparison. We start off by setting up the bare-bones features into which we incorporate real rigidities. The baseline general equilibrium setup is based on Backus, Kehoe & Kydland (1995) model.

### 2.1 Common Features

Time is discrete,  $t = 0, 1, 2, \dots, \infty$ , and there are two ex-ante symmetric countries labeled *domestic* and *foreign*. Each country is populated by identical and infinitely lived households which supply labor and physical capital, consume goods, trade assets, and accumulate physical capital. Tradable intermediate goods are country-specific; that is, good  $d$  is sourced from the domestic country (at least at some level of aggregation), and  $f$  is sourced from the foreign country. The source of uncertainty is country-specific productivity shocks, and demand shock when applicable.

In terms of notation, we distinguish foreign country related variables from the domestic ones using an asterisk. The history of shocks up to and including period  $t$  is denoted by  $s^t = (s_0, s_1, \dots, s_t)$ , where the initial realization  $s_0$ , as well as the time invariant probability measure  $\mu$  over the compact shock space  $S$  are assumed given. In the presentation of the model, whenever possible, we exploit symmetry of the two countries and present the model from the domestic country's perspective.

#### 2.1.1 Households

We start from household problem, which is completely standard.

Each country is populated by a unit measure of identical and infinitely lived households. Households supply production factors to domestic producers, accumulate physical capital, trade financial assets, and consume goods. After each history  $s^t$ , the stand-in household chooses the allocation, which consists of the level of consumption  $c$ , investment in physical capital  $i$ , labor supply  $l$ , and purchases of a set of one-period  $s_{t+1}$ -contingent bonds  $b(s_{t+1}|s^t)$  to maximize the expected discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \int_{S^t} u(c(s^t), l(s^t)) \mu(ds^t). \quad (1)$$

Asset markets are complete (for simplicity), and the budget constraint of the domestic

household is given by

$$P(s^t) (c(s^t) + i(s^t)) + \int_S Q(s_{t+1}|s^t) b(s^{t+1}) = b(s^t) + w(s^t) l(s^t) + r(s^t) k(s^{t-1}) + \Pi(s^t),$$

$$k(t+1) = (1 - \delta) k(s^t) + i(s^t) \text{ all } s^t.$$

In the above budget constraints, we assume that the composite consumption good in each country is the numéraire. We do so by normalizing the level of prices  $P(s^t)$  in each country to unity; the resulting ideal CPI price indexes in each country are thus equal to one on a state by state basis.<sup>9</sup>

The expenditure side of the budget constraint consist of purchases of the consumption and investment goods and purchases of one-period-forward  $s_{t+1}$ -state contingent bonds. The income side consists of income from maturing bonds purchased at history  $s^{t-1}$ , labor income, rental income from physical capital, and the dividends paid out by local firms. The foreign budget constraint, due to a different numéraire, additionally involves a price  $x(s^t)$  that translates the foreign numéraire to the domestic numéraire in the bond purchases term. By definition of the numéraire in each country, this price is the (welfare-based) real exchange rate<sup>10</sup>; it crucially integrates the domestic and foreign asset markets into one world asset market.<sup>11</sup>

The model implies a set of usual first order conditions, which, in turn, imply that the real exchange rate is governed by the perfect risk sharing condition stated below:

$$x(s^t) = \frac{u_c^*(s^t)}{u_c(s^t)}. \tag{2}$$

This condition says that households behave like a benevolent family, and fully share risk internationally, by equalizing MRS from consumption across the borders, with the relative price of the final consumption good across countries  $x$ . Since we evaluate pricing implications of the models in relation to the underlying movements of the real exchange rate, the fact that

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<sup>9</sup>The ideal-CPI is defined by the lowest cost of acquiring a unit of composite consumption ( $c$  in the domestic country,  $c^*$  in the foreign country)

<sup>10</sup>In the data real exchange rate is measured using fixed-weight CPI rather than ideal CPI indices. Quantitatively, this distinction turns out not to matter in this particular class of models.

<sup>11</sup>Since the foreign budget constraint is expressed in the foreign country numéraire, and so is  $b^*$ , in order to use  $Q$  as the intertemporal price, the term  $x(s^{t+1})b^*(s_{t+1}|s^t)$  first translates the purchase value of the foreign bonds to the domestic country numéraire units, and then  $Q(s_{t+1}|s^t)/x(s^t)$  expresses the price of this purchase again in terms of the foreign numéraire.

the equation implies partly counterfactual properties of the real exchange rates has little bearing on our results, and we will verify robustness of our results when applicable.

We next describe production technology, and trade.

### 2.1.2 Production

Goods are traded at three levels of aggregation: *intermediate level*, *sectoral level*, and *final level*. International trade takes place between intermediate good producer and sectoral good producers, and this is the stage at which all action will be coming from in terms of pricing to market.

Intermediate goods are denoted by  $d_i$ , and  $f_j$ , where  $d$  and  $f$  indicate the source country (domestic, foreign), and  $i$  and  $j$  stand for a particular producer index (index of a firm). At the sectoral level, which is within each country, sectoral producers aggregate these intermediate goods into single composite good  $D$  and  $F$ . Finally, on the final good level, local final good producers combine these composite goods  $D$  and  $F$  into a final consumption/investment good which are sold to the local households in a perfectly competitive market.

### 2.1.3 Final Good Producers

Final good producers produce a composite consumption and investment goods  $c$  and  $i$  that are supplied to households, and turned by them into  $c$  and  $i$ .

To producer their output, they purchase composite goods  $D$  and  $F$  from the intermediate good producers at home and abroad, and then aggregate them into the final consumption/investment good (non-tradable). These goods are aggregated according to a CES function given by

$$G(D, F) = \left( \omega D^{\frac{\gamma-1}{\gamma}} + (1 - \omega) F^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \quad (3)$$

where  $\gamma$  is the elasticity of substitution (Armington elasticity) and  $\omega$  is parameterizing home bias. The aggregated output is then used for consumption and investment, is purchased by household in a competitive market. There is no added value in this sector, and it can well be interpreted as part of HH preferences.

Given the aggregation function above, the instantaneous profit of an aggregating firm is given by

$$P(s^t) G(D, F) - P_D(s^t) F(s^t) - P_F(s^t) F(s^t).$$

Hence, the optimality conditions for the final good producers are given by  $P_D(s^t) =$

$G_D(s^t) P(s^t)$ ,  $P_F(s^t) = G_F(s^t) P(s^t)$ , while the aggregation constraint is given by (3).

#### 2.1.4 Sectoral Good Producers

Sectoral good producers are importers in our model. They produce composite non-tradable varieties  $D(s^t)$  and  $F(s^t)$ , respectively. In general, their role will be to aggregate a set of country specific intermediate goods sourced from each country intermediate good producers.

The aggregation is governed defined over a set of varieties from each country. The details will be specified later, and the problem may also include additional technological constraints, such as search. The sectoral goods are always resold to final good producers through a competitive market.

#### 2.1.5 Intermediate Good Producers

Intermediate good producers are exporters in our setup, and they interact with sectoral good producers, who are importers. They produce individual varieties denoted by  $d(i, s^t)$  and  $f(i, s^t)$ , where notation  $i$ , if used, indicates product differentiation by the identity of a given producer (otherwise these goods are homogenous within a country).

To produce output, they have access to a country specific constant returns to scale production function  $z^F(k, l)$ , where for now we assume away heterogeneity in  $z$ . This technology uses country-specific capital  $k$  and labor  $l$  that are immobile, and productivity  $\hat{z} \equiv \log(z)$  follows an exogenous AR(1) process

$$\hat{z}(s^t) = \psi \hat{z}(s^{t-1}) + \varepsilon_t, \quad \hat{z}^*(s^t) = \psi \hat{z}^*(s^{t-1}) + \varepsilon_t^*, \quad (4)$$

where  $0 < \psi < 1$  is a common persistence parameter, and  $s_t \equiv (\varepsilon_t, \varepsilon_t^*) \in S$  is an i.i.d. normally distributed random variable with zero mean. Output may be a differentiated commodity, depending on the context (or homogenous).

The production function of each individual producer is identical unless otherwise noted, and assumes to be constant returns to scale.

It will be convenient to simplify notation a bit and summarize the production process by an economy-wide marginal cost  $v$  faced by all producers. Given domestic factor prices  $w$ ,  $r$  and domestic shock  $z$ , the marginal cost, also equal to per unit cost, is given by:

$$v(s^t) \equiv \min_{k,l} \{w(s^t) l + r(s^t) k \text{ subject to } z(s^t) F(k, l) = 1\}. \quad (5)$$

The instantaneous profit function  $\Pi$  of an particular intermediate good producer from selling  $d$  quantity in domestic market and  $d^*$  in the foreign market is given by

$$\Pi(s^t) = (p_d(s^t, d) - v(s^t))d + (x(s^t)p_d^*(s^t, d^*) - v(s^t))d^* \quad \forall s^t,$$

where individual prices  $p$  and  $p^*$  at which these goods are sold at home and abroad, respectively, depend on quantity. The specifics how export market works will pin down the functional form of the demand faced by the producers, and affect pricing decisions of the producers. Here note that we assume that intermediate good producers are monopolists over their respective variety, and they can segment international markets.

We next describe the key aspects how intermediate producers and sectoral producer interact through the process of exporting and importing. The feasibility conditions of the economy as a whole are straightforward and will be briefly described in the context of each setup introduced below.

### 3 Baseline Sources of Real Rigidities

Here we lay out the feature that will imply price discrimination across markets by monopolistic intermediate good producers (pricing to market). Each setup is based on the papers mentioned earlier in the introduction. We focus on the description of the problem faced by exporters and importers, and how additional features we introduce affects economy-wide feasibility and market clearing (which is standard in the baseline case and has been omitted).

#### 3.1 Costly Search

The setting here modifies the formulation of this model relative to the original paper; in our formulation search occurs on the final good producer's level rather than on the consumer level. This modification changes the interpretation of the original friction, which here should be interpreted as business-to-business search friction rather than a *consumer search*. We introduce this modification to improve the performance of the model on the quantity side without sacrificing prices. Later, we also discuss a version of the model that adheres closely to the original.

### 3.1.1 Problem of Importers

Sectoral good producers are importers and purchase intermediate goods of type  $d$  and  $f$  from producers in each country to turn them into composite aggregate quantity  $D$  and  $F$  sold to final good producers in their respective country of residence. The intermediate goods are all identical (perfectly substitutable) when sourced from a given country, and so  $D$  and  $F$  aggregator is simply given by the integral over individual quantities purchase from producers from domestic country, and foreign country, respectively. The key element of the model is that these purchases must be acquired through the process of costly search, which we describe below.

Search friction can be overcome by sending a measure of atomless representatives of the final good firm—employed at the economy-wide marginal cost  $v$ . Specifically, it is assumed that in every period, the firm (headquarters thereafter) sends measures  $s_d$  and  $s_f$  of reps to purchase  $1/\theta$  units of goods of type  $d$  and  $f$ , respectively. The total cost of this activity for the firm is  $(s_d + s_f)v(s^t)$ .

With probability  $q$ , each atomless rep can direct its search to a given country, and can obtain one price quote of the commodity that it was directed to search for ( $d$  or  $f$ ); with probability  $1 - q$  it manages to obtain two competing quotes, and not more than two quotes are ever obtained.

The headquarter must send multiple reps to purchase supplies, simply because, after accepting the price, each rep can only purchase  $1/\theta$  of a particular commodity at an atomless level with respect to headquarters. Purchase is always conditional that such price does not exceed the reservation prices  $r_d(s^t)$  and  $r_f(s^t)$  set a priori by the headquarters. Since in equilibrium no posted price by any intermediate level producer will ever exceed the reservation price set by headquarters for its reps (searching for  $d$  or  $f$ , respectively), we can assume without loss of generality that the rep always makes a purchase, i.e., all posted prices are lower than the reservation price.

Given measures of the searching reps, each facing a capacity constraint how much they can purchase from a single producer they find, the total amount of goods of a given type purchased by the entire firm is given by  $d = \frac{s_d(s^t)}{\theta}$  and  $f = \frac{s_f(s^t)}{\theta}$ .

The distribution of prices at which goods are purchased depends on the distribution of prices posted by firms  $F(p; s^t)$ , and is given by (distribution of the lowest of the two draws

from  $F$ ):

$$H_d(p; s^t) = qF_d(p; s^t) + (1 - q) [1 - (1 - F_d(p; s^t))^2],$$

The average price paid by the searching reps (for commodity  $d$ ) can then be obtained by integrating over  $H$

$$p_d(s^t) = \int_{P_l}^{P_h} p \frac{dH_d(p; s^t)}{dp} dp,$$

where the bounds of the integration will be defined later. The price of good  $f$  can be derive analogously.

### 3.1.2 Problem of Exports

Intermediate good producers are exporters and produce homogenous goods  $d$  and  $f$  that are purchased by the sectoral good producers from the home country and the foreign country. The production cost is equal to the marginal cost  $v$ , as stated earlier.

Even though goods from a given country are homogenous, the key element that generates downward sloping demand from producer perspective is brought about by the fact that the intermediate good producer; when giving price quote to a searching rep, does not know how many quotes a given rep has observed. This feature, as demonstrated by Burdett & Judd (1982), is generally sufficient to give rise to a unique equilibrium featuring an endogenous price dispersion of physically identical goods. Burdett & Judd (1982) show that the support of the prices posted by all firms selling in a given market is compact and connected, and thus can be represented by a closed interval  $[P_h, P_l]$ . Moreover, the optimal probability distribution  $F(\cdot)$  is uniquely pinned down by the condition that, given other producers also draw from  $F(\cdot)$ , the intermediate good producer must be indifferent between all the prices from the support of  $F(\cdot)$ , after taking into account endogenous winning probability (probability of making sales).

Formally, for any  $p \in [P_l(s^t), P_h(s^t)]$ , the condition tht pins down  $F$  requires that the probability  $F(p; s^t)$  that a posted price is lower than  $p$  must satisfy:

$$(p - v(s^t))(q + 2(1 - q)(1 - F(p; s^t))) = (P_h(s^t) - v(s^t))q. \quad (6)$$

The condition is implies by the fact that expected profits from posting an arbitrary price  $p$ , factoring in an endogenous probability of making sales given by  $(q + 2(1 - q)(1 - F(p; s^t)))$ , must be identical to the profits from posting the highest price possible  $P_h$  (equal to reservation

price of the buyer). At this highest price, by definition, the intermediate producer is making sales if and only if the representative has only a single price quote (which happens with probability  $q$ ), which makes probability of making sales equal to  $q$ .

Clearly, given bounds of the support,  $P_h, P_l$ , equation (6) defines distribution function  $F$ . These bounds can be derived as follows. The upper bound of the distribution  $P_h$  is determined by the condition that the final good producer must be indifferent between buying the good for  $P_h$  and instructing representatives who have a draw  $P_h(s^t)$  to abort the purchase and instead direct more reps to purchase the same good at the average price  $p_d$ , i.e.,  $\theta v(s^t) = P_h(s^t) - p_d(s^t)$ . The lowest bound can be found by plugging in  $P_l$  for  $p$  to (6).

The full characterization of equilibrium prices in the model is as follows, and can easily be calculated from the conditions listed above:

$$\begin{aligned} P_h(s^t) &= v(s^t) + \frac{\theta}{1-q}v(s^t), \\ P_l(s^t) &= \frac{P_h(s^t)q + 2(1-q)v(s^t)}{2-q} = v(s^t) + \frac{q\theta}{2-3q+q^2}v(s^t), \\ F(p) &= 1 - \frac{1}{2} \frac{q}{1-q} \frac{P_h(s^t) - p}{p - v(s^t)}, \end{aligned}$$

and the expected price paid by a representative looking for good  $d$  is:

$$p_d(s^t) = v(s^t) + \frac{\theta q}{1-\theta}v(s^t).$$

### 3.1.3 Effects on Aggregate Feasibility

At every date and state, the feasibility condition for intermediate goods additionally includes the cost of search:  $d + d^* + \theta(d + f) = zk^\alpha n^{1-\alpha}$ . The remaining feasibility conditions are straightforward and will be omitted.

## 3.2 Costly Distribution

In this section, we lay out a version of the monopolistic competition model due to Corsetti & Dedola (2005), which offers one of the simplest modeling of pricing to market available. The key element here is that each producer is a monopolist over an atomless country-specific variety of good as in the Dixit-Stiglitz model, but for each good a local distribution cost has to be incurred before the good is delivered to the final consumer. The existence of an explicit

distribution cost (or sector), governed by a Leontief production function, makes the demand for each variety of good depend not only on the price of that good charged by intermediate good producers, but also on the local cost of distribution — in effect delivering what is perceived by intermediate good producers as time-varying price elasticity of demand<sup>12</sup>. We first setup the baseline version of the model, and then later extend it to introduce an explicit non-tradable sector, adhering more closely the original formulation.

### 3.2.1 Problem of Importers

The sectoral good producers here aggregate a variety of differentiated intermediate goods purchased from measure one of intermediate good producers from each country. In addition to paying the purchase price for the tradable good to the intermediate good producer, each sectoral producer has to incur a distribution cost denominated in the local good (fixed per unit of good)<sup>13</sup>. In particular, in order to sell a good in the domestic market, each sectoral producer has to pay a distribution cost  $\xi$ . Having purchased all the goods, sectoral producer aggregates different varieties of goods into a composite good  $d$  and  $f$ , respectively. The aggregator over varieties is CES, with an elasticity  $\theta$ ,

$$D(s^t) = \left( \int_0^1 d(i, s^t)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

Aggregation of the foreign goods (in the domestic country) is analogous.

The instantaneous profit function of the sectoral producers is given by

$$P_D(s^t) D(s^t) - \int_0^1 [p_d(i, s^t) + \xi v(s^t)] d(i, s^t) di - \int_0^1 [p_f(i, s^t) + \xi v(s^t)] f(i, s^t) di.$$

Markets for composite goods  $d$  and  $f$  are competitive, and hence the prices of composite goods charged by sectoral good producers are given by

$$P_j(s^t) = \left[ \int_0^1 (p_j(i, s^t) + \xi v(s^t))^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

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<sup>12</sup>Leontief specification of distribution is a critical assumption. If, for example, the production function in the distribution sector is Cobb-Douglas, as for example in Eaton and Kortum (2002), there is no pricing to market in this model. If elasticity is higher than unity, pass-through coefficient would be counterfactually negative.

<sup>13</sup>We can think of the sectoral good producers as just employing capital and labor to produce distribution services according to the economy-wide technology.

where  $j = D, F$ .

Here we observe that the demand function for good  $d(i, s^t)$ , and analogously for good  $f(i, s^t)$ , takes the form

$$d(i, s^t) = \left( \frac{p_d(i, s^t) + \xi v(s^t)}{P_d(s^t)} \right)^{-\theta} D(s^t). \quad (7)$$

It thus crucially departs from the usual Dixit-Stiglitz formulation. Unlike in the standard Dixit-Stiglitz model, here demand for each variety of good not only depends on the price charged by each intermediate good producer ( $p_d(i, s^t)$  and  $p_f(i, s^t)$ ), but also on the local marginal distribution cost  $v$ . Hence, depending on the level of sales, the perceived elasticity of demand by monopolistic intermediate good producers varies, which will lead to pricing to market.

### 3.2.2 Problem of Exporters

Intermediate good producers are indexed on a unit interval by  $i$  and produce differentiated commodities that are also country specific. As before, their marginal production cost is given by (5).

A producer of variety  $i \in [0, 1]$  chooses home wholesale price  $p_d(i, s^t)$  and dock export price  $p_d^*(i, s^t)$  to maximize profit

$$(p_d(i, s^t) - v(s^t))d(i, s^t) + (x(s^t)p_d^*(i, s^t) - v(s^t))d^*(i, s^t),$$

subject to demand equations at the sectoral level (given by (7), and analogous ones for the export market).

For later use, we observe that the producer prices solve the problem stated above, and are given by

$$\begin{aligned} p_d(i, s^t) &= \frac{\theta}{\theta - 1}v(s^t) + \frac{\xi}{\theta - 1}v(s^t), \\ p_x(s^t) &\equiv x(s^t)p_d^*(i, s^t) = \frac{\theta}{\theta - 1}v(s^t) + \frac{\xi}{\theta - 1}x(s^t)v^*(s^t). \end{aligned} \quad (8)$$

### 3.2.3 Effects on Aggregate Feasibility

Feasibility additionally includes the cost of distribution:  $d + d^* + \xi(d + f) = zk^\alpha n^{1-\alpha}$ .

### 3.3 Deep Habits

In this section, we introduce ‘relative deep habits’ into the model. Just like in the standard Dixit-Stiglitz model, the setup involves a measure of atomless intermediate good producer who are monopolists over a country-specific variety, but unlike in the Dixit-Stiglitz model, consumers develop an external habit to each type of variety they consume<sup>14</sup>. This habit multiplicatively shifts demand from the monopolistic producer’s perspective — and thus can boost sale price (for any fixed quantity sold).

Specifically, habit is developed with respect to a continuum of ‘deep’ varieties of country specific intermediate goods  $d(i, s^t)$  and  $f(i, s^t)$ .

#### 3.3.1 Problem of Importers

The key departure here is that sectoral good producers, who are importers, aggregate intermediate varieties using *habit-adjusted* quantities,  $d^h(j)$  and  $f^h(j)$ , respectively, rather than pure quantities that they purchase. The aggregation from this point on is standard, and takes the form of a CES aggregator governed by an elasticity  $\varphi$ :

$$D(s^t) = \left[ \int_0^1 d^h(j, s^t)^{\frac{\varphi-1}{\varphi}} dj \right]^{\frac{\varphi}{\varphi-1}} \quad \text{and} \quad F(s^t) = \left[ \int_0^1 f^h(j, s^t)^{\frac{\varphi-1}{\varphi}} dj \right]^{\frac{\varphi}{\varphi-1}}.$$

For each good  $j \in [0, 1]$ , the habit-adjusted quantity  $d^h(j)$  is assumed to be determined by the level of habit in the previous period,  $h_d(j, s^{t-1})$ , and the purchases of the good today,  $d(j, s^t)$ , according to the formula

$$d^h(j, s^t) = \frac{d(j, s^t)}{h_d(j, s^{t-1})^\theta},$$

where  $\theta$  parameterizes the strength of the habit ( $\theta$  assumed to be less than zero, following the convention introduced by Ravn, Schmitt-Grohe & Uribe (2006)). Habit evolves according to:

$$h_d(j, s^t) = \rho h_d(j, s^{t-1}) + (1 - \rho) \bar{d}(j, s^t),$$

---

<sup>14</sup>Evolution of habits is driven by average purchases of each variety and not purchases of any individual household. For simplicity, here we model habit as being formed at the level of producers, which is homomorphic to the setup in which it is directly incorporated into the consumer problem.

where  $\bar{d}$  assures producers do not internalize their own decision on accumulation of habit, giving rise to external habit. It stands for the *average* level of purchases of good  $j$  in the economy. Analogous equations hold for good  $f$ .

The instantaneous profit function of the sectoral producers is given by

$$P_D(s^t) D(s^t) + P_F(s^t) F(s^t) - \int_0^1 p_d(i, s^t) d(i, s^t) di - \int_0^1 p_f(i, s^t) f(i, s^t) di,$$

where  $p_d(i, s^t)$  and  $p_f(i, s^t)$  are prices charged by the monopolistically competitive intermediate good producers.

The sectoral good producer's problem can be simplified by solving a temporal decision of allocating purchases across the individual intermediate  $j$  varieties. For any amount of the aggregate goods desired,  $D$ , given prices,  $p_d(j)$ , there is a unique expenditure-minimizing allocation of  $d(j, s^t)$  — which defines demand from the intermediate producer's perspective — and it is given by

$$d(i, s^t) = \left( \frac{p_d(i, s^t)}{P_D(s^t)} \right)^{-\varphi} h_d(i, s^{t-1})^{\theta(1-\varphi)} D(s^t), \quad (9)$$

where the aggregate price index is

$$P_D(s^t) \equiv \left[ \int h_d(j, s^{t-1})^{\theta(1-\varphi)} p_d(j, s^t)^{1-\varphi} dj \right]^{\frac{1}{1-\varphi}}.$$

Equations for quantities and prices of good  $f$  can be expressed analogously. It is clear from the above formula how habit shifts demand relative to a Dixit Stiglitz formulation (equivalent to  $h_d = 1$ ).<sup>15</sup>

### 3.3.2 Problem of Exporters

In each country, there is a measure of goods produced by unit measure of monopolistic intermediate good producers. Their unit production cost is given by (5).

Producers take the demand relations defined by (9) as given. Intermediate producers fully internalize the effect of making sales on the formation of habit in a given market. Specifically,

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<sup>15</sup>To save on notation, the setup here is a simplified version of the setup that we use in the quantitative analysis section. There, we exclude investment  $i$  from the habit formation process, as it seemed more natural for us to think of investment as being denominated in physical units rather than habit adjusted units. Habit is only imposed on consumption goods. We have verified that this distinction would not make any significant difference for any of the result reported throughout the paper.

they take into account the law of motion for habit as laid out in (3.3.1), but with  $\bar{d}$  replaced by their respective production choice for each market: with slight abuse of notation  $d(i, s^t)$  at home, and  $d^*(i, s^t)$  abroad.

The optimization problem of each intermediate good producer is to maximize the present discounted stream of profits, stated before (see common features), where the discount factor is implied by the consumer's stochastic discount factor. However, the optimization is inherently dynamic, as it is subject to demand equations that involve habit, and thus internalized law of motion for habit. The statement of the dynamic problem is straightforward and will be omitted.

### 3.3.3 Effects on Economy-wide Feasibility

The model does not imply any additional restrictions on aggregate feasibility constraint.

## 3.4 Sectoral Aggregation

This section develops a setup that introduces a two tier aggregation on the sectoral level, characterized by a different elasticity of substitution across the two levels, which we refer to as commodity level and firm level.

As a result, the price elasticity of demand as perceived by each individual firm selling in a given market reflects the two elasticities to a varying degree, which quite surprisingly turns out to depend on its respective market share. This feature endogenously gives rise to a variation in markups with fairly minimal structure, that blends in well with the way commodities appear in data.

To turn the original formulation due to Atkeson and Burstein (2008), which is a fully blown trade model with heterogeneous producers, we simplify their setup here quite a bit. In contrast to the original paper, we exogenously fix the number of firms in the economy and remove some of the producer heterogeneity. Specifically, we all producers identical within their respective category, and there are two types of categories of intermediate good producers: exporters or non-exporters. To simplify the analysis, there are no non-tradable goods, but there will be non-traded goods by assumption (non-exporters). Productivity of exporters and non-exporters will differ, and exporters will be bigger. None of these features matter qualitatively, and we choose parameters to 'design' a favorable environment to deliver pricing to market. Atkeson and Burstein (2008) show that this structure is broadly consistent with

new generation of trade models.

### 3.4.1 Problem of Importers

On the sectoral level of our model, there is two tier industry that deals simultaneously with both imported and domestic output; it features two levels of aggregation: the highest we refer to as commodity level, and the lowest as firm level. The usual interpretation applies: primitive goods are brands or varieties of a particular type of commodity, while on the industry level we talk here about general commodity types such as computers, cars etc...

The commodity level aggregation is CES, and is characterized by elasticity  $\gamma$ . The crucial assumption here is that this elasticity is relatively low, which will make firms perceive it with a varying degree:

$$D(s^t) + F(s^t) = \left[ \int_0^1 (y(j, s^t))^{1-\frac{1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}. \quad (10)$$

Note that since sectora output can be flexibly converted into sectoral good of type  $D$  or  $F$  depending on demand, the final good aggregator elasticity of substitution has no bite here.

Given prices of compositie commodities,  $P(j, s^t)$ , we observe that the inverse demand of the commodity aggregating firm is given by

$$\frac{P(j, s^t)}{P(s^t)} = \left( \frac{y(j, s^t)}{c(s^t) + i(s^t)} \right)^{\frac{-1}{\gamma}},$$

where  $P(s^t)$  is the price index (lowest cost of acquiring a unit of the final investment/consumption good), given by

$$P(s^t) = \left( \int_0^1 (P(j, s^t))^{1-\gamma} dj \right)^{1/(1-\gamma)}.$$

Recall that this last is normalized in our model to serve as the numéraire, and can be eliminated.

The firm level aggregation is also CES, and aggregates across all domestic and exporting firms selling in a given market. The crucial assumption here is that the elasticity of substitution is relatively high, again matching up nicely with the idea that differentiation between brands is more elusive than between types of commodities. This elasticity is denoted by  $\rho$ ,

and the respective aggregator is given by

$$y(j, s^t) = \left[ \sum_{k=1}^n (d(k, j, s^t))^{\frac{\rho-1}{\rho}} + \sum_{k=n+1}^{n_X} (f(k, j, s^t))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}.$$

The instantaneous profit function of the firm level aggregating firm is given by

$$P(j, s^t) y(j, s^t) - \sum_{k=1}^n p_d(k, j, s^t) d(k, j, s^t) - \int_0^1 p_f(k, j, s^t) f(k, j, s^t),$$

and the inverse demand for a domestic intermediate good  $k$  in sector  $j$  is

$$\frac{p_d(k, j, s^t)}{P(j, s^t)} = \left( \frac{d(k, j, s^t)}{y(j, s^t)} \right)^{\frac{-1}{\rho}}.$$

The sectoral price index  $P(j, s^t)$  is taken by the producers as given, and it is defined by

$$P(j, s^t) = \left[ \sum_{k=1}^n (p_d(k, j, s^t))^{1-\rho} + \sum_{k=n+1}^{n_X} (p_f(k, j, s^t))^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$

Since the firm's problem is effectively static, note sectoral producers maximize the instantaneous profit function given prices, and subject to the aggregation constraint. Since all firms of the same type across all sectors within a country are identical, the subscripts  $j$  and  $k$  are redundant once we restrict attention to 'type-identical' allocations.

### 3.4.2 Problem of Exporters

We assume that there are  $n$  home and  $n_X$  foreign intermediate good producers operating within each sector  $j$ , indexed by  $k$ . Conversely, within each country and sector,  $n - n_X$  producers sell only at home, and  $n_X$  sell in both markets (by assumptions here, endogenously in the original paper). Clearly, we only refer to the latter producers as exporters, and so unlike in other models not all intermediate producers are exporters here. Producers employ local labor and capital to produce their respective variety of good. Their unit production costs are given by (5) and are the same across varieties within their respective country of residence. They are Cournot competitors.

The problem of the domestic country producer  $k$  selling in sector  $j$  of in the home country

market is given by choosing price  $p$  and quantity  $d$  to maximize profit (at all  $s^t$ ):

$$\pi_d(j, s^t) = (p - v(s^t)) d$$

subject to demand equations implied by sectoral and final good producer demands

$$p = \left( \frac{d}{y(j, s^t)} \right)^{-\frac{1}{\rho}} \left( \frac{y(j, s^t)}{c(s^t) + i(s^t)} \right)^{-\frac{1}{\gamma}}.$$

The solution of the above maximization problem implies:

$$p = \frac{\varepsilon(j, s^t)}{\varepsilon(j, s^t) - 1} v(s^t),$$

where the elasticity  $\varepsilon(j, s^t)$  is a function of the market share of intermediate good producer  $k$  and involves the choices of all other producers (Cournot competition):

$$\varepsilon(j, s^t) = \left[ \frac{1}{\rho} (1 - S(j, s^t)) + \frac{1}{\gamma} S(j, s^t) \right]^{-1}$$

where

$$S(j, s^t) = \frac{p_d(k, j, s^t) d(k, j, s^t)}{\sum_{\kappa=1}^n p_d(\kappa, j, s^t) d(\kappa, j, s^t) + \sum_{\kappa=1}^{n_X} p_f(\kappa, j, s^t) f(\kappa, j, s^t)}$$

The maximization for other markets is defined in an analogous way. The only modification is that to produce for exports, a producer has to incur an exporting cost of  $\tau$  units of own production per unit exported.

### 3.4.3 Effects on Aggregate Feasibility

Feasibility here is  $nd(s^t) + n_X d^*(s^t) (1 + \tau) = z(s^t) k(s^t)^\alpha l(s^t)^{1-\alpha}$ , with analogous conditions applying to other markets.

## 4 Alternative Specifications

### 4.1 Consumer Search\*

For comparison, we include also a version of the model that follows closely the original formulation, which does not readily fit our framework. This version introduces search into the household problem, and hence there are no final good producers in this formulation. The intermediate good producers' problem is identical to the one stated above. In what follows

next, we highlight the key differences with respect to our baseline formulation of the search friction.

The key difference between the previous setup and the original setup is that the search friction requires that households need to search in order to purchase goods  $d$  and  $f$ . Specifically, it is assumed that the household can send measures  $n_d$  and  $n_f$  of searchers who can purchase  $\bar{z}$  units of good  $d$  and  $f$ , respectively. These measures are counted against the total time endowment of the household, and so the total amount of labor entering the utility function is given by

$$l(s^t) = n(s^t) + \theta(d(s^t)/\bar{z} - f(s^t)/\bar{z}),$$

where  $n$  is the time devoted to work in production of goods,  $\theta$  is dis-utility from shopping, and  $\bar{z}$  is the number of units purchased by each searcher (implying  $d = n_d * \bar{z}$ ).

Following the original paper, we use linear utility,

$$u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \kappa l,$$

and introduce shocks to  $\kappa$  (labor-wedge shock). In calibration, the shock is backed out from the data on real wages and consumption, and labor-leisure choice first order condition implied the household's problem,

$$\kappa = \frac{w_t}{P_t} c^{-\sigma}.$$

Business cycle volatility of this object is about 4.5% in the US data, and it allows the model to match closely the volatility of the real exchange rate. Following the original setup, we also include a convex cost of capital adjustment (governed by parameter  $\phi$ ), which tames the otherwise too high volatility investment.

We should emphasize that while qualitatively similar, the calibration of this model crucially differs from the baseline specification outlined before. Unlike in our version of the setup, here shopping time is counted as home production rather than market production. This imposes an additional restriction on the parameter values, requiring, for example, that search time is consistent with the American Time Use Survey (ATUS) number of about 25% of work time. Our formulation allows for a more general interpretation that includes B2B search.

## 4.2 Costly Distribution\*

Here, we extend the baseline friction outlined above to incorporate non-tradable goods. This specification follows closely the original paper. We include it for comparison purposes, and argue that this departure qualitatively does not matter for our conclusions. For details, refer to the original paper. Here we briefly highlight the key differences with respect to the earlier setup.

Since now each country produces and consumes tradable and non-tradable goods, and the latter are used up in the process of distribution, the pricing formulas additionally include the relative price of non-tradable goods  $P^N$ :

$$\begin{aligned} p_d(i, s^t) &= \frac{\theta}{\theta - 1} v^T(s^t) + \frac{\xi}{\theta - 1} P^N(s^t), \\ p_x(i, s^t) &\equiv x(s^t) p_d^*(i, s^t) = \frac{\theta}{\theta - 1} v^T(s^t) + \frac{\xi}{\theta - 1} x(s^t) P^{N*}(s^t). \end{aligned}$$

The specification of the forcing process is sector specific, and so the baseline period of the later quantitative specification of this model is 1 year.

## 5 Discussion of Included PTM Mechanisms

Here we present a qualitative analysis of pricing-to-market mechanism of each model.

### 5.1 Prices and Markups in Costly Search Model

Formulas for prices in the *Costly Search* model depend on the local search cost. Specifically, the export and home price of the domestic good in the model are given by

$$p_x(s^t) = v(s^t) + \frac{\theta q}{1 - q} x(s^t) v^*(s^t) \quad \text{and} \quad p_d(s^t) = v(s^t) + \frac{\theta q}{1 - q} v(s^t).$$

By dividing these two prices, we can link the the deviations from the law of one price in the model to the cost-based real exchange rate  $x(s^t) v^*(s^t) / v(s^t)$  by

$$p_d^x(s^t) \equiv \frac{p_x(s^t)}{p_d(s^t)} = \frac{1 + \frac{\theta q}{1 - q} \frac{x(s^t) v^*(s^t)}{v(s^t)}}{1 + \frac{\theta q}{1 - q}}.$$

The above equation implies that the magnitude of pricing to market generated by the model crucially depends on the level of producer markups, here given by  $\frac{\theta q}{1 - q}$ . Below, we discuss

the quantitative implication of this feature of the model for different levels of (steady-state) markups.

### 5.1.1 Implications for PTM

As noted in the previous section, the model can generate pricing-to-market and incomplete pass-through of exchange rates due to costly search denominated in local units. Here we assess the size of these effects by deriving the theoretical pass-through coefficient implied by the model. We do so by evaluating the elasticity of the real export price with respect to the real exchange rate, which is given by:

$$PT \equiv \frac{\partial \log(p_x)}{\partial \log(x)} = \frac{\frac{\theta q}{1-q} \frac{v^*(s^t)x(s^t)}{v(s^t)}}{1 + \frac{\theta q}{1-q} \frac{x(s^t)v^*(s^t)}{v(s^t)}} \Big|_{ss} = \frac{\frac{\theta q}{1-q}}{1 + \frac{\theta q}{1-q}}.$$

By definition, this number tells us by how much in percentage terms the real export price moves in response to a 1% change of the real exchange rate  $x$ .

As we can see, the theoretical pass-through in the model crucially depends on the level of producer markups, captured by the ratio  $\frac{\theta q}{1-q}$ . In particular, for markups equal to 30%, we find that the PT coefficient generated by the model is 24%, for the level of markups equal to 50%, it is 33%, and for the level of markups as high as 100%, it is 50%.

The evidence on the degree of pass-through varies widely in the literature, but most studies, while controlling for costs and other factors, estimate the empirical pass-through coefficient to be in the interval<sup>16</sup> 35%-50%. We refer to this range as the ‘consensus view’. The calculation above suggests that for the model to be consistent with the consensus range of estimates, it requires markups of at least 50%. This is a relatively high number given the number typically used in the literature — but may not be in the presence of large fixed or sunk costs of production. However, we can say that such features should be identified as a necessary precondition for pricing to market. From the business cycle theory perspective, it is not clear how these features would then restrain models’ performance in terms of business cycle statistics, and additional theoretical and empirical research is required here.

<sup>16</sup>Most estimates are centered around 60% for import prices, which would imply 40% to export prices (=100%-60%). For example, Campa & Goldberg (2005) find pass-through to import prices in OECD to be around 46%. Goldberg & Knetter (1997) report number closer to .6 for import prices and thus .4 for export prices.

## 5.2 Prices and Markups in Costly Distribution Model

As we can see from (8), the formulas for prices in the case of the *Costly Distribution* model are similar to the ones implied by the *Costly Search* model. However, there is one crucial difference that we should point out. In the case of *Costly Distribution*, as opposed to *Costly Search*, only part of the markup is denominated in the local numéraire units, reducing pricing-to-market implied by the model. This conclusion follows immediately from an analogous derivation of the theoretical exchange rate pass-through to export prices, which in this case is given by

$$PT \equiv \frac{\partial \log(p_x)}{\partial \log(x)} = \frac{\frac{\xi}{\theta-1} \frac{x(s^t)v^*(s^t)}{v(s^t)}}{1 + \frac{\xi}{\theta-1} \frac{x(s^t)v^*(s^t)}{v(s^t)}} \Big|_{ss} = \frac{\frac{\xi}{\theta-1}}{1 + \frac{\xi}{\theta-1}},$$

and is determined by a term that is lower than the total producer markup<sup>17</sup> given here by  $\frac{\theta}{\theta-1} + \frac{\xi}{\theta-1}$ .

## 5.3 Prices and Markups in Sectoral Aggregation Model

The key implication of this setup is the perceived demand elasticity by the monopolistic intermediate good producers depends on market shares, which varies the markup. Specifically, the export price of the home good is given by

$$p_x(s^t) \equiv x(s^t)p_d^*(s^t) = \frac{\varepsilon_d^*(s^t)}{\varepsilon_d^*(s^t) - 1} (1 + \tau) v(s^t)$$

where

$$\begin{aligned} \varepsilon(s^t) &= \left[ \frac{1}{\rho} (1 - S_d^*(s^t)) + \frac{1}{\gamma} S_d^*(s^t) \right]^{-1} \\ S_d^*(s^t) &= \frac{p_d^*(s^t) d^*(s^t)}{n p_f^*(s^t) f^*(s^t) + n_X p_d^*(s^t) d^*(s^t)} \end{aligned}$$

<sup>17</sup>In the original paper by Corsetti and Dedola (2005) the non-tradable sector is assumed less productive than the tradable sector—requiring a lower setting of  $\zeta$ . This assumption, however, does not resolve the issue. In the quantitative model, the share of distribution cost in the final retail price is tightly disciplined by the available estimates by Burstein, Neves & Rebelo (2003) and effectively it does not matter whether it comes from a parameter, or a ‘productivity inflated price’ of the non-tradable input.

while the home price of the same good is given by

$$p_a(s^t) \equiv p_d(s^t) = \frac{\varepsilon_d(s^t)}{\varepsilon_d(s^t) - 1} v(s^t)$$

where ( $\tau$  stands for the iceberg cost of exporting)

$$\begin{aligned} \varepsilon(s^t) &= \left[ \frac{1}{\rho}(1 - S_d(s^t)) + \frac{1}{\gamma} S_d(s^t) \right]^{-1} \\ S_d(s^t) &= \frac{p_a(s^t) d(s^t)}{n p_d(s^t) d(s^t) + n_X p_f(s^t) f(s^t)}. \end{aligned}$$

As we can see from the above formulas, markup in each market is crucially influenced by the perceived endogenous elasticity of demand  $\frac{\varepsilon^*}{\varepsilon^* - 1}$  in the export market and  $\frac{\varepsilon}{\varepsilon - 1}$  in the domestic market, respectively. Any variation in markups in the model will hence work through the variation in these elasticities. Formally, elasticities will co-vary with the term  $\frac{1}{\rho}(1 - S_d^*) + \frac{1}{\gamma} S_d^*$ , as is clear from the expressions above. How large this variation will be crucially depends on the difference between elasticities  $\gamma$  and  $\rho$  (for a given change in  $S_d$ ), and the variability of the market share  $S_d^*$  over the business cycle.

To illustrate this point, following Atkeson and Burstein (2008), we explicitly derive the elasticity of exporter's markup with respect its market share:

$$\frac{d \log\left(\frac{p_x}{v}\right)}{d \log(S_d^*)} = \frac{S_d^* \left(\frac{1}{\gamma} - \frac{1}{\rho}\right)}{1 - \frac{1}{\rho}(1 - S_d^*) - \frac{1}{\gamma} S_d^*}.$$

As is clear from the formula, the relative size of the exporting firm is important prerequisite for PTM to arise, as the elasticity of the markup with respect to the market share turns out strictly increasing in market share  $S_d^*$ . This implies that having large firms in the model makes markups sensitive to changes in their market share, amplifying pricing-to-market. The other element that drives PTM is the home-bias in trade — the fact that overall the share of good  $d$  firms is much smaller abroad than at home. Home bias is important because it makes the market share of a firm abroad more sensitive to change in the quantity sold, as other firms are not doing the exact same thing at exact same time. This should all be clear from the included formula for  $S_d^*(s^t)$ , according to which the numerator must increase relative to denominator following the shock.

Atkeson and Burstein (2008) quantitatively meet the above requirement for PTM by

matching the size distribution of firms in the data, and the share of exporting firms abroad (home-bias). In our quantitative analysis, since all firms are identical, we satisfy ‘the large exporting firm’ requirement by assuming the most extreme possibility of just one exporting firm ( $n_X = 1$ ). In addition, the exporting firm is relative small at home in our model, which is less so in Atkeson and Burstein’s calibration. This approach is consistent with the idea of finding the upper bound of PTM this model is capable of delivering.

We next investigate the potential of this model to deliver a plausible range of pricing to market.

### 5.3.1 Implications for PTM

As noted above, market share movements in response to exchange rates are critical for pricing to market to arise in this model. In this section, we ask how large these market share movements need to be for the model to be consistent with the consensus estimates of the empirical pass-through of exchange rates into export prices of at least 35%, and the overall business cycle frequency volatility of the real exchange rate of 3.97% (US quarterly data, 1984-2009).

To answer this question, we assume that only market shares co-vary with the real exchange rates—an assumption justified by the fact that other components of the prices are typically controlled for in the empirical estimates of exchange rate pass-through.

The next note that the theoretical pass-through coefficient implied by the model (at the steady state) is

$$PT \equiv \frac{d \log p_x}{d \log x} = \frac{S_d^* \left( \frac{1}{\gamma} - \frac{1}{\rho} \right)}{1 - \frac{1}{\rho} (1 - S_d^*) - \frac{1}{\gamma} S_d^*} \bigg|_{ss} \frac{d \log \hat{S}_d^*}{d \log x},$$

Now, by plugging in the required value of pass-through ( $PT = 0.35$ ), we can obtain the lower bound for the elasticity of market shares with respect to exchange rates

$$\frac{d \log \hat{S}_d^*}{d \log x} \geq 0.35 \times \frac{1 - \frac{1}{\rho} (1 - S_d^*) - \frac{1}{\gamma} S_d^*}{S_d^* \left( \frac{1}{\gamma} - \frac{1}{\rho} \right)} \bigg|_{ss}.$$

The right-hand side of the above formula tells us how much the foreign market shares must move (in percentage terms) in response to a 1% change of the real exchange rate change to deliver exchange rate pass-through of 35% to export prices. Note that the derived lower bound depends only on the elasticity parameters and steady state market shares. Hence, we

can evaluate it using the values of calibrated parameters and the targeted aggregate import shares.

Since higher  $S_d^*$  helps the model to generate more PTM, in order to give it the best chance, we follow our parameterization by assuming  $n_X = 1$ —implying that the market share of an average exporting firm is equal to an aggregate import share of a country. Furthermore, we choose sectoral import share  $S_d^*$  of 16.5% based on the numbers reported by Atkeson and Burstein (2008)<sup>18</sup>, as well as their elasticity parameters<sup>19</sup>.

Given the aforementioned values of all the parameters, our evaluation implies that the market shares in the model must be at least 1.8 more volatile than the real exchange rate. As mentioned above, we think that this is the most favorable calculation to the model, as the same estimate using our baseline parameter values is as high as 4.4 ( $S_d^* = .12, \rho = 8.7, \gamma = 1.52$ ). Comparing to the data, 1.8 is still a bit too high. In the US data, the relative volatility of the (aggregate) import share relative to the real exchange rate is about 1.20<sup>20</sup>.

Our conclusion is that this friction can deliver high levels of exchange rate pass-through to export prices for a reasonable range of producer markups, but on the business cycle frequency, this implication is subject to the caveat of volatile market shares.

## 5.4 Prices and Markups in Deep Habit Model

The formulas for prices in the *deep habit model* model depend on the target market specific shadow value of habit  $\psi$ . Specifically, the real export and home prices of good  $d$  are given by

$$p_x(s^t) = v(s^t) + \Delta_d^*(s^t) - (1 - \rho)\psi_d^*(s^t) \quad \text{and} \quad p_d(s^t) = v(s^t) + \Delta_d(s^t) - (1 - \rho)\psi_d(s^t),$$

where

$$\Delta_d(s^t) = \frac{p_d(s^t)}{\phi} \quad \text{and} \quad \Delta_d^*(s^t) = \frac{x(s^t)p_d^*(s^t)}{\phi}, \quad (11)$$

<sup>18</sup>Average of exports and imports of manufacturing goods in the US divided by gross manufacturing output

<sup>19</sup>In AB, these elasticities have been chosen to align the quantitative model with the producer markups of 30% and an equal expenditure share across industries. In our specification, we target the same level of markups and predicted theoretical pass-through of 40%.

<sup>20</sup>To calculate this number we have rescaled nominal series for GDP by a constant equal to the share of non-service sectors in GDP (for year 2000). Then, we have subtracted exports of goods to obtain the domestic absorption as measure of domestic sectoral output. For imports we have used series for imports of goods to the US. The time period is 1984:2009. Volatility of the real exchange rate for this period is 3.97% (IMF IFS), and it has been used to relate it.

and

$$\psi_d(s^t) = \sum_{s_{t+1}} Q(s_{t+1}, s^t) \mu(s^{t+1}|s^t) \left[ \rho \psi_d(s^{t+1}) + \Delta_d(s^{t+1}) \frac{\theta(1-\varphi) \bar{d}(j, s^{t+1})}{h_d(j, s^t)} \right], \quad (12)$$

$$\psi_d^*(s^t) = \sum_{s_{t+1}} Q(s_{t+1}, s^t) \mu(s^{t+1}|s^t) \left[ \rho \psi_d^*(s^{t+1}) + \Delta_d^*(s^{t+1}) \frac{\theta(1-\varphi) \bar{d}^*(j, s^{t+1})}{h_d^*(i, s^t)} \right]. \quad (13)$$

In the above expressions,  $\Delta$  represents the shadow cost of selling an additional unit of output in a given market, and it is given by the loss of markup on existing sales induced by a downward sloping demand faced by the monopolistic producers (exporters).  $\psi$  represents the shadow value of habit induced by the dependence of price on quantity ( $\delta > 0$ ), and due to habit persistence, it is given by a recursive asset pricing equation involving an exogenous depreciation of habit determined by  $1 - \rho$ , and a per unit ‘dividend’ from habit in the form of  $\Delta$ . Note that the expression  $\theta(1-\varphi) \bar{d}(j, s^{t+1})/h_d(j, t)$  that appears in the formula for  $\psi$  is equal to the derivative of the demand function faced by the monopolist w.r.t.  $h_d$ .

As is clear from the above formulas, the dynamics of the shadow value of habit determines pricing to market implications of the model. To see this, note that after substituting  $\Delta$  into the expressions above, the export price is set as a constant markup over the marginal cost net of the shadow value of habit:

$$x(s^t) p_d(s^t) = \frac{\phi}{\phi - 1} [v(s^t) - (1 - \rho) \psi_d^*].$$

In fact, a good analogy of  $\psi$  would be Tobin’s  $q$  in the investment literature, except that here habit is accumulated by selling more goods.

What then determines the evolution of the shadow value of habit? Qualitatively, many factors, including the discount factor and future evolution of prices; but, quantitatively, the following effect seem most relevant. When the marginal cost  $v$  falls persistently in the model, which is the case after a positive productivity shock at home, the home producers realize that they will be likely selling more today and into the future. This will apply to both markets, but due to initial home bias, while lower in absolute terms, increase of sales will be larger abroad. As a result, due to higher expected future sales, the value of habit persistently goes up in absolute terms and in relative terms abroad (relative to home market). Again, the reason is exactly analogous to why in  $q$ -theory of investment  $q$  goes up after a positive

productivity shock. Since there is a need for more capital, frontloading accumulation is an optimal response — which boosts the  $q$ . Here we can see the same effects by observing that the term  $\theta(1 - \varphi)\bar{d}(j, s^{t+1})/h_d(j, s^t)$  goes up because  $\bar{d}$  does. Note that this term corresponds to the marginal product of habit, and it goes up both in absolute terms and in relative terms more in the foreign market.

#### 5.4.1 Implications for PTM

With productivity shocks, the model generates a *negative* pass-through of exchange rates to export prices. After a persistent positive productivity shock in the home country, home firms expect to sell more abroad in the future (in relative terms), which makes habit abroad more valuable today (in relative terms), and results in lower markups on exported goods relative to goods sold at home. The same conclusion applies when the underlying shocks are standard demand shocks multiplying the utility function of a representative consumer. The only known exception is the specification of demand shocks proposed by Ravn et al. (2007)<sup>21</sup>.

Based on this analysis, we conclude that the model requires a particular correlation between real exchange rate movements and the value of habit<sup>22</sup>.

## 6 Parameterization

All models are parameterized in a uniform way (whenever possible). The common targets we use to parameterize the models are: (i) imports/GDP ratio of 12% (US data), (ii) 30% producer markups, (iii) 30% work hours relative to time endowment, and (iv) short-run elasticity of trade flows of 0.7 (except for the *industry aggregation* friction; see Appendix for details).

The parameters  $\beta, \sigma, \delta, \alpha$ , are parameterized in a standard way. Productivity shock process is common across all models, and symmetric across countries. The persistence parameter is .91, volatility of measured TFP residuals is 0.00608, and the correlation between measured TFP residuals is .28. In the case of Corsetti & Dedola (2005) the process has been taken directly from Corsetti, Dedola & Leduc (2008). (See the Appendix for more details.)

<sup>21</sup>In the setup proposed by Ravn et al. (2007) additive demand shocks generate incomplete pass-through because they are critically combined with an additional habit formation imposed on the government consumption generating demand shocks. As a result, government expenditures creates an externality on lower markups in the entire economy. This effect flips the correlation of the real exchange rate with the shock—which in this case depreciates following a positive demand shock at home rather than appreciates.

<sup>22</sup>We do not report here the results from the additive formulation of the *deep habits* model, but we have studied this variation as well, and the same conclusions apply.

Table 1: Parameter Values in the Models.

Parameter	Baseline	Variation (model w/ asterisk)
Common parameters		
$\sigma$	2.0	
$\beta$	0.99	
$\alpha$	0.36	
$\delta$	0.025	
Price Dispersion, Price Dispersion*		
$\eta$	0.325	n.a.
$\omega$	0.643	0.7562
$\theta$	1.73	11.193
$\bar{z}$	n.a.	5.46
$\gamma$	2.0	1.76
$q$	0.148	0.1051
$\phi$	n.a.	8.5
$\kappa$	n.a.	0.033
Deep Habits		
$\theta$	-0.1	
$\rho$	0.85	
$\gamma$	0.675	
$\bar{l}$	10	
$\varphi$	3.48	
Costly Distribution, Costly Distribution*		
$\omega$	0.6680	n.a.
$\theta$	8.7	8.0
$\xi$	1.33	0.8
$\gamma$	1.63	1
$\delta$	as above	.1 (annualized)
$\beta$	as above	.96 (annualized)
$\bar{z}^N$ (rel. prod. of NT-sector)	n.a.	0.5
Industry Aggregation		
$n_D$	4	
$n_X$	1	
$\rho$	8.7	
$\gamma$	1.52	
$\tau$	0.1525	

We now turn to the description of the specifics of the calibration unique to each friction. The values of all parameters are summarized in Table 1.

**Costly Search** We parameterize this friction by requiring that the following additional targets from the original paper are met: (i) shopping to time spent working of 25% as reported by Alessandria (2009), and (ii) coefficient of variation of the posted prices of 25%.

Because our baseline specification of the model differs from the original paper, we no longer target the share of search in time endowment of households as Alessandria (2009) does. An analog of this share in our model is the fraction of GDP that is used in the distribution sector. In our setup, it is endogenously implied by the model, and accounts for 60% of the total value added produced in the economy. Since higher values help the model, and we are seeking an upper bound in terms of PTM implications, we adopt this number. Nevertheless, one should keep in mind that this number is a bit large, for example, relative to the size of retail and distribution sectors as a whole (38% of total value added in non-service sector in the US data for year 2000).

**Consumer Search\*** The parameterization of this setup follows closely the original paper. All targets of the original paper are met, but the exact values of the parameters are slightly different because the value of the common parameters are different.

**Costly Distribution** We parameterize the friction by requiring that distribution costs constitute 50% share of ‘non-tradable’ inputs in retail prices (as implied by Burstein, Neves & Rebelo (2003)).

**Costly Distribution\*** The model has been parameterized similarly as in the original paper. The baseline period is one year, and all parameters have been adjusted to this frequency. The stochastic process has been taken from Corsetti, Dedola & Leduc (2008). All targets of the original paper are met and all main parameters take the same value. However, unlike the original model, this version of the model does not include monetary shocks and sticky wages.

**Sectoral Aggregation** The most important parameters for pricing-to-market are: the number of firms chosen in the model, and the difference between the elasticities,  $\gamma$  and  $\rho$ . In terms of the number of firms, as pointed out by Atkeson & Burstein (2008), the presence of

large firms is crucial for generating pricing to market. In the original paper, there is a large number of firms (40), but firm size heterogeneity implies that only a few large ones really matter for (export) prices. In our specification of their friction, due to assumed representativeness, all firms need to be of equal size. Therefore, we set the total numbers of firms to a low number of firms ( $n = 4$ ), and make an extreme assumption that only one firm exports ( $n_X = 1$ ) firm that exports. The setting of the two parameters is consistent with the ratio ( $\frac{n_X}{n}$ ) taken from AB (in fact, this ratio lead us to choose  $n=4$ )<sup>23</sup>.

To calibrate the value of the elasticity parameters  $\gamma$  and  $\rho$ , we note that the difference between these elasticities maps onto the degree of pass-through generated by the model, and their weighted average determines the average level of producer markups. Consequently, we choose these numbers to match a coefficient of pass-through of exchange rates to export prices of 40% and producer markups of 30% (as in the original paper).

**Deep Habits** As for the habit parameters,  $\theta$  and  $\rho$ , we adopt the values from Ravn et al. (2006).

## 7 Quantitative Analysis

### 7.1 Data

To compare prices in the models to the data, we focus here on selected moments characterizing the dynamics of export prices. Our moments are motivated by the following decomposition of the export price movements:

$$p_x \equiv \frac{P_X}{P} = \underbrace{\frac{P_X}{P_D}}_{p_d^x} \times \underbrace{\frac{P_D}{P}}_{p_d}, \quad (14)$$

where  $P_X$  denotes the home currency based price of exported good (basket),  $P_D$  denotes a home currency based producer price of a comparable tradable good (basket) sold at home, and  $P$  is some measure of the overall price of aggregate consumption in the home country (we chose CPI for convenience).

This decomposition allows us to split the movements of the export prices into pure deviation from LOP  $p_d^x$ , and the residual relative price movements specific to the home market  $p_d$ .

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<sup>23</sup>It would be possible to incorporate several different firm sizes, but we conjecture that this would not matter much for the result.

Table 2: Moments summarizing deviations from LOP.

Statistic	Description
$\sigma(p_d^x)/\sigma(x)$	Relative magnitude of deviations from LOP
$\sigma(p_d)/\sigma(x)$	Relative volatility of the price of tradable goods at home
$\rho(p_d^x, x)$	Correlation of dev. from LOP w/ the real exchange rate
$\rho(p_d, x)$	Correlation of home prices w/ the real exchange rate

$\sigma$  denotes the standard deviation of logged and HP filtered data;  $\rho$  denotes the correlation coefficient.

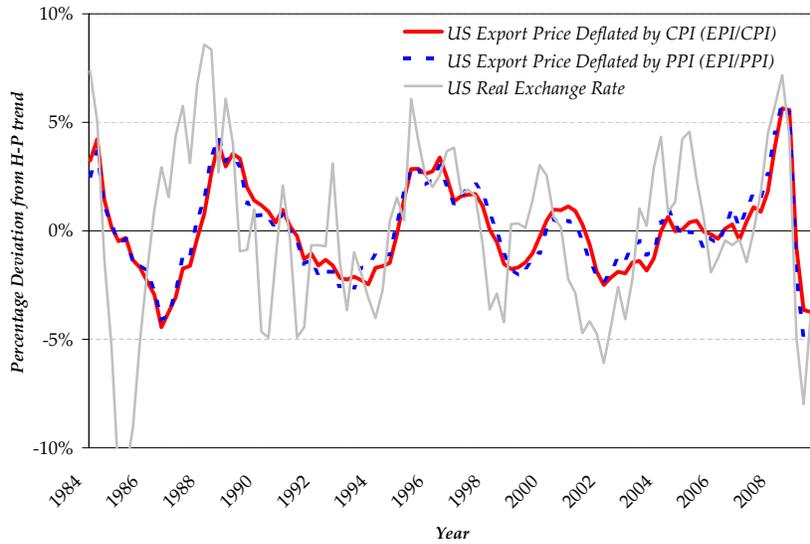


Figure 1: Dynamics of Aggregate Export Price in the US.

In most models,  $P_X$  and  $P_D$  will correspond to the price of the same commodity, and so  $p_d^x$  will be a good test of the model's capability to generate deviations from LOP. As we will see below, in the data  $p_d^x$  is highly volatile and highly positively correlated with the real exchange rate. Standard models imply that this term is constant, and all our friction move it in some way. The particular moments of the data we focus on are listed in Table 2.

Table 3: Deviations from LOP in Aggregate Data.

Statistic	Value
<i>A. Properties of Aggregate Real Export Price</i>	
$\sigma(p_x)/\sigma(x)$	.52
$\rho(p_x, x)$	.47
<i>B. Deviations from LOP</i>	
$\sigma(p_d^x)/\sigma(x)$	.53
$\rho(p_d^x, x)$	.51
<i>C. Residual</i>	
$\sigma(p_d)/\sigma(x)$	.13
$\rho(p_d, x)$	-.18

$\sigma$  denotes the standard deviation of logged and HP filtered data,  $\rho$  denotes the correlation coefficient.

### 7.1.1 Aggregate Evidence

To document properties of prices in the aggregate data, we first use US data on producer export price index (EPI) from BEA to measure export prices, and a comparable index for producer prices of finished products excluding food and energy (PPI) to measure home prices of comparable tradable basket.<sup>24</sup> We have excluded food and energy because these components are likely to be volatile for reasons unrelated to business cycle fluctuations. To measure the price of aggregate consumption, we use CPI that excludes food and energy for consistency (the results do not depend on this). All our data is quarterly for the time period (1983-2010), and statistics are based on first logged and then HP filtered time series (smoothing 1600). We do not look at other countries here, but the facts and regularities that we use have been widely documented in the literature and we omit any additional robustness checks on this basis.

Table 3 summarizes all the results, which we also illustrate in Figure 1. Panel A of table 3 shows that aggregate real export price in the US is relatively volatile, and also highly positively correlated with the real exchange rate. On the basis of our decomposition, and the statistics included in panel B of Table 3, we conclude that most of the movements in the real export price are attributable to the deviations from LOP. In fact, the deviations from LOP captured by the relative price  $p_d^x$  are at least 4 times more volatile than the residual  $p_d$ , and are almost the sole driver of the observed positive correlation between the aggregate

<sup>24</sup>The PPI data confounds both export prices and domestic prices, and is only an approximate measure of  $P_D$ . However, for our particular application this is sufficient, as it will only imply that our conclusions establish the lower bound for the underlying deviations from LOP implied by the aggregate data.

Table 4: Deviations from LOP in Disaggregated Data.

Statistics	Median Value	Quartile [ $Q_1, Q_3$ ]	bracket
<i>A. Properties of disaggregated real export prices</i>			
$\sigma(p_{x,i})/\sigma(x)$	.88	[ .54, .99 ]	
$\rho(p_{x,i}, x)$	.82	[ .50, .89 ]	
<i>B. Deviations from LOP</i>			
$\sigma(p_{d,i}^x)/\sigma(x)$	.90	[ .55, .99 ]	
$\rho(p_{d,i}^x, x)$	.84	[ .67, .89 ]	
<i>C. Residual</i>			
$\sigma(p_{d,i})/\sigma(x)$	.23	[ .11, .33 ]	
$\rho(p_{d,i}, x)$	-.14	[ -.25, .07 ]	

$\sigma$  denotes the standard deviation of logged and HP filtered data,  $\rho$  denotes the correlation coefficient.

real export price and the real exchange rate. These findings are consistent with the idea of a positive but incomplete pass-through of exchange rates to export prices.

### 7.1.2 Supporting Micro Evidence

To support our aggregate findings, here we offer an alternative look at the disaggregated data at a commodity classification level. The data-set we use comes from Bank of Japan and includes prices of 30 most heavily traded manufacturing commodities disaggregated at roughly 4 digit level<sup>25</sup>. The prices come from a producer level survey of the actual contractual agreements. The time period is 1995-2005. Again, this dataset is at this point one of many that researchers have used, and findings of even much finer datasets are broadly consistent with our findings here.

Table 4 reports the results. As we can see, the patterns on the commodity level are consistent with the aggregate evidence listed in the previous table. Real export prices of the majority of commodity classifications are volatile and highly positively correlated with the Japanese real exchange rate, just like the aggregate price was. Moreover, the analogous decomposition into a set of component statistics reveals that the movements of export prices are attributable to deviations from the law of one price. The relative price  $p_{x,i}^r$  is highly volatile and positively correlated with the Japanese real (effective) exchange rate, whereas

<sup>25</sup>The examples of included commodity categories are: computers, ball bearing, agricultural tractors, silicon wafers etc...

the home price of the same commodity classification is much less volatile, and actually slightly negatively correlated with the real exchange rate.

We next turn to the quantitative analysis of the calibrated models.

## 7.2 Quantitative Findings

Quantitative predictions of the parameterized models are reported in Tables 5 and 6. As we can see from Table 5 (Row 4 of Panel B), all models generate some degree of pricing-to-market, as expected. Overall, compared to US data, most statistics look well qualitatively, but the models fall short in generating enough incompleteness of pass-through. Furthermore, as pointed out before, the Ravn, Schmitt-Grohe & Uribe (2006) model generates pricing to market that gives rise to generally counterfactual correlations of the aggregate price indices (Panel A of Table 5).

Below, we briefly discuss each friction's quantitative predictions.

**Costly Search and Consumer Search** This calibration of the model generates more pricing to market as compared to the *Costly Distribution* friction. As discussed above, this is a robust feature of this friction and comes from the fact that in the case of *Consumer Search* the entire markup is denominated in the foreign unit, as opposed to only part of it in the case of *Costly Distribution*.

Despite a low degree of theoretical pass-through predicted by the model, the model implies large deviations from LOP as measured by the relative price:  $p_d^x$ . At the same time, the real export price  $p_x$  is not that volatile relative to the volatility of the real exchange rate  $x$ . The reason for that is that the model implies a relatively volatile home marginal cost  $v$  relative the real exchange rate  $x$ . As we can see from column 4 of Table 5, *Consumer Search*\* model resolves this issue by introducing an additional shock, and by matching the volatility of the real exchange rate. We conclude that this particular property is more of an artifact of too low volatility of the real exchange rate relative to marginal cost.

The baseline model does exceptionally well in accounting for the quantity-related statistics. However, part this success is can be attributed to our particular specification of the friction that effectively makes a large share of GDP to be a non-tradable distribution/search sector. In our baseline specification more than half of the GDP is accounted for by the value

Table 5: International Prices: Comovement and Relative Volatility<sup>a</sup>

Predictions of the PTM Theories									
Statistic	Data <sup>b</sup>	Perfect Competition	Costly Search	Consumer Search*	Costly Distribution	Costly Distribution*	Costly Distribution* <sup>d</sup>	Sectoral Aggregation	Deep Habits
<i>A. Correlations</i>									
$p_x, x$	0.47	-1.00	1.00	0.82	1.00	0.52	0.98	0.98	-0.95
$p_d^x, x$	0.51	0.02	1.00	0.98	1.00	0.98	1.00	1.00	-0.83
$p_d, x$	-0.18	-1.00	-1.00	0.11	-1.00	0.27	-1.00	-1.00	-0.97
<i>B. Standard deviations relative to<sup>c</sup> <math>x</math></i>									
$x$	3.97	0.45	0.55	3.67	0.54	1.86	0.31	0.31	0.51
$p_x$	0.52	0.16	0.18	0.26	0.04	0.83	0.23	0.23	0.36
$p_d^x$	0.53	0.00	0.30	0.20	0.17	0.24	0.35	0.35	0.12
$p_d$	0.13	0.16	0.12	0.14	0.13	0.76	0.11	0.11	0.17
<i>C. X-Rate Pass-through</i>									
	35%-50%	0%	23%	18%	15%	20%	40%	40%	n.a.
<i>D. Producer Markups</i>									
	30%	0%	30%	22%	30%	40%	30%	30%	30%

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>Data for the US, 1984:1-2009:4.

<sup>c</sup>Ratio of corresponding standard deviation to the standard deviation of the real exchange rate  $x$ .

<sup>d</sup>The model has been calibrated to annual frequency and the statistics generated are not readily comparable to the ones listed in data column.

Table 6: Quantities - Comovement and Relative Volatility<sup>a</sup>

Predictions of the PTM Theories									
Statistic	Data <sup>b</sup>	Perfect Competition	Costly Search	Consumer Search*	Costly Distribution	Costly Distribution* <sup>d</sup>	Sectoral Aggregation	Deep Habits	
<i>A. Correlations</i>									
<i>domestic with foreign</i>									
<i>Measured TFP</i> <sup>c</sup>	0.30	0.30	0.34	0.44	0.33	0.54	0.31	0.30	
GDP	0.40	0.36	0.40	0.50	0.38	0.56	0.17	0.37	
Consumption	0.25	0.33	0.30	0.61	0.32	0.54	0.73	0.40	
Employment	0.21	0.49	0.52	0.43	0.50	0.55	0.05	0.55	
Investment	0.23	0.19	0.24	0.56	0.23	0.31	0.32	0.26	
<i>GDP with</i>									
Consumption	0.83	0.95	0.95	1.00	0.96	0.99	0.94	0.95	
Employment	0.85	0.98	0.98	0.96	0.98	0.91	0.99	0.97	
Investment	0.93	0.67	0.67	0.73	0.67	0.45	0.64	0.67	
Net exports	-0.49	-0.57	-0.54	-0.49	-0.56	-0.87	0.58	-0.56	
<i>Terms of trade with</i>									
Net exports	-0.17	-0.84	-0.86	-0.86	-0.86	-0.77	0.98	-0.93	
<i>B. Standard deviations</i>									
<i>relative to GDP<sup>d</sup></i>									
GDP	1.33	1.13	1.18	2.45	1.15	2.05	1.55	1.08	
<i>Measured TFP</i>	0.60	0.70	0.67	0.32	0.69	0.89	0.51	0.74	
Consumption	0.74	0.33	0.36	0.84	0.36	0.78	0.44	0.38	
Investment	2.79	3.24	3.76	2.76	3.77	3.00	2.78	3.81	
Employment	0.81	0.47	0.50	1.30	0.47	0.17	0.75	0.41	
Net exports	0.30	0.14	0.13	0.04	0.13	0.11	0.21	0.13	

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>US data for the period 1980:1-2004:1.

<sup>c</sup>Calculated using the actual national accounting formulas; due to time varying markups measured TFP slightly differs from the TFP coefficient fed into the models.

<sup>d</sup>Ratio of corresponding standard deviation to the standard deviation of *GDP*.

<sup>e</sup>The model has been calibrated to annual frequency and the statistics generated are not readily comparable to the ones listed in data column.

added generated in the final good sector (search). Such large share would help all models to come closer to the data in this respect.

In contrast, the *Consumer Search*\* model, by treating shopping as home production and by adding an extra shock, is no longer neutral to quantities. The model results in excess output volatility and excess employment volatility.<sup>26</sup> Our conclusion is that the *Consumer Search* specification is not fully neutral to business cycle dynamics of quantities.

**Costly Distribution, and Costly Distribution\*** The model results are similar to *Price Dispersion* model, but due to a slightly different dependence of export prices on markups, the performance of the *Costly Distribution* model for prices is strictly worse.

**Sectoral Aggregation** While the model does well on prices, that is, it generates a high degree of exchange rate pass-through to export prices, quantity statistics look worse when compared to the standard model. This should not surprise given a high setting of international elasticity between home and foreign goods needed for pricing to market to arise. Specifically, the model implies a too low international comovement of GDP relative to TFP, and an excess international comovement of consumption. In addition, despite capital accumulation being present, it implies a positive correlation of net exports and output, versus a negative one in the data and in the standard model.

Finally, we also note that since pricing to market arises due to market share movements, once the model is required to match real exchange rate volatility from the data, it may be harder than in the case of other models to tame quantities by introducing additional features. To retain the same degree of PTM, for example, it would require market shares to be more volatile by an order of magnitude.

**Deep Habits** The prices look worse than in the standard model. Statistics for quantities look similar to the standard model, and in some dimensions even better.

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<sup>26</sup>Moreover, we have also studied a specification of *Consumer Search* friction featuring CRRA utility function and only productivity shocks (no labor wedge shock), which we do not report here. This specification performed strictly worse than the frictionless model due to a negative international comovement of employment and counterfactually low international comovement of output.

## 8 Conclusions

In this paper, we provide the first unified study of several leading pricing to market models to advance our understanding of the driving forces behind PTM and structure the existing contributions from an applied researcher's perspective. We evaluate these models from the point of view of DSGE models that are required to be consistent with a broad range of statistics, and ask the question how useful are these model to bring prices closer to the data without compromising model performance in other dimensions. We also identify key predictions of these model and requirements for PTM to arise, which we view as useful guide in identifying preconditions for PTM on empirical grounds (e.g. large fixed cost that justify high mark-ups).

# Appendix

## A1. National Accounts in the Model

Real GDP is real GDP in constant prices ( $\equiv$  steady state prices). Since the price of consumption/investment good is normalized to one, consumption and investment in period zero prices are  $c$  and  $i$ . Employment index is measured by  $l_{i,t}$ .

In deflation of the prices and measurement of real exchange rate, we have used the ideal CPI. This does not make any difference except for Price Dispersion friction. In the case of this model, the transacted prices differ from the posted price (CPI measurement should be based on posted prices). However, since in the data such distinction does not make any difference for any of the patterns we focus on<sup>27</sup> we have decided to use the ideal CPI to abstract from any implications of the model coming from this channel.

## A2. Estimation of the Productivity Shock Process

To construct the TFP residuals  $z$  from the data we follow a similar procedure to Heathcote & Perri (2004), and include physical capital. Physical capital has been constructed from the gross-fixed capital formation series using perpetual inventory method with exogenously assumed depreciation rate of  $\delta = 0.025$ . For the US we have used total hours worked, and for the rest of the world civil employment index instead. Given the quarterly data-set from 1980.1 to 2004.3 for the aggregate of main 15 European countries, Japan, Canada, Switzerland, and Australia, we have constructed the series of  $z$  from the following equation

$$\log(z) = \log(y) - 0.36 \log(k) - 0.64 \log(n),$$

where  $y$  denotes GDP in constant prices, and the coefficient 0.64 denotes the assumed share of labor income in GDP - consistent with the parameterization of the model and the values estimated for the developed countries. We linearly detrend the series for  $\log(z)$ , and estimate the parameters of the underlying productivity process. In the case of Corsetti & Dedola (2005) the process has been taken directly from Corsetti, Dedola & Leduc (2008)—which limits our analysis to annual frequency.

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<sup>27</sup>I.e exactly analogous patterns can be documented by instead using PCE deflator or GDP deflator,

### A3. Measurement of Short-run Price Elasticity of Trade

Short-run elasticity of trade flows measures how trade flows between countries respond to a relative price changes seen in the time-series. Here, we use the so called volatility ratio to assess the lower bound for this elasticity.

When the demand for domestic and foreign good is modeled by a CES aggregator of the form

$$G(d_t, f_t) = \left( \omega_t d_t^{\frac{\gamma-1}{\gamma}} + (1 - \omega_t) f_t^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}},$$

the import ratio  $\frac{f_t}{d_t}$  is intimately related to the relative price of domestic and imported goods  $\frac{p_{d,t}}{p_{f,t}}$ .

$$\log \frac{f_t}{d_t} = \gamma \log \frac{p_{d,t}}{p_{f,t}} + \log \frac{\omega_t}{1 - \omega_t}. \quad (\text{A1})$$

Thus, in the case of time-varying weights  $\omega$ , the above approach gives the upper bound value for the value of this parameter:

$$\gamma = \sigma(\log \frac{f_t}{d_t}) / \sigma(\log \frac{p_{d,t}}{p_{f,t}} + \frac{1}{\gamma} \log \frac{\omega_t}{1 - \omega_t}) \leq \sigma(\log \frac{f_t}{d_t}) / \sigma(\log \frac{p_{d,t}}{p_{f,t}}) = VR. \quad (\text{A2})$$

Based on the median value for OECD countries, we this way obtain the upper bound on elasticity  $\gamma$  to be .7 on quarterly frequency. In models with frictions, VR does not map onto  $\gamma$ , and so we instead construct the analogous object in the model, and set  $\gamma$  so that the model implied VR is .7.

### A4. Data Sources

Export prices and PPI data comes from BEA. Real exchange rate data comes from International Monetary Fund, International Financial Statistics Database, 2010. Prices for Japan come from Bank of Japan and have been compiled by the authors from flat files available online. To construct TFP residuals used in the estimation of the stochastic process, we have used nominal GDP data from World Development Indicators, World Bank, Gross Fixed Capital Formation, GDP in constant prices and Civil Employment from Source OECD.org, Quarterly National Accounts, series for physical capital have been constructed using the perpetual inventory method with a constant depreciation of 2.5%, and aggregate GDP for blocks of countries has been computed from growth rates of GDP in constant prices (recent years, varies by country) weighted by the nominal GDP of each country in 2000 (we applied the

growth rates backwards). Statistics pertaining to quantities that appear in the paper have been calculated from the same data.

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