The Trade-Comovement Puzzle*

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Abstract

Standard international transmission mechanism of productivity shocks results in a weak relation between trade and business cycle synchronization relative to the data: a result known as the *trade-comovement puzzle*. We characterize the forces responsible for the puzzle and show the analysis points to two basic modifications that can largely resolve the puzzle without fundamentally changing the model's transmission mechanism or its shock structure: Greenwood-Hercowitz-Huffman (GHH) preferences and low trade elasticity modeled as dynamic elasticity consistent with low short-run and high long-run trade elasticity. We show that financial market frictions are much less effective than these alternatives.

Keywords: trade-comovement puzzle, elasticity puzzle, business cycle comovement, international shock spillovers

JEL codes: E32, F44, F47, F32

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A central question in international macroeconomics is how foreign shocks spillover onto other countries. Empirically, trade linkages have been found relevant, suggesting that trade plays a crucial role in the endogenous transmission of business cycle fluctuations across countries.[1] In an influential paper, Kose and Yi (2006) have shown that this basic proposition is at odds with how productivity shocks are transmitted in the standard international macro model: a result known as the *trade-comovement puzzle.* While the failure of the standard model has been well-documented quantitatively, to date little is known about the underlying mechanism through which trade affects business cycle transmission in the standard theory. In particular, it is not clear which features are critical for the model's failure, and hence which structural modifications may be promising in remedying the issue.

In this paper, we accomplish two goals. First, we close the gap in the literature by providing a foundational analysis of the trade-comovement puzzle; that is, we lay out the basic forces underlying the the puzzle and trace these forces back to the model's structural assumptions. Second, we show the analysis of the puzzle points to two basic modifications of the standard theory that can largely resolve the puzzle without fundamentally changing the model's transmission mechanism or its shock structure: Greenwood-Hercowitz-Huffman (GHH) preferences and low trade elasticity modeled as dynamic elasticity that is low in the short-run but progressively higher in the longer run. We provide a complete quantitative evaluation of these resolutions and show how dynamic elasticity works differently than simply lowering CES elasticity in a standard Armington aggregator – a case considered by Kose and Yi (2006). We conclude that the trade-comovement puzzle is best interpreted as imposing empirically viable parametric and structural restrictions on the standard transmission mechanism rather than rejecting it outright.

The first contribution of our paper is to provide a semi-analytic characterization of the basic forces that govern the effect of trade on standard transmission mechanism of productivity shock across the border. Our baseline analytic setup follows closely the model by Backus et al. (1995) and allows us to explore such features as the role of risk aversion, elasticity of substitution between home and foreign goods or different functional forms for the preferences for consumption and leisure, as well as to examine the distinct roles of capital and labor inputs in shock transmission. We show that the

[1] An extensive empirical literature documents a tight link between bilateral trade intensities and business cycle comovement across countries. By running cross-country regressions, Frankel and Rose (1998), Clark and van Wincoop (2001), Calderon et al. (2002), Otto et al. (2001), Baxter and Kouparitsas (2005), Kose and Yi (2006) and Inklaar et al. (2008) all find that, among bilateral country pairs, more trade is associated with more synchronized business cycle fluctuations. In an important paper, Johnson (2014) shows that the effect of trade on output is prenouced even after controlling for the relation between measured TFP and trade. See also diGiovanni and Levchenko (2010) for a related disaggregated industry-level analysis.

predictions of the model are driven by the interaction of two basic channels of shock transmission that have offsetting effects on the theory-implied trade-comovement relation. We label these channels as the *substitution channel* and the *income channel*.[2]

Consider first the effect of trade via the *substitution channel*. This channel leads to a positive transmission of foreign shocks into home country's output and in the standard theory it is a robust source of a positive relation between the steady state level of trade between countries and the comovement of their output over the business cycle.

Intuitively, due to the built-in complementarity between differentiated home and foreign goods, a positive productivity shock in the foreign country lowers the price of the foreign good relative to the home good. Since consumption and investment in the home country also involves the foreign good, and labor and capital exclusively produce the home good, this raises the relative price of home labor and capital in terms of the home consumption/investment good. Through the usual substitution effect between labor and leisure, and due to lower cost of investment, this raises home country's labor supply, investment and output. Importantly, since trade corresponds to the share of foreign good in home country's consumption and investment spendings, this relative price effect is stronger the more countries trade, implying a positive relation between trade and the strength of the transmission of the foreign productivity shock into home country's output.[3]

Consider now the *the income channel*. With the exception of very low levels of the elasticity of substitution between home and foreign goods, the income channel is a potent source of a negative relation between trade and output comovement. In a usual parameterization of the standard model, the effect of trade via the income channel weakens or even reverses the positive relation between trade and comovement implied by the substitution channel, resulting in the trade-comovement puzzle.

The income channel is brought about by the fact that a positive productivity shock abroad has a positive income effect on the home country due to the appreciation of home country's terms of trade and equilibrium net asset payout associated with international borrowing and lending (in baseline model under complete markets). This effect raises home consumption, and due to the built-in complementarity between consumption and leisure, it reduces home labor supply. In addition, due to built

[2] Our naming convention departs from that introduced by Kose and Yi (2006), who labeled these channels as *complementarity channel* and *risk-sharing channel*, respectively. Since complementarity between home and foreign goods would also be a source of risk-sharing (income channel) even under financial autarky, we chose more generic labels.

[3] Trade also changes the response of terms of trade to shocks. This, however, turns out to be a secondary factor because this second effect is proportional to trade, which in bilateral context is very small (median imports-to-GDP ratio across all bilateral pairs in our sample is 0.85%).

in complementarity between labor and capital, it also reduces investment. Crucially, the magnitude of this effect depends on trade, making it relevant for the trade-comovement pattern. Intuitively, transfers are beneficial in equalizing the marginal utility from consumption across countries, but they are also distortionary because they further contribute to the excess supply of foreign good after the shock in proportion to trade. Asset prices and exchange rates internalize this distortion, and hence trade discourages transfers. Consequently, *more* trade implies *less* comovement.

The second contribution of our paper is to identify the set of modifications of the standard theory that can suppress the effect of trade via the income channel without fundamentally changing the transmission mechanism or the shock structure. In particular, we consider and analyze the following modifications of the baseline theory: i) financial autarky; ii) GHH preferences; and iii) low trade elasticity modeled as a dynamic elasticity that is low in the short-run elasticity but progressively higher in the longer run.[4]

The first modification shuts down endogenous asset trade, and hence suppresses the effect of trade via the income channel. The second modification eliminates the income effect on labor supply, which is at the heart of the offsetting effect of trade via the income channel. Finally, the third modification has the effect of a decaying distortionary effect of the unbalanced supply of the two differentiated goods, since in the longer run home and foreign goods become closer substitutes. This feature delays transfers and crucially interferes with capital accumulation.

We analyze these modifications and evaluate them quantitatively vis-à-vis the baseline model. First, our financial autarky results show that the restrictions on asset trade are unlikely to be quantitatively promising – a finding that echoes that by Kose and Yi (2006) and others. The main issue is that risk sharing is largely driven by the income effect of terms of trade, which financial autarky fails to eliminate, as first shown by Cole and Obstfeld (1991). What we show here is that the income effect of terms of trade is *proportional to trade*, since terms of trade affects the value of imports. Accordingly, terms of trade-implied transfers similarly lead to an offsetting effect of trade via the income channel. The quantitative results confirm this insight: the financial autarky model accounts for only 25% of the trade-comovement relationship in the data, compared to 20% in the baseline complete markets model.

Second, we consider GHH preferences, which turn out to be a more effective way of eliminating

[4]Here we use a simple approach of adding a local convex adjustment cost. This approach can be microfounded via search frictions a la Drozd and Nosal (2012). It can also be interpreted as the effect of deep habit, as in Mazzenga and Ravn (2002).

the offsetting effect of trade via the income channel than asset trade restrictions. In contrast, GHH preferences eliminate the mechanism through which the income channel affects trade-comovement relation, rather than the source of income effects. Quantitatively, we find that GHH preferences can account for as much as 60 percent of the relation between trade and comovement in the data, which more than doubles the strength of the relation vis-à-vis the case of financial autarky and triples it relative to the baseline complete markets setup. With productivity shocks, however, GHH preferences come at a cost of deteriorating the model's ability to deliver countercyclical and volatile current account – which is also too weak in the baseline model.[5]

Finally, we study the implications of low trade elasticity modeled as dynamic elasticity. We find that this modification works best quantitatively. For our target for the short-run trade elasticity of 1.17, which is conservative, the model accounts for 60 percent of the trade-comovement relation. For a less conservative but still viable targets, it can go above 80%. Since this result is independently grounded in trade elasticity evidence,[6] and it improves the model's business cycle performance as far as countercyclicality of net exports is concerned, we conclude that the standard theory can be modified at virtually no cost in terms tractability and performance to effectively address the trade-comovement puzzle.

Intuitively, low trade elasticity generates a positive relation between trade and comovement even in the standard setup by increasing the distortionary effect of transfer-implied excess supply of foreign good after the shock, hence attenuating the negative effect of trade via the income channel. But, as noted by Kose and Yi (2006), lowering the elasticity parameter in the CES Armington aggregator falls short of quantitatively accounting for the trade-comovement puzzle. Dynamic trade elasticity improves upon this benchmark because the aforementioned costly distortion of transfers applies in the short-run but not in the longer run. This, on business cycle frequency, reverses the direction of transfers by incentivizing investment in foreign capital to merely soak up the surplus of foreign goods in the interim. This incentive comes from the fact that, at a later date, the benefits of increased

[5]For more details, see the discussion in Raffo (2008).

[6]Esimates of trade elasticity, that is, elasticity of substitution between imported and domestically produced goods, are widely different depending on the empirical strategy. In particular, trade elasticity measured with respect to permanent tariff changes over a longer time horizon (several years), known as long-run trade elasticity estimates, are quite high (see for example Head and Ries (2001), Eaton and Kortum (2002), Clausing (2001), Anderson and van Wincoop (2004) or Romalis (2007)). In contrast, business cycle frequency estimates that utilize time-series of quantities trade and prices point to much lower trade elasticities (Reinert and Roland-Holst (1992), Blonigen and Wilson (1999)), often below one. Standard CES setup can not account for this discrepancy by implying equal short- and long-run trade elasticities. A convex adjustment cost in our model reconciles these estimates by implying gradual adjustment of trade. For further discussion, refer to Ruhl (2008). For microfoundations of such an approach, refer to Drozd and Nosal (2012).

supply of the foreign good can be more efficiently shared between the countries than in CES case with low elasticity. Crucially, the reason why this affects the model-implied trade-comovement relation is because trade weakens this effect. A negative transfer alleviates rather than exacerbates the oversupply of the foreign good arising after the shock, and as explained earlier, this effect is amplified by trade regardless of its direction. Since here it is beneficial, trade makes transfers more negative and because negative transfers are a source of *positive* rather than negative comovement, more trade implies *more* rather than less comovement.[7]

Our quantitative results are based on an experiment of increasing trade that conceptually follows closely that in Kose and Yi (2006). Specifically, we similarly use the cross-country correlation of output as a measure of comovement and distinguish between trade openness and bilateral trade intensity by introducing a large third country that serves as the relative rest-of-the-world.[8] We calibrate all models to match both trade openness and bilateral trade intensity of a median country pair in our dataset. We parameterize the stochastic productivity process so that the initial comovement of output, output volatility, and its persistence are consistent with the median country and its relative rest of the world. We then consider an experiment of raising trade intensity within the calibrated country pair and also raising trade openness to match the transition from 50th to 90th percentile of country pairs by trade intensity. We evaluate the impact of such a change on the implied correlation of output vis-à-vis those by a regression between trade and comovement across country pairs.[9]

Related literature. — Our paper is closely related to Kose and Yi (2006), who were the first ones to document the trade-comovement puzzle and provided the first analysis of the standard theory. Here, we provide an analytic characterization of the mechanism underlying the puzzle and point out two modifications of the standard theory that go beyond those considered by them. Our paper is also related to the literature that explores the role of other forces that can link trade and comovement. Most notable among these efforts are theories that generate endogenous TFP spillovers that are proportional to trade, from which we abstract and which are independently relevant. In this regard,

[7]As an additional effect, when there is more trade, foreign capital is no longer as useful to soak up the excess of foreign goods in the short-run. This leads to even more asymmetric responses which in the future result in bigger imbalance in the supply of the foreign good. Having dynamic elasticity also weakens this effect, which further enhances the result.

[8]Our modeling of trade assumes more standard specification of having different preference weights rather than quadratic iceberg costs.

[9]Similar results are obtained by generating all 190 pairs in the model by matching trade intensity in the bilateral pair and the relative rest of the world and running analogous regression on model-generated data. Our baseline regression coefficient implies that moving from the 10th to the 90th percentile of the bilateral trade intensity distribution raises the predicted GDP correlations by 0.2, which is similar to the magnitude reported by Kose and Yi (2006) and other studies.

Liao and Santacreu (2015) develops a theory in which trade leads to larger technology spillovers. De Soyres (2016) takes a different approach and argues that it is the presence of markups that leads the measured TFP to comove more when there is more trade. While TFP and markups are certainly features worth exploring, Johnson (2014) argues that TFP correlation is only a partial remedy. In particular, Johnson (2014) shows that service sectors exhibit exact same trade-comovement pattern but measured TFP fails to be correlated with trade as in the good producing sectors. Input-output linkages turn out insufficient to close the gap for service sectors and hence output overall.[10] These findings call for an endogenous mechanism that correlates comovement with trade independently of the correlation between trade and comovement of TFPs, such as the ones we provide.

The rest of the paper is organized as follows. Section 1 lays out our theoretical results. Section 2 discusses data, presents our quantitative model, describes parameterization, and discusses our quantitative findings. Section 3 concludes.

1 Theory

In this section, we characterize the mechanism through which trade affects business comovement in a standard two-country international business cycle model a la Backus et al. (1995). Informed by this analysis, we explore several modifications of the theory and discuss how they can potentially resolve the trade-comovement puzzle. We assess them quantitatively in Section 2.

Our analytic setup is simplified to gain tractability and it is meant to illuminate mechanisms rather than to quantify them. Relative to our later quantitative setup, here we i) focus on a two country bilateral pair, abstracting from trade with the rest of the world, and ii) assume full depreciation of physical capital with no time to build in the accumulation of capital. Our analysis is semi-parametric in the sense that the less pertinent parameters assume a numeric value typically used in the literature.

1.1 Baseline model

Below, we lay out the setup. In doing so, we explore symmetry and streamline notation by focusing our exposition on the home country. We drop time subscripts and history-dependent notation whenever all variables pertain to the concurrent period t. Foreign analogs of home country variables are differentiated by an asterisk. If a bar is placed over a variable, it indicates deterministic steady state

[10] See also the work by Lev and Radhakrishnan (2003), who provides related empirical evidence.

value. A hat placed over a variable indicates log-deviation from the steady state.

The world economy consists of two symmetric countries, referred to as home and foreign, which trade country specific intermediate goods and assets. Each country is populated by a large number of competitive firms and households. Firms have access to country-specific technology and produce country-specific tradable intermediate goods from locally supplied labor and capital. Households supply labor, accumulate capital, and purchase both types of intermediate goods to aggregate them into a final non-tradable good used for consumption and investment. Law of one price holds and the asset market is complete (unless otherwise noted).

Production Firms employ labor l and rent capital k from local households to produce the home good d. The production function is Cobb-Douglas in capital and labor, that is

$$y = Ak^{\alpha}l^{1-\alpha}, \tag{1}$$

where A is total factor productivity, assumed to follow a country-specific mean-reverting stochastic process. The cost of employing labor is w and the cost of renting capital is r (in units of good d). Firms take prices as given and choose capital and labor to maximize profits:

$$\Pi = Ak^{\alpha}l^{1-\alpha} - rk - wl, \tag{2}$$

Constant returns to scale technology implies that profits are zero in equilibrium and hence factor prices satisfy:

$$r = \alpha A(\frac{l}{k})^{1-\alpha}, \quad w = (1-\alpha)A(\frac{k}{l})^{\alpha}. \tag{3}$$

Consumption Households purchase home and foreign goods from firms in a centralized Walrasian market and aggregate these goods via a standard CES aggregator:

$$G(d,f) = (\omega^{\frac{1}{\rho}}d^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}}f^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}, \tag{4}$$

where ω determines the importance of each good in the consumption basket and ρ determines the elasticity of substitution between home and foreign goods – henceforth *Armington elasticity.* This final good is nontradable and it is either consumed or invested in capital. Capital accumulation is simplified in that it assumes full depreciation and no time-to-build, hence

$$c + k = G(d,f). \tag{5}$$

Unless otherwise noted, the utility function is CRRA with risk aversion σ and Cobb-Douglas in consumption and leisure,[11]

$$u(c, l) = \frac{(c^{\psi}(1-l)^{1-\psi})^{1-\sigma}}{1-\sigma}. \tag{6}$$

Household trade a complete set of state-contingent bonds in a centralized Walrasian market. Let s^t denote the history of productivity shocks A, A^* up to and including period t. Let the price of good d in terms of good f be p. Then, following any history s^t, the budget constraint of home country's representative household is

$$d(s^t) + f(s^t)/p(s^t) + \sum_{s^{t+1}} Q(s^{t+1})B(s^{t+1}) = B(s^t) + w(s^t)l(s^t) + r(s^t)k(s^t), \tag{7}$$

where $B(s^t)$ denotes bond holdings in state s^t that pay one unit of good d per bond. Bonds are purchased one period in advance at the worldwide price $Q(s^{t+1})$. There is no borrowing constraint other than the one that excludes Ponzi-schemes. For clarity, the foreign household budget constraint is

$$f^*(s^t)/p(s^t) + d^*(s^t) + \sum_{s^{t+1}} Q(s^{t+1})B^*(s^{t+1}) = B^*(s^t) + (w^*(s^t)l^*(s^t) + r^*(s^t)k^*(s^t))/p(s^t). \tag{8}$$

Home country households choose consumption c, investment i, capital k, labor supply l, purchases of individual goods d and f, and bond holdings $B(s^{t+1})$ to maximize

$$\sum_t \sum_{s^t} \beta^t Prob(s^t) u(c(s^t), l(s^t)), \tag{9}$$

subject to (4), (5) and (7). First order conditions – in addition to budget constraints – comprise of marginal conditions that govern: i) labor supply:

$$wG_d(d, f) = -\frac{u_l(c, l)}{u_c(c, l)}, \tag{10}$$

ii) demand for goods:

$$p = \frac{G_d(d, f)}{G_f(d, f)}, \tag{11}$$

iii) investment in capital:

$$rG_d(d, f) = 1, \tag{12}$$

and iv) issuance of state contingent bonds:

$$\frac{u_c(c, l)}{u_c^*(c^*, l^*)} = \frac{c^* + k^*}{d^* + f^*/p} \frac{d + f/p}{c + k}. \tag{13}$$

[11]Standard separable utility function gives similar results.

The last condition is known as the *perfect risk-sharing condition.* It implies that home and foreign households effectively act as a family that shares business cycle risk by equalizing the marginal rate of substitution of consumption across countries to the marginal rate of transformation of consumption across countries, that is, the ideal real exchange rate.

Market clearing and equilibrium Finally, feasibility in the goods market requires

$$\begin{aligned} d + d^* &= y, \\ f + f^* &= y^* \end{aligned} \tag{14}$$

and in the bonds market it is $B^*(s^t) + B(s^t) = 0$. The formal definition of equilibrium is straightforward and will be omitted. By welfare theorems, the equilibrium allocation solves the planning problem of maximizing the joint utility of home and foreign household subject to (1), (4), (5), (14), and analogous conditions for the foreign country. The planning problem is effectively static because the objective function is additive across all histories and all constraints of the planning problem are concurrent. Accordingly, all equilibrium policy functions are functions of the pair of exogenous productivities (A, A^*), which makes our setup well-suited for analytics.

1.1.1 Basic definitions

Here, we define the key objects of interest needed to analyze the trade-comovement relation implied by the model.

1. **Bilateral trade:** Empirical studies of trade-comovement relation use long-term average trade between partner countries. We thus associate these measures with the ratio of steady state value of imports to the steady state value of GDP ($A = A^* = 1$):

$$\bar{x} := \frac{\bar{f}}{\bar{y}} = \frac{\bar{f}}{\bar{f} + \bar{d}}. \tag{15}$$

Trade in steady state is endogenous and it is determined by parameter ω through the relation:

$$\bar{x} = 1 - \omega. \tag{16}$$

Accordingly, examining the effect of trade on comovement amounts to examining the effect of $1 - \omega$ on shock transmission. Under homothetic preferences, other parametric approaches, such as an iceberg cost, deliver quite similar results and will not be considered here.

2. **Comovement:** The standard measure of business cycle synchronization in the trade-comovement literature is the correlation coefficient between home and foreign country output. The correlation coefficient is not a tractable object and our setup allows to instead focus on a simpler measure that is monotonically related to the correlation coefficient;[12] namely, the relative elasticity of home country's output to foreign productivity shocks:

$$\mathcal{S}(\bar{x}) := \left(\frac{\partial \log y(A, A^*)}{\partial \log A^*}\right)\left(\frac{\partial \log y(A, A^*)}{\partial \log A} + \frac{\partial \log y(A, A^*)}{\partial \log A^*}\right)^{-1}, \tag{17}$$

where $y(A, A^*)$ is the equilibrium output as function of the state (A, A^*).

3. **Trade-comovement relation:** Finally, we define the theory-implied trade-comovement relation as

$$\mathcal{L}(\bar{x}) := \frac{d\mathcal{S}(\bar{x})}{d\bar{x}}. \tag{18}$$

Intuitively, $\mathcal{L}$ measures how, on the margin, the relative elasticity of home country's output changes with trade. $\mathcal{L}$ is the key object of interest and here it maps onto the correlation-based trade-comovement relation.[13]

1.2 Decomposition of shock transmission mechanism

Next, we develop a framework to study the effect of trade on comovement in our model. Specifically, we isolate the key channels of shock transmission and set up an extensive form system that helps uncover the mechanism through which trade affects shock transmission across borders.

1.2.1 Channels of shock transmission

In the first step, we augment the equilibrium system so as to isolate effective income transfers between countries in each state. We define:

$$T = \underbrace{(1 - p(s^t)^{-1})f(s^t)}_{T_p} + B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}), \tag{19}$$

where T is the total income transfer and T_p is the part of the transfer attributed to the income effect of terms of trade. It is easy to verify that this definition indeed identifies zero-sum income transfers

[12]This is shown in the Online Appendix, Section I.
[13]See Online Appendix, Section I.

between the two countries. Plugging in to (7) and (8), the above definition implies

$$d + f = y + T \tag{20}$$

$$d^* + f^* = y^* - T. \tag{21}$$

Accordingly, as required, T soaks up the effect of prices and net asset payout from the budget constraints. (We used the fact that in equilibrium $y = rk + wl$.)

With this definition in hand, we next log-linearize the model's equilibrium conditions in steps to separate the substitution effect of the terms of trade p on home country's output and the joint income effect of terms of trade and asset payout; that is, the income effect associated with transfer T as defined in (19). To that end, we keep both p and T as exogenous stochastic processes and trace back their effect on each country's output. Since terms of trade and asset payouts is all that connects the two countries, the effect on each country can be considered in isolation. Accordingly, for the home country, we log-linearize conditions comprising equations (3)-(5), (10)-(12), and the budget constraint (7) replaced by (20) to introduce transfer payment T to the system. We then solve the system to express the log-deviation of home country's output $\hat{y}$ in terms of $\hat{p}$ and T:[14]

$$\hat{y}(\hat{A}; \hat{p}, T) = \frac{\hat{A}}{1-\alpha} + \underbrace{\bar{x}\frac{1-\psi+\alpha}{1-\alpha}\hat{p}}_{\text{substitution channel}} - \underbrace{\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\frac{1-\psi}{\psi}T}_{\text{income channel}}. \tag{22}$$

Equation (22) shows that home country's output is affected by foreign country's productivity shock A^* through the terms of trade and equilibrium net asset payout (last two terms), with the terms of trade having both a substitution effect and an income effect that is part of T. Accordingly, we label the second term as the *substitution channel* of shock transmission, and the last term as the *income channel* of shock transmission.

By construction, the *substitution channel* measures the substitution effect of terms of trade on home country's output due to its effect on the home price of final consumption/investment good relative to home labor. Intuitively, terms of trade affects this relative price because home labor and capital produce the home good d and the final good involves the foreign good in (4). The term $1/(1-\alpha)$ represents the effect of investment in capital, as the supply of labor remains unchanged due to offsetting substitution and income effects (see equation for $\hat{l}$ in Appendix A.2.). As the expression shows, *the effect of terms of trade on home country's output via the substitution effect is proportional to trade.* Intuitively, this is because trade determines the share of foreign good in home consumption

[14] Derivations are in Mathematica files available online. The remaining equations are stated in Appendix A.2. Conditions for the foreign country follow by symmetry.

and investment spendings.

The *income channel* of shock transmission corresponds to the the income effect of transfer T on home country's output due to its effect on home country's labor supply and, indirectly, its effect on investment due to the built-in complementarity between capital and labor. Intuitively, a positive transfer raises consumption in the home country, by (20), and due to the built-in complementarity between consumption and leisure in (6), it lowers home labor supply and hence home investment and output. Importantly, *the income effect of transfers on home country's output is independent of trade* because preferences are homothetic and technology is constant returns to scale.

1.2.2 General equilibrium forces of market clearing and risk sharing

The last two steps close the above partial equilibrium system by determining $\hat{p}$ and T to satisfy the market clearing conditions in (14) and risk-sharing condition (13).

Market clearing Here we keep treating T as an exogenous stochastic process and use market clearing condition (14) to endogenize p, alongside the system from the first stage for both countries. Since the first stage of the decomposition involves budget constraints (20) and (21), we only need one market clearing condition by Walras law. To facilitate the interpretation of the results, we use market clearing for good f in (14) and express it in terms of the excess demand for the imported good f in the home country; that is:

$$\frac{f}{y^* - f^*} - 1 = 0. \tag{23}$$

Formally, we log-linearize equation (23) (linearize with respect to T since $\bar{T} = 0$) and plug in policy functions derived in Section 1.2.1. We then solve for $\hat{p}$ in terms of $\hat{A}$, $\hat{A}^*$, and T:

$$\hat{p}(\hat{A}, \hat{A}^*; T) = \frac{-\frac{\hat{A}-\hat{A}^*}{1-\alpha} + \frac{\alpha^{\frac{\alpha}{\alpha-1}}}{\psi}\left(\frac{1}{\bar{x}} + 2\frac{\alpha-\psi}{1-\alpha}\right)T}{2\left(\rho(1-\bar{x}) + \bar{x}\frac{1-\psi+\alpha}{1-\alpha}\right)}. \tag{24}$$

By construction, the numerator in (24) corresponds the impact of productivity and transfers on excess demand as defined by the left-hand side of (23). Intuitively, relative productivity in isolation ($T = 0, \hat{p} = 0$) raises demand for good f in the home country in proportion to the increase in the home country's output y, which in percentage terms goes up by $\hat{A}/(1-\alpha)$. By (20) and homothetic preferences, in percentage terms, d and f increase by as much as output y ($T = 0$, $\hat{p} = 0$). Similarly, in the foreign country, $y^* - f^*$ goes up by $\hat{A}^*/(1-\alpha)$ in percentage terms, as both y^* and f^* increase by this much.

The effect of transfer payments T, corresponding to the second term in the numerator, is crucial for our results. The key observation is that, when the home country receives a positive net transfer from the foreign country ($T > 0$), its demand for foreign good f and home good d barely changes, and analogously in the foreign country. Formally, this is clear from equations for $\hat{d}$ and $\hat{f}$ in Appendix A.2. Intuitively, this is because T has two offsetting effects: One, $T > 0$ increases consumption in the home country by relaxing the budget constraint (7) and this raises demand for goods d and f proportionally (by homotheticity of G and u). Two, through its negative income effect on home labor supply, it reduces labor and investment, which in turn reduces demand for goods d and f due to lower income and less demand for capital. The net effect turns out proportional to the term $\alpha - \psi$ multiplying T and hence it is zero when α equals ψ – which is the case in the typical calibration of the standard theory – as we later assume. Consequently, in a typical calibration of the standard theory, the transfers barely move d, f, d^* and f^*.

The key effect of transfer $T > 0$ is thus its income effect on foreign country labor supply and its associated effect on foreign investment. The analog of equation (22) for the foreign country implies that both effects combined raise foreign output y^*, since seen from the foreign country's perspective, $T < 0$. Consequently, the increase in the supply of the foreign good for exports, i.e., $y^* - f^*$ in the denominator of (23), for fixed f^*, is inversely proportional to steady state trade $\bar{x}$ when expressed in percentage terms relative to its steady state value. Mechanically, this follows from the fact that in the steady state $\bar{y}^* - \bar{f}^* = \bar{y}\bar{x}$. *This property implies that, ceteris paribus, when a country pays a transfer, in equilibrium this payment leads to an excess supply of its own good, making this good cheaper in the international goods market.*

Finally, the denominator of (24) captures the market clearing effect of terms of trade $\hat{p}$. Since it affects both countries, all effects are multiplied by a factor of two. Intuitively, terms of trade affects the excess demand in (23) for two reasons. First, it has the usual expenditure switching effect on consumption of each good because it is the relative price of the two goods. This effect is naturally proportional to the elasticity of substitution, and it maps onto the term $\rho(1 - \bar{x})\hat{p}$ in the expression above.[15] Second, as captured by the next term, terms of trade affects the relative price of the final consumption/investment good in terms of home country's labor, since home labor and capital produce the home good and the final good requires the foreign good. This effect is analogous to the one discussed in the context of equation (22).

[15]In particular, the first stage of our decomposition implies the following relation between trade and terms of trade: $\hat{x} = \rho(1 - \bar{x})\hat{p}$, where $\hat{x}$ corresponds to logarithm of trade defined analogously to its steady state value in (15).

Risk sharing In the last step, we derive the equilibrium condition for T by keeping p as an exogenous stochastic process and using risk sharing condition (13) in combination with the first stage policy functions from Section 1.2.1 to endogenize T – as this condition is key to the determination of transfer payments in equilibrium.

Specifically, we pin down how processes for productivity A, A^*, transfers T, and terms of trade p jointly affect the risk-sharing condition, which we rewrite as follows:

$$\frac{u_c(c,l)}{u_c^*(c^*,l^*)} - \frac{c^*+k^*}{d^*+f^*/p}\frac{d+f/p}{c+k} = 0. \tag{25}$$

Formally, we log-linearize (25) and plug in home and foreign country policy functions derived in Section 1.2.1 (also Appendix A.2.) We then calculate T as a function of $\hat{A}$, $\hat{A}^*$, and $\hat{p}$ to get:

$$T(\hat{A},\hat{A}^*;\hat{p}) = \frac{-(1+\psi(\sigma-1))\frac{\hat{A}-\hat{A}^*}{1-\alpha} - (1+2\bar{x}\frac{\psi-\alpha}{1-\alpha})\hat{p}}{2\sigma\alpha^{\frac{\alpha}{\alpha-1}}/(1-\alpha)} \tag{26}$$

Finally, we use equation (19) and separate the contribution of terms of trade by deriving:

$$T_p(\hat{p}) = \bar{x}\alpha^{\frac{\alpha}{1-\alpha}}\psi\hat{p}. \tag{27}$$

The first term in the numerator of (26), $-(1+\psi(\sigma-1))(\hat{A}-\hat{A}^*)/(1-\alpha)$, corresponds to the effect of the relative productivity on the left-hand side of (25). Relative productivity raises consumption of the country with higher productivity and distorts the risk sharing condition. Productivity exclusively affects the marginal rate of substitution u_c/u_c^* in (25), as the ideal real exchange only depends on trade and terms of trade through the expression $\hat{p}(1-2\bar{x})$.

The second term in the numerator of (26) corresponds the effect of terms of trade on the left-hand side of risk sharing condition (25). Terms of trade has an effect on both the marginal rate of substitution and the real exchange rate. The dominant effect, however, is its effect on the real exchange, and hence the effect of terms of trade is negative. Formally, higher terms of trade makes the final good in the foreign country cheaper than in the home country, and by (25) this attenuates transfers (or even reverses their direction in the extreme). Intuitively, asset prices convey the information that one of the goods is in excess supply and the marginal benefit from it is lower, discouraging asset positions that induce transfers that would make this situation worse. This follows from the first welfare theorem, which generally implies that prices lead to behaviors consistent with efficient allocation of resources.

Finally, as before, the denominator captures the effect of transfer T on risk sharing condition (25). A positive transfer raises consumption in the home country and lowers consumption in the foreign country, which has a positive effect on the marginal rate of substitution of the final good across countries, i.e., u_c/u_c^* in (25). The direction of this effect is clear from (22), (7) and the utility function.

General equilibrium Equations (24)-(26) define a fixed point in $\hat{p}$ and T. In what follows, we keep (24)-(26) implicit to facilitate the analysis and for later use denote the solving functions by $\hat{p}(\hat{A}, \hat{A}^*)$ and $T(\hat{A}, \hat{A}^*)$. Together, (22)-(26) pin down equilibrium dynamics of the model.

1.2.3 Parameter domain

To gain tractability and simplify the algebra, we restrict the values of the parameters that are less pertinent to our analysis.[16] In particular: 1) we use a numeric value for $\alpha = 1/3$ and $\psi = 1/3$, as such values are typically assumed in the literature as part of calibration of the standard theory. 2) We restrict attention to risk aversion parameter value for which the intertemporal substitution effect dominates the income effect, i.e. $\sigma \geq 1$. Finally, 3) we focus on trade levels satisfying $0 < \bar{x} \leq \min\{1/(1+\sigma/2), 1/3\}$. In a bilateral context, featuring low bilateral trade levels, this is an innocuous assumption for not too high values of the risk aversion parameter σ.[17]

1.3 Analysis of the effects of trade on comovement

We now use our framework to analyze the effect of trade on comovement. All our results pertain to first order approximation of equilibrium dynamics given by (22)-(27).

We evaluate (18) using (22)-(26) to obtain the following decomposition:

$$\mathcal{L} = \underbrace{(1-\psi+\alpha)\left(\frac{\partial \hat{p}(A, A^*)}{\partial A^*} + x\frac{\partial^2 \hat{p}(A, A^*)}{\partial A^* \partial \bar{x}}\right)}_{\text{effect of trade via substitution channel: } \mathcal{L}_S} - \underbrace{\alpha^{\frac{\alpha}{\alpha-1}} \frac{1-\psi}{\psi} \frac{\partial^2 T(A, A^*)}{\partial A^* \partial \bar{x}}}_{\text{effect via income channel: } \mathcal{L}_I}. \tag{28}$$

The first term, labeled $\mathcal{L}_{\mathcal{S}}$, corresponds to the effect of trade via the *substitution channel*. As expected, it crucially depends on the response of terms of trade to foreign shock, i.e., $\frac{\partial \hat{p}(A,A^*)}{\partial A^*}$. The second term $\mathcal{L}_{\mathcal{I}}$ corresponds to the effect of trade via the *income channel*. It solely depends on how trade affects transfers, as implied by the cross-partial derivative $\frac{\partial^2 T(A,A^*)}{\partial A^* \partial \bar{x}}$.

As we show next, for most parameter values, these two channels have an offsetting effect on the model-implied trade-comovement relation, giving rise to trade-comovement puzzle. They are also

[16] These values imply empirically plausible values for the labor share and the share leisure in time endowment.

[17] In what follows, we prove all results by plugging in numeric values for α and ψ. These results generalize but analytically the conditions are more cumbersome to handle.

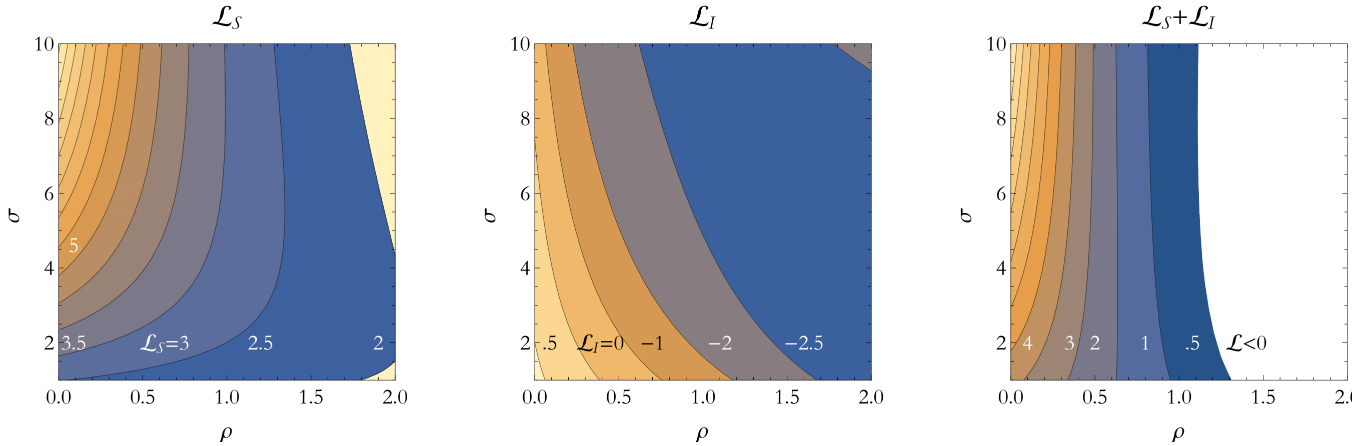


Figure 1: Decomposition of model-implied trade-comovement relation ($\bar{x} = 5\%$).

Notes: The figure illustrates the decomposition of model implied trade-comovement relation $\mathcal{L}$ as implied by equation (28) in the space of parameters σ and ρ (rightmost panel). The leftmost panel correspond to the contribution of the substitution channel, $\mathcal{L}_S$, and the middle panel corresponds to the income channel, $\mathcal{L}_I$. The figure assumes $\bar{x} = 5\%$.

driven by distinct structural assumptions, which is key to our analysis in the following sections.

1.3.1 The role of substitution channel

Figure 1 (left panel) illustrates the effect of trade operating via the substitution channel $\mathcal{L}_S$ for all values of free parameters ρ and σ and a fixed level of trade $\bar{x} = 5\%$. As is clear from the figure, the substitution channel is a robust source of positive relation between trade and shock spillovers. Proposition 1 generalizes this result, which follows from Lemma 1 and equation (28).

Proposition 1 $\mathcal{L}_S > 0$.

Lemma 1 *Equilibrium response of terms of trade implies:* $\frac{\partial \hat{p}(A,A^*)}{\partial A^*} > 0$ *and* $\frac{\partial \hat{p}(A,A^*)}{\partial A^*} + \bar{x}\frac{\partial^2 \hat{p}(A,A^*)}{\partial A^* \partial \bar{x}} > 0$.

Mechanically, the reason why the substitution channel leads to a positive association between trade and shock spillovers is because, (i) trade increases the sensitivity of home country's output to terms of trade p, giving rise to the term $\frac{\partial \hat{p}(A,A^*)}{\partial A^*}$ in equation (28), and (ii) the terms of trade appreciates after a positive productivity shock abroad – which is the dominant effect. Trade also affects how much terms of trade responds, which is captured by $\bar{x}\frac{\partial^2 \hat{p}(A,A^*)}{\partial A^* \partial \bar{x}}$ However, since this effect is proportional to trade, which is small in a bilateral context (median is 0.85%), this effect turns unimportant for all practical purposes and we do not discuss it further.

Intuitively, the reason why trade increases the sensitivity of home country's output to the terms of trade is because terms of trade affects the relative price of the final consumption/investment good relative to home labor and capital in proportion to trade. This is because home labor and capital produce exclusively the home good and the final consumption/investment requires the foreign good in proportion to trade.

The reason why terms of trade appreciates after the shock is because a positive productivity shock in the foreign country leads to an increased supply of the foreign good, which lowers its price relative to the home good. This is clear from the system that pins down the joint response of terms of trade and transfers, i.e., equation (24) and (26), which under parametric restrictions imposed here boil down to (For simplicity, since we focus on foreign productivity shock, we set $\hat{A} = 0$):

$$\begin{aligned} \hat{p} &= \frac{1}{3\bar{x} + 2(1-\bar{x})\rho} \left(\frac{3}{2}\hat{A}^* + \frac{3\sqrt{3}}{\bar{x}} T \right) \\ T &= \frac{1}{3\sqrt{3}\sigma} \left(\frac{2+\sigma}{2}\hat{A}^* - \hat{p} \right). \end{aligned} \tag{29}$$

Mechanically, absent the feedback mechanism between p and T, the above system implies that the shock by itself leads to an appreciation of terms of trade by equation for $\hat{p}$ (assuming $T = 0$), and it also leads to a positive transfer payment $T > 0$ by equation for T (assuming $\hat{p} = 0$). These two basic effects are tied by the general equilibrium feedback mechanism whose strength is inversely proportional to trade through the term $(1/\bar{x})T$ in equation for $\hat{p}$. This feedback mechanism implies that a positive transfer furthers the appreciation of terms of trade and this reduces the size of the transfer T in general equilibrium. Importantly, as Lemma 2 below shows, without terms of trade appreciation, transfers can only be positive, and hence terms of trade always further appreciates after the shock.

What is the intuition behind all these effects? As discussed in Section 1.2.2, a positive transfer T exacerbates the excess supply of foreign good that arises after the shock, which is why a positive transfer furthers the appreciation of terms of trade after the shock (as implied by equation for $\hat{p}$). As also discussed in Section 1.2.2, the feedback effect is inversely proportional to trade because the foreign output increased by the transfer raises the supply of foreign good for exports in proportion to the level of export. Again, asset prices internalize this distortion in equilibrium so as to discourage trades that imply such transfers – as formally implied by the right hand side of the risk-sharing condition in (13).

1.3.2 The role of income channel

Figure 1 (middle panel) illustrates the effect of trade operating via the income channel $\mathcal{L}_I$. As is clear from the figure, the income channel is a potent offsetting force except for very low values of

Armington elasticity ρ, where it reverses direction. Proposition 2 summarizes this result, which follows from Lemma 2 below and equation (28). In contrast to the substitution channel, the key to this result is how transfers change with trade, i.e., the cross-partial derivative $\frac{\partial^2 \hat{T}(A,A^*)}{\partial A^* \partial \bar{x}}$.

Proposition 2 *For* $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$, $\mathcal{L}_I < 0$.

Lemma 2 *For* $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$, *equilibrium response of transfer* T *implies:* $\frac{\partial \hat{T}(A,A^*)}{\partial A^*} > 0$, $\frac{\partial^2 \hat{T}(A,A^*)}{\partial A^* \partial \bar{x}} > 0$.

As Lemma 2 shows, for trade elasticity that is not too low, it is the foreign country that pays the home country after the shock. The intuition is analogous to the discussion above and that laid out in Section 1.2.2. Specifically, a positive transfer paid by the foreign country, while being beneficial in equalizing the marginal utility from consumption across countries in (13), is also distortionary because it exacerbates the oversupply of the foreign good that arises after the shock. The less countries trade, the stronger this effect is. In equilibrium, asset prices internalize this and discourage trades that imply distortionary transfers. Since positive transfers from the foreign country are a source of negative comovement, for reasons explained in the context of (22) in Section 1.2.2, trade has a *negative* effect on comovement due to the income channel.

Interestingly, when trade elasticity ρ is very low, this dynamic reverses because the response of terms of trade is so pronounced that the direction of transfers becomes negative.[18] This can be seen in Figure 1, which shows reversed sign of the income effect for low values of ρ. It is clear from (29) that for a sufficiently low value of ρ this may indeed be the case.

The reason for reversed effect for low elasticity can be easily understood by applying our reasoning in reverse. First, a negative transfer is a source of *positive* rather than negative comovement and a negative transfer *alleviates* rather than exacerbates the oversupply of the foreign good after the shock. In general equilibrium, trade amplifies this now beneficial effect of transfers through the solution to the system (29), making transfers even more negative when countries trade more. Consequently, trade has a *positive* rather than negative effect on comovement. Lemma 3 establishes this important connection between the direction of transfers and the effect of trade on their size.

Lemma 3 $\frac{\partial \hat{T}(A,A^*)}{\partial A^*} < 0$ *implies* $\frac{\partial^2 \hat{T}(A,A^*)}{\partial A^* \partial \bar{x}} < 0$ *and hence* $\mathcal{L}_I > 0$.

[18] This is sufficient condition but not necessary. However, in practice significant effect only arises when the direction reverses.

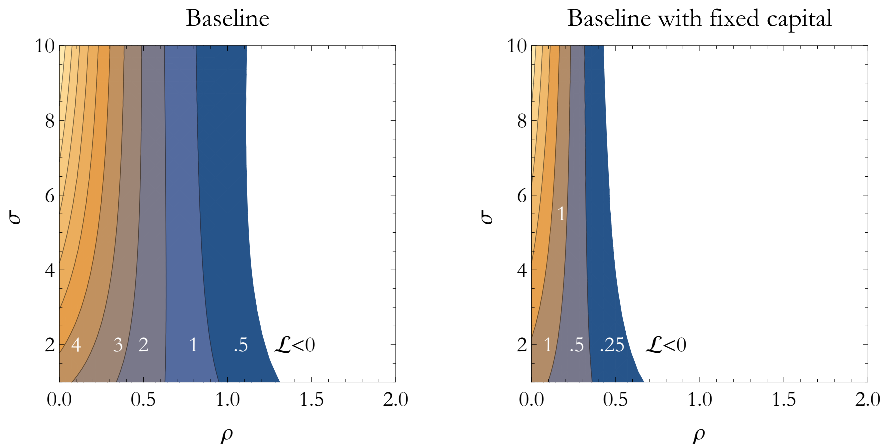


Figure 2: Contribution of adjustment of capital to trade-comovement relation ($\bar{x} = 5\%$).

Notes: The figure compares trade-comovement relation $\mathcal{L}$ in the baseline model with and without adjustment of capital. Fixed capital specification assumes an extreme cost of adjusting capital, as described in Section 1.3.3. The figure assumes $\bar{x} = 5\%$. The leftmost panel is the same as that in Figure 2.

1.3.3 The role of capital

Both channels affect simultaneously labor and capital, and thereby output. But what is the role of labor and capital in isolation?

To address this question, here we consider a version of the model that incorporates an extreme friction on the adjustment of capital so that capital always is equal to the steady state value $\bar{k}$ from the frictionless model. In particular, we replace feasibility by

$$c + k + \chi(k - \bar{k})^2 = G(d, f),$$

and assume χ is infinitely large so as to ensure $k = \bar{k}$. Other than that, the model is identical.

As Figure 2 shows, the presence of capital exacerbates the puzzle. For comparison, the analog of equation (22) in this case is

$$\hat{y}(\hat{A}; \hat{p}, T) = \frac{\hat{A}}{1 + \alpha(1 - \psi)} + \bar{x}\frac{(1 - \alpha)(1 - \psi)}{1 + \alpha(1 - \psi)}\hat{p} - \frac{\alpha^{\frac{\alpha}{\alpha - 1}}\frac{1 - \psi}{\psi}}{1 + \alpha(1 - \psi)}T. \tag{30}$$

As the expression shows, the key difference is the effect of trade via the substitution channel, which is determined by the term $\bar{x}(1 - \alpha)(1 - \psi)\hat{p}$, as opposed to $\bar{x}(1 + \alpha - \psi)\hat{p}$ in equation (22). Intuitively, this difference comes from the fact that the terms of trade affects the relative cost of investment, since investment requires the foreign good in proportion to trade and capital produces exclusively the home good. The additional effect of shutting off adjustment of capital is that the distortionary effect of positive transfers on the oversupply of the foreign good after the shock is further reinforced and similarly inversely proportional to trade. Intuitively, without capital the home country's demand for goods d and f (f^* and d^* abroad) – which, recall, were unaffected by transfers in the baseline case – here respond in the direction of transfers. This leads to even greater oversupply of the foreign good inversely proportionally to the level of trade.

1.3.4 Trade-comovement puzzle

Figure 1 (right panel) shows the net effect of the income and substitution channels on the trade-comovement relation $\mathcal{L}$. As we can see, except for low values of Armington elasticity ρ, the effect of trade via the income channel completely offsets or significantly weakens the otherwise positive effect of trade via the substitution channel. This is the essence of the trade-comovement puzzle, as most parameterizations of the model use Armington elasticity ρ above one. As shown by Kose and Yi (2006), while low levels of elasticity improve model's performance, they still fall short quantitatively.

1.4 Candidate resolutions of trade-comovement puzzle

Building on the developed intuitions, we now explore three basic modifications of the standard theory that work by suppressing the effect of trade via the income channel in different ways. In particular, we consider: i) financial autarky; ii) GHH preferences; and iv) dynamic trade elasticity that is low in the short-run (i.e., on business cycle frequency) but progressively higher in the longer run. Table 1 shows how these modifications affect the key equations of our decomposition. Figures 2-4 illustrate the trade-comovement relationship in each case. The figures consider both the baseline specification and the model with fixed capital, as discussed in Section 1.3.3.

Table 1: Decomposition (22)-(27) across model specifications.

Baseline		Autarky	GHH
$\hat{y} \propto$	$\hat{A} + \bar{x}\hat{p} - 2\sqrt{3}T$	$\hat{A} + \frac{1}{3}\bar{x}\hat{p}$	$\hat{A} + \frac{2}{3}\bar{x}\hat{p}$
$\hat{p} =$	$\frac{1}{3\bar{x}+2(1-\bar{x})\rho}\left(-\frac{3}{2}(A - \hat{A}^*) + \frac{3\sqrt{3}}{\bar{x}}T\right)$	same	$\frac{3}{4\bar{x}+2(1-\bar{x})\rho}\left(-(\hat{A} - \hat{A}^*) + \frac{\sqrt{3}(1-2\bar{x})}{2\bar{x}}T\right)$
$T =$	$\frac{1}{3\sqrt{3}\sigma}\left(-\frac{2+\sigma}{2}(A - \hat{A}^*) - \hat{p}\right)$	$\bar{x}\frac{\hat{p}}{3\sqrt{3}}$	$\frac{1}{6\sqrt{3}}\left(-(\hat{A} - \hat{A}^*) - \frac{1-2\bar{x}}{3\sigma}\hat{p}\right)$
$T_p =$	$\bar{x}\frac{\hat{p}}{3\sqrt{3}}$	$\bar{x}\frac{\hat{p}}{3\sqrt{3}}$	$\bar{x}\frac{\hat{p}}{3\sqrt{3}}$

Notes: The table derives system (22)-(27) for all model specifications from Section 1.4 under parametric restrictions laid out in Section 1.2.3. Column correspond to model specifications described in the heading and rows corresponds to respective equations that replace (22)-(27).

1.4.1 Financial autarky and asset market frictions

We start from financial autarky, which is an extreme form of asset trade restriction. In terms of modeling, financial autarky imposes a market clearing condition requiring that bond holdings in each country, $B(s^t), B^*(s^t)$, are zero in equilibrium.

As is clear from Table 1 (column two) and Figure 3, financial autarky leads to an overall positive relation between trade and comovement but the net effect is still much smaller than the effect of trade via the substitution channel alone. This result shows that the restrictions on asset trade in general are unlikely to be quantitatively promising.

The reason why this is the case is apparent from the equation for T_p in Table 1 (column two, row three). The equation shows that the bulk of risk sharing transfers in the model are associated with the income effect of terms of trade, which financial autarky fails to eliminate. This point is reminiscent of Cole and Obstfeld (1991), who were the first to show the risk sharing role of terms of trade fluctuations. What we show here is that the income effect of terms of trade is also proportional to trade and hence terms of trade-implied transfers similarly result in an offsetting effect of trade via the income channel of shock transmission.

The adjustment of capital crucially affects the direction of the effect. The key reason, apparent from the equations in Table 1 (column two), is that both income and substitution channel are linked to terms of trade. Consequently, it is the relative importance of the substitution versus income channel that matters here, and suppressing the response of capital, as discussed before, reduces the relative importance of the substitution channel.

1.4.2 GHH preferences

GHH preferences are potentially promising because they eliminate the effect of income on labor supply decisions of households – which is at the heart of offsetting income channel.

Here, we consider a numerical example of a model with GHH preferences by replacing (6) with

$$u(c, l) = \frac{(c - \psi \frac{l^2}{2})^{1-\sigma}}{1-\sigma}. \tag{31}$$

This utility function assumes a Frisch elasticity of labor supply of one – which is consistent with macro evidence. We also assume a numeric value for ψ consistent with the share of labor in time endowment of $1/3$, as is the case in the baseline model.[19]

[19]Here it requires $\psi = 1/\sqrt{3}$ and is part of calibration in the quantitative model.

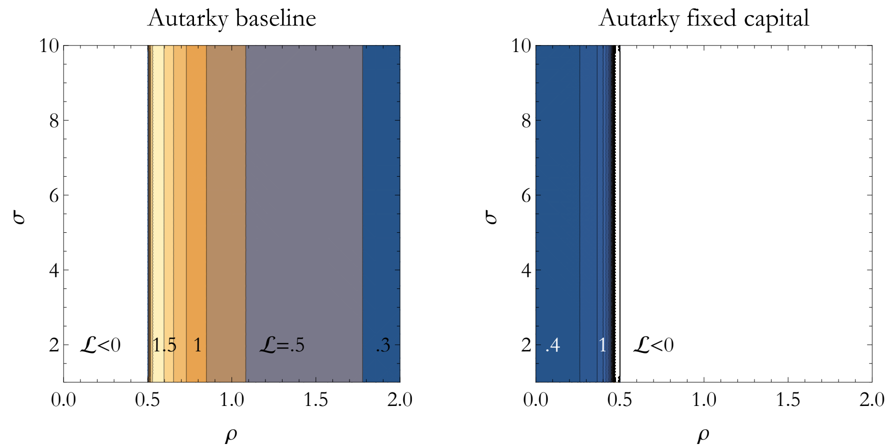


Figure 3: Trade-comovement relation $\mathcal{L}$ under financial autarky ($\bar{x} = 5\%$).

Notes: The figure illustrates the model-implied trade-comovement relation $\mathcal{L}$ in the baseline model under the assumption of financial autarky, as discussed in Section 1.4.1. The right panel corresponds to the model without adjustment of capital; that is, the baseline model that assumes an extreme adjustment cost that in effect fixed capital. See Section 1.3.3 for more details about this specification. The figure assumes $\bar{x} = 5\%$.

Table 1 (column three) confirms that the effect of trade via the income channel is indeed absent in this case (row one). At the same time, the substitution channel works similarly as in the baseline model, which makes GHH case promising in resolving the puzzle. Figure 4 confirms that this is the case. The figure shows a robustly positive and strong relation between trade and comovement for all parameter values. But, as for other specifications, frictions that sever adjustment of capital weaken the result considerably. Accordingly, realistic modeling of capital is required to assess the quantitative potential of GHH preferences.

1.4.3 Low trade elasticity

Figure 2 shows that low Armington elasticity ρ implies a positive relation between trade and comovement because the effect of trade via income channel is small or even positive. This phenomenon occurs when Armington elasticity ρ is close to one or below one. However, as already mentioned, this is not enough to account for the trade-comovement puzzle. In addition, using a very low value for the Armington elasticity is problematic for several other reasons.[20]

[20] First, having low trade elasticity makes sense only in the short-run (on business cycle frequency). Numerous studies find that long-run elasticity of trade with respect to permanent tariff changes is large. See references in footnote 6. Second, Armington elasticity below one reverses the relation between trade flows and trade costs, such tariffs. In the context of our model, imposing an trade cost $\tau > 0$ that is explicitly measured as a tariff in national accounting implies that measured imports are $f(1+\tau)$, and in such a case lower trade cost τ implies less trade for Armington elasticity below one. The contradicts the evidence on the effect of trade liberalization episodes. Our dynamic specification does not create such problems.

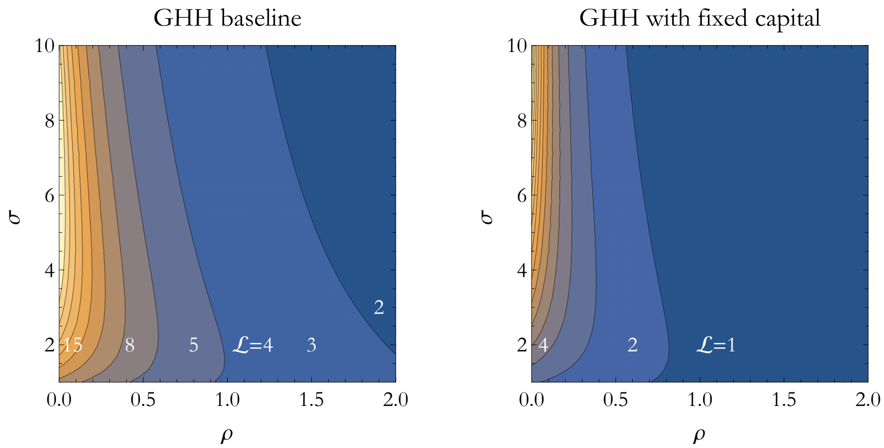


Figure 4: Trade comovement relation $\mathcal{L}$ under GHH preferences ($\bar{x} = 5\%$).

Notes: The figure illustrates the model-implied trade-comovement relation $\mathcal{L}$ in the baseline model under GHH preferences (31). The right panel considers the baseline setup but without adjustment of capital, as implied by extreme convex adjustment cost (see description in Section 1.3.3). The figure assumes $\bar{x} = 5\%$.

As we find, the qualitatively promising performance of simply lowering the Armington elasticity parameter can be greatly enhanced by instead having high Armington elasticity and modeling the dynamics of trade elasticity as resulting from a convex and dynamic adjustment cost, so as to imply low elasticity in the short-run and progressively higher in the longer run. The advantage of such an approach is that it is well-grounded in measurement of trade elasticity in the data and avoids the issues associated with low Armington elasticity.[21] Such a modification goes beyond our analytic framework, and we explore it in the quantitative section only. There, we consider a high value for ρ in Armington aggregator and impose a dynamic convex adjustment cost on the share of each good in production of final good. This is implemented in equations (33) and (34) for our three-country quantitative model. The key reason why this modification works is because it more forcefully reverses the direction of transfers on business cycle frequency, which as discussed in Section 1.3 have a *positive* rather than negative effect on trade-comovement relation. We discuss the intuition why it is the case in Section 2.

2 Quantitative analysis

This section provides a quantitative evaluation of our candidate resolutions of the trade-comovement puzzle using a calibrated three-country model. Before we proceed, we discuss the ways we generalize

[21] See discussion in footnote 20.

the model, then we set the quantitative goal for the theory by updating standard trade-comovement data regressions, and finally we discuss the results.

2.1 Generalized three-country setup

We generalize the setup of Section 1 along three key dimensions, while keeping the remaining assumptions analogous to the baseline two-country setup. We state them from home country perspective, as the setup in other countries is symmetric.

1. We distinguish between trade openness and bilateral trade intensity by considering a three country system: a large country, referred to as *rest of the world*, and a symmetric bilateral pair of two smaller countries: *home* and *foreign*. We assume that the composite final good in each country is a weighted CES of three goods:

$$G(d,f,g) = \left(\omega_d d^{\frac{\rho-1}{\rho}} + \omega_f f^{\frac{\rho-1}{\rho}} + \omega_g g^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{32}$$

   We model country sizes by assuming that the measure of households is L times higher in the large country than in each of the small countries.

2. We introduce *dynamic trade elasticity* to the model by adding a convex adjustment cost on trade shares that depends on previous period trade shares. The convex adjustment cost is paid in final consumption and it is given by

$$\Phi(d,f,g) = \frac{\phi}{2}\left\{\left(\frac{f/d}{f_{-1}/d_{-1}} - 1\right)^2 + \left(\frac{g/d}{d_{-1}/g_{-1}} - 1\right)^2\right\}. \tag{33}$$

3. We assume capital is durable and depreciates at a constant rate $\delta < 1$ per model period. Accordingly,

$$c + i + \Phi(d,f,g) = G(d,f,g), \tag{34}$$

   and the law of motion for capital is

$$k = (1-\delta)k_{-1} + i - \chi\left(\frac{\delta k_{-1}}{i} - 1\right)^2, \tag{35}$$

   where k is the capital stock, i is investment, δ is depreciation rate of capital and χ parameterizes the convex adjustment cost.

4. We assume that the country-specific productivity shock $A(s^t)$ follows an AR(1) process with

no cross-country spillovers:

$$\log(A_i(s^t)) = \zeta_i \log(A_i(s^{t-1})) + \varepsilon_i(s_t),$$

where $i = 1, 2, 3$, and the residuals ε_i are assumed to be normally distributed with zero mean, standard deviation σ_i^2, and correlation matrix μ_{ij}. The parameters are specified symmetrically for the two small countries, resulting in identical processes, and separately for the single large country.

2.2 Trade-comovement relation in the data

To set the quantitative target for the model to account for, we follow Kose and Yi (2006) and use data for 20 industrialized countries over the period 1980Q1-2011Q4 to back out the implied trade-comovement relation from a cross-country regression. Countries in our sample constitute about 59% of world GDP and 53% of world trade (as of 2011).[22] The regression results serve as a reference point to assess model's performance.[23] Our baseline regression specification is

$$corr(GDP_i, GDP_j) = \alpha + \beta_x trade_{ij} + X_i + X_j + E_{ij} + \varepsilon_{ij}, \tag{36}$$

where $corr(GDP_i, GDP_j)$ is the correlation between countries i and j of the logged and HP-filtered series of real GDP. X_i and X_j are country dummies, and E_{ij} is the European dummy, which takes the value of 1 if both countries in the pair are both in the European Union. The variable $trade_{ij}$ is a symmetric measure of bilateral trade intensity of countries i and j, measured at the beginning of the sample (in 1980), and given by the log of

$$\max\{\frac{IM_{ij}}{GDP_i}, \frac{IM_{ji}}{GDP_j}\}, \tag{37}$$

where IM_{ij} are nominal imports (in US dollars) by country i from country j and GDP_i is the nominal GDP (in US dollars) of country i, both measured in 1980. All of the results are robust to picking other years as the base year for the bilateral trade measure.

The measure of trade defined in (37) varies in our sample from 0.03% (Korea with Portugal) to 27% (Ireland with the United Kingdom). Notably, it is symmetric and robust to having trade partners of very different sizes. For example, if the U.S. is an important trading partner for Canada,

[22] For a complete list of data sources, see the Online Appendix. The country list includes: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[23] This result has also been confirmed by other studies for a variety of specifications – see, for example Kose and Yi (2006), Baxter and Kouparitsas (2005) or Clark and van Wincoop (2001), among others.

but Canada is not as important for the U.S., equation (37) still returns a high number.[24]

Table 2 reports our results. We include OLS results as well as results from an IV regression in which the instruments are common border, common language and distance. Both OLS and IV regressions give highly significant positive coefficients, which suggest a strong effect of bilateral trade on comovement of GDP. The estimates imply that moving from the 10th to the 90th percentile of the bilateral trade spectrum increases the GDP correlations by 0.21 (IV) or 0.11 (OLS). Relative to median GDP correlation of 0.52 in our sample, this is an economically significant effect.

Table 2: Trade-comovement relation in cross-country data.

	Dependent Variable: GDP correlation	
	OLS	IV
$trade_{ij}$	0.034**	0.065***
	(0.016)	(0.024)
E_{ij}	0.060	-0.028
	(0.093)	(0.106)
Country FE	yes	yes
R-squared	0.694	0.684

Notes: The table shows estimated cross-country pair regression between bilateral GDP correlation, bilateral trade intensity (trade_{ij}), EU membership dummy (E_{ij}), and country fixed effects. **,*** denote significance at the 5% and the 1% levels. Numbers in parentheses are standard errors.

2.3 Parameter values and model specifications

This section describes parameter values and functional forms for all model specifications. As a general strategy, we choose parameter values that match moments characterizing the median country in our sample and its relative rest of the world. However, to set up the exercise of varying the level of trade in the bilateral pair, we add several moment conditions to ensure that the levels of trade and output correlations are consistent with a median country pair that we choose as starting point. In the exercise of varying trade, we change utility weights ω to match the 90th percentile pair's bilateral and rest of the world trade. Parameter values for each model specification are summarized in Tables 3-5. The baseline period length is one quarter.

[24]This contrast with measures expressed as averages, such as $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$. Such asymmetric measures give small numbers when trade partners have asymmetric sizes, i.e., small countries trade with big countries. For example, our measure is 8 times higher than $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$ for the Germany-Austria pair, 6 times higher for the US-Canada pair, and 15 times higher for the UK-Ireland pair. The results do not crucially depend on how we measure trade intensity. Conceptually symmetric measure is consistent with the fact comovement is a symmetric measure.

Baseline model We start from the baseline specification, which is similar to that used by Kose and Yi (2006) and defines the trade-comovement puzzle . The baseline specification assumes $\phi = 0$, utility function given by (6), and complete markets. Parameter values for the baseline specification can be found in Table 3.

Table 3: Parameter values.

Parameter		Value
	Baseline model	
ρ	elasticity of substitution	1.17
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.8, 0.01665, 0.18335
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.18335, 0.18335, 0.6333
η	leisure weight in utility	0.332
σ	risk aversion	2
β	time discount factor	0.99
α	capital share	0.36
δ	depreciation of physical capital	0.025
ϕ	cost of adjustment of goods ratio	0
χ	capital adjustment cost	2.35
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.82, 0.82, 0.90$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.5, 0.65,0.65
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0088,0.0088,0.00607
	Baseline with financial autarky (if different than in baseline model)	
$\zeta_1, \zeta_2, \zeta_3$	persistence of the preference shock	$0.83, 0.83, 0.905$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of preference shocks	0.49, 0.49,0.59
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of preference shocks	0.0076,0.0076,0.0053
χ	capital adjustment cost	0

Notes: The table shows calibrated parameter values for each model specification in Section 2.3. Country 1 and 2 are two symmetric small countries and country 3 is relative rest of the world.

In terms of basic preference and technology parameters, we adopt the standard values as used in Backus et al. (1995). This includes: the risk aversion parameter $\sigma = 2$; discount factor $\beta = 0.99$, capital share in production $\alpha = 0.36$; consumption share in utility $\psi = 0.33$, and depreciation rate of capital of 2.5% per quarter.[25] In terms of country sizes, we assume that the population size of the rest of the world is 20 times that in each of the two small countries. This matches the population difference between a median country in our sample and its relative rest of the world.

[25]The discount factor implies that the average real interest rate in the model is 4 percent (return on capital), capital share implies that labor share of income is 64 percent, and depreciation of capital implies that investment to GDP ratio hovers at around 25 percent, which is higher then current OECD average of less than 20 percent but consistent with historic values during the sample period.

We choose the parameters governing the stochastic process in (36) ($\zeta_i, \mu_{ij}, \sigma_i$), the share of consumption in the utility function (ψ), the Armington elasticity ρ, and the adjustment cost in the law of motion for capital χ to jointly account for the median behavior of real GDP in our sample, median volatility of investment of 3.04%, and the median upper-bound measure of the short-run trade elasticity as volatility ratio of 1.17, following the approach and measurement in Drozd and Nosal (2012).[26]

As far as the behavior of GDP goes, we set the parameters of the stochastic productivity process to be symmetric across the bilateral pair in the model, set to match the median autocorrelation and standard deviation of real GDP in our sample (0.83 and 1.41%, respectively). We also match the autocorrelation and the standard deviation of real GDP for the median country's relative rest of the world (0.89 and 1.05%, respectively). This first part captures the closed economy properties of TFP process of median country and relative rest of the world.

To be consistent with the correlation of output in the median country pair by trade intensity – which is the basis of our exercise of varying trade intensity in the model – we assume correlation of shocks μ_{ij} to match the correlation of real GDPs within the median pair and also the correlation with their relative rest of the world. In particular, we match the correlation of pair of small country GDPs of 0.52 and the correlation of GDPs of the country pair with their relative rest of the world of 0.66.

Financial autarky This specification modifies the asset structure of the baseline model by assuming complete financial autarky. We set parameter values analogously as in the baseline case. Parameter values can be found in Table 3.

GHH preferences This specification modifies the utility function of the baseline model by instead assuming GHH utility function given by (31). This utility function ensures that the Frisch elasticity of labor supply is one and we pick the parameter ψ to ensure that the share of labor in time endowment is the same as in other specifications. We set the other parameter values by following the exact same

[26] The volatility ratio measures the volatility of the ratio of quantity indices of imports to domestic absorption relative to the volatility of their respective price deflator. It is an upper bound in the sense that any regression-based measure of elasticity between quantities in the numerator and relative price in the denominator would have to be scaled down by the correlation coefficient, which is always less than one. To construct the *volatility ratio* we use constant and current price values of imports and domestic absorption DA, given by $DA = (C + G) + I - IM$. The prices are taken to be their corresponding price deflators. Denoting the deflator price of domestic absorption by P_{DA} and the deflator price of imports by P_{IM}, the *volatility ratio* is then defined as $\sigma(\frac{IM}{DA})/\sigma(\frac{P_{DA}}{P_{IM}})$, where σ refers to the standard deviation of the logged and Hodrick-Prescott filtered quarterly time series.

procedure as in the baseline case. Parameter values can be found in Table 4.

Table 4: Parameter values continued.

Parameter		Value
GHH (whenever different from baseline model)		
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.789, 0.01625, 0.19475
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.19475, 0.19475, 0.6105
η	inverse Frisch elasticity	1
θ	multiplier on disutility from labor	0.447
χ	capital adjustment cost	7.1
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.81, 0.81, 0.885$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.49, 0.6,0.6
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0079,0.0079,0.0054
GHH plus dynamic elasticity (whenever different from dynamic elasticity model)		
η	inverse Frisch elasticity	1
θ	multiplier on disutility from labor	0.1703
ϕ	cost of adjustment of goods ratio	2.95
χ	capital adjustment cost	11.3
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.81, 0.81, 0.875$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.48, 0.62,0.62
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0081,0.0081,0.0054

Notes: As in previous table.

Dynamic elasticity model The dynamic elasticity model assumes $\phi > 0$, set to account for the difference between the short- and long-run trade elasticity estimates in the data.[27] In particular, in the dynamic elasticity model, the long-run response of trade x to the relative price of the domestic good versus the foreign good is exclusively determined by the Armington elasticity ρ. For example, the response of trade to a permanent tariff reduction of ΔT percent, which is equivalent to a permanent change in the price of good d relative to f, will be unaffected by the friction and determined by Armington elasticity ρ. Intuitively, in the long-run, the adjustment cost friction is slack, and the response of trade is governed by the elasticity of substitution between domestic and foreign good. Since an analogous equation has been estimated in the trade literature to measure the elasticity parameter, we adopt these estimates and set ρ equal to 15, which is close to the upper limit of the values reported in the trade literature. We then calibrate ϕ so as to match the same 1.17 target for

[27]See discussion in footnote 6.

Table 5: Parameter values.

Parameter		Value
	Dynamic elasticity model (if different than in baseline model)	
ρ	elasticity of substitution	15
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.3943, 0.2912, 0.3144
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.3144, 0.3144, 0.3711
ϕ	cost of adjustment of goods ratio	4.932
χ	capital adjustment cost	5.15
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.82, 0.82, 0.90$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.5, 0.65,0.65
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0088,0.0088,0.00607
L_D, L_F, L_W	population sizes	$1, 1, 20$
	Dynamic elasticity model w/ low elasticity target (if different than above)	
ϕ	cost of adjustment of goods ratio	18.1
χ	capital adjustment cost	5.415
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.825, 0.825, 0.90$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.47, 0.624,0.624
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.00895,0.00895,0.00615

Notes: As in previous table.

the volatility ratio as in the baseline specification. The remaining parameter values are calibrated analogously to the baseline specification. In addition, we also present results for the version of the dynamic elasticity model in which the volatility ratio target has been set to a lower value of 0.5 (the *dynamic elasticity model w/ low elasticity target* case). Since the empirical estimates of short-run trade elasticity cover a very wide range of values, and many of them are significantly below our target of 1.17, we view it as informative to show the potential of the model if parameterized to a lower value of the short-run elasticity. Parameter values for this model can be found in Table 5.

GHH and dynamic elasticity As an additional exercise, we also discuss a parameterization that combines GHH and dynamic elasticity. The parameterization is analogous to dynamic elasticity case but with GHH preferences in (31). Parameter values are reported in Table 4.

Table 6: Business Cycle Statistics: Data and Models[a]

Statistic	Data [b] Median[b]	Dynamic elasticity	Baseline	Baseline financial autarky	GHH	GHH + dynamic elasticity	Dynamic elasticity low target
A. Correlation							
domestic with foreign							
TFP (measured)	0.44	0.50	0.51	0.49	0.49	0.48	0.47
GDP	0.52	0.52	0.52	0.52	0.52	0.52	0.52
Consumption	0.41	0.55	0.56	0.63	0.84	0.54	0.48
Employment	0.42	0.64	0.56	0.59	0.62	0.065	0.73
Investment	0.50	0.38	0.47	0.57	0.58	0.44	0.38
GDP with							
Consumption	0.71	0.99	0.97	0.91	0.85	0.96	0.99
Employment	0.60	0.99	0.99	0.99	0.99	0.99	0.96
Investment	0.71	0.98	0.99	0.99	0.99	0.98	0.98
Net exports	-0.20	-0.62	-0.68	0.04	0.42	-0.54	-0.66
Terms of trade with							
Net exports	-0.31	-0.92	-0.83	0.04	0.71	-0.84	-0.97
B. Volatility relative to GDP							
Consumption	0.79	0.46	0.34	0.25	0.22	0.39	0.48
Investment	3.04	3.04	3.04	2.97	3.04	3.04	3.04
Employment	0.71	0.34	0.44	0.49	0.46	0.44	0.32
Net exports	0.59	0.26	0.10	0.003	0.02	0.31	0.29

Notes:The table reports business cycle statistics for each model specifications described in Section 2.3 relative to the values in the data.
[a]Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, data column refers to the median in our sample of countries for the period 1980Q1-2011Q4.

2.4 Findings

This section presents our quantitative results. Before we discuss model implications for the trade-comovement puzzle, we establish that the modifications we propose do not crucially deteriorate business cycle performance.

2.4.1 Business cycle properties

Table 6 documents each of the model's performance relative to a standard set of business cycle statistics. As is clear from the comparison to the data (column one), the dynamic elasticity model improves upon the baseline model and other models in an important respect: volatility and countercyclicality of net exports, which is a known shortcoming of the baseline model.[28] Specifically, in the dynamic elasticity model, the volatility of net exports relative to output is 26% versus 59% in the data, and hence the model still falls short of the data, but it is only 10% in the baseline model. Not surprisingly, financial autarky performs poorly in this respect.

The table also shows that the the main weakness of GHH preferences in comparison to dynamic elasticity is also the behavior of net exports. The GHH model implies a very low volatility of net exports relative to GDP, only 2%. It also implies a counterfactual positive correlation of net exports with terms of trade and real GDP.[29] Interestingly, combining GHH preferences with dynamic elasticity successfully resolves this issue (see column 6).

2.4.2 Trade-comovement puzzle and candidate resolutions

To compute each model's prediction for the trade-comovement relation, we generate an increase in trade in the bilateral pair to match the 90th percentile of the bilateral trade distribution in our sample, equal to 3.85%. We also change trade with rest of the world (third country) to match the higher level of trade openness in the data of the 90th percentile pair (total imports to GDP ratio).[30] We then compute the implied slope of the trade-comovement relationship by dividing the change in the correlation of real GDPs within the bilateral pair by the change in their log imports to GDP ratio – which corresponds to our measure of trade intensity in the empirical exercise. Table 7 reports the fraction of the data slope explained by each of the models, where data slope corresponds to OLS

[28] See the discussion in Raffo (2008).

[29] To be precise, given extremely low variability of the net exports, these statistics are essentially undefined. However, they are robustly negative in the data and this indicates a failure of the model.

[30] We vary trade by varying utility weights ω_{ij} – just like in the analytical model.

coefficient in Table 2.[31]

Table 7: Implied Slope of Trade-Comovement: Fraction relative to Data.

Model specification	Implied slope relative to data
Baseline	20%
Financial autarky	25%
Dynamic elasticity	64%
GHH preferences	60%
GHH preferences + dynamic elasticity & $\rho = 15$	65%
Dynamic elasticity w/ low elasticity target	80%

Notes: The table reports the implied slope between trade and comovement (output correlation) by each model specifications described in Section 2.3 relative to the value for the data. Data value is derived from the implied OLS regression coefficient reported in Table 2. The slope value for the models has been calculated by increasing bilateral trade intensity from the calibrated median value of bilateral trade to 90th percentile, and accordingly adjusting trade openness with rest of the world.

As is clear from the first two lines of Table 7, the baseline model and financial autarky fall short of accounting for the empirical trade-comovement relationship, which is the essence of the trade-comovement puzzle. The gap between theory and data is quite significant, as the slope implied by the baseline model in either case is at most a quarter of the relationship in the data. As discussed in Section 1.4.1, the reason for the relatively poor performance of financial autarky is that it does not preclude risk-sharing, as a big part of it comes from terms of trade movements.

In contrast, rows 3-5 show that both the model with GHH preferences and dynamic elasticity model come close to accounting for the puzzle. In particular, GHH preferences account for 60% of the trade-comovement relation in the data, confirming the intuition developed in Section 1. The dynamic elasticity model performs even better, accounting for 64% of the trade-comovement relationship in the data for our conservative setting of short-run trade elasticity. This triples the fraction of data coefficient accounted for by theory relative to baseline model's 20%. The last row of table 7 shows that for a more aggressive – but still empirically supported – setting of short-run elasticity of 0.5, the model can account for 80% of the trade-comovement relation, underscoring the quantiative potential of the dynamic elasticity model in matching the data. Importantly, the model's business cycle properties do not deteriorate.

[31] We obtain virtually identical conclusions from an exercise of generating 190 trade pairs, as in our data, and running a cross-sectional regression on model generated data, identical to our data regression. For clarity of exposition, here we focus on implied slope using 50th and 90th percentile.

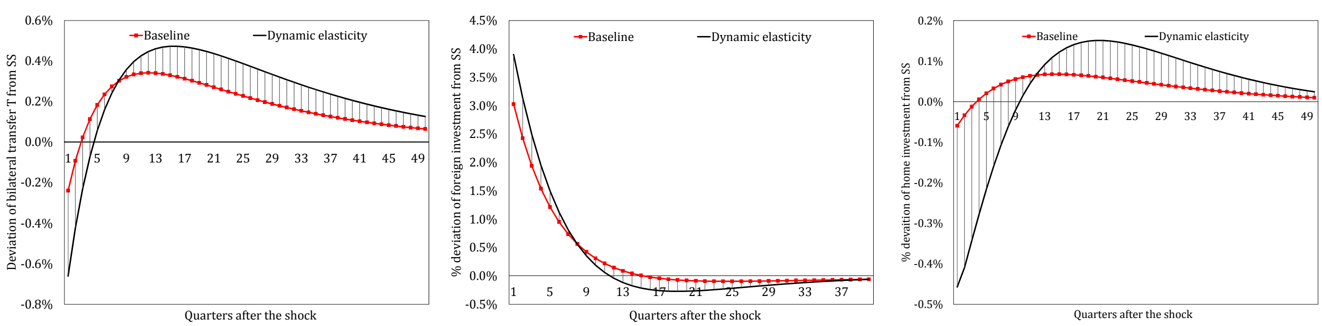
2.4.3 Mechanism of dynamic elasticity

The results presented in Table 7 show that the dynamic elasticity model is successful in generating the results relative to the baseline model. Here, we explain the intuition behind this result and relate it to the improved countercyclicality of net exports.

Analogously to our two country setup of Section 1, we back out the implied transfers in the three country setup and compare them across the dynamic elasticity and baseline models. Specifically, we define the zero-sum transfer between the two small countries, denoted by T, and additionally define a symmetric transfer paid to both countries by the rest of the world, denoted by T^w. The home and foreign country's analogs of budget constraints in (20)-(21) that define these transfer payments become:

$$\begin{aligned} d + \bar{p}_f f + \bar{p}_g g &= wl + rk + T + T^w \\ d^* + \bar{p}_f f^* + \bar{p}_g g &= (w^* l^* + r^* k^*) p_f - T + T^w, \end{aligned}$$

where $\bar{p}$ with subscripts pertains to steady state prices. It is clear that the two transfers are sufficient to implement any reallocation of resources between the three countries.



(a) Transfer T (b) Foreign Investment (c) Home Investment

Figure 5: Impulse response function of transfer T and foreign country investment to a productivity shock in foreign country.

Notes: The figure shows impulse responses to a positive productivity shock in the foreign country for the baseline model and the dynamic elasticity model, as described in Section 2.3.

Figure 5 shows the impulse response of the bilateral transfer T and investment after a positive productivity shock in the foreign country. It compares the baseline model and the dynamic elasticity model.

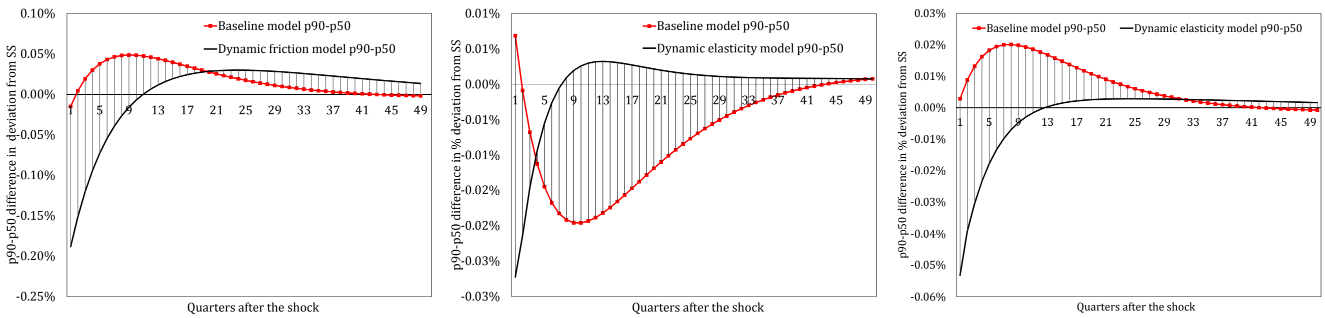
Consider first the baseline model. The impulse responses illustrate the classic S-curve or J-curve effect of capital first documented in Backus et al. (1994). Specifically, as the foreign country receives the shock, it accumulates more capital and at the same time the home country de-accumulates its own capital, while on net transferring resources to the foreign country. This dynamic is driven by the fact that foreign capital is more productive and hence it pays to build it up in the short-run. As expected, these flows reverse at longer time horizon, typically beyond business cycle frequency, and foreign country eventually makes a transfer to the home country, which helps rebuild its depreciated capital stock. In our analytic model, this effect of capital also arises, except that its effect it is not dynamic and nets out contemporaneously.

As impulse responses in Figure 5 show, dynamic elasticity amplifies these responses. Intuitively, this is because dynamic elasticity allows for an adjustment of trade shares at longer time horizons, at which point home and foreign goods are closer substitutes. Consequently, the dynamic elasticity model is more forgiving in initially taking advantage of the foreign productivity shock through accumulation of foreign capital, as it is easier to later accommodate the additional excess supply of the foreign country's good relative to home country's good – which in the baseline model with low elasticity is costly. This has two important ramifications for the theory-implied trade comovement relation.

First, as we have explained in Section 1.3 (income channel), negative transfers are a source of *positive* relation between trade and comovement. Since transfers are much more robustly negative in the dynamic elasticity model, and for longer, the positive effect of reversed direction of transfers will also be more pronounced.

The second reason is the presence of accumulation of capital itself that interacts with this friction. When there is little trade, i) the on-impact accumulation of foreign capital is beneficial because it soaks up the additional supply of the foreign good, but it is also costly because ii) it results in excess supply of the foreign good in the future, which may be difficult to take advantage of and share across countries. The key effect of having dynamic elasticity is that, while i) remains important, ii) becomes less important because, over time, trade shares can adjust and unbalanced supply of the two goods can be efficiently accommodated.

To confirm this intuition, Figure 6 shows how changing trade in our exercise from median to 90th percentile changes the illustrated responses in the dynamic elasticity and the baseline model. Specifically, it plots the difference in impulse responses of the p90 and median bilateral trade calibration. As the difference in impulse responses shows, initially transfers become more negative and for longer in the dynamic elasticity model. As the figure also shows, both foreign and home investment respond less in the 90th percentile case relative to median on business cycle frequency (see panels (b)-(c) of Figure 6). On net, these responses imply that in the dynamic elasticity model there is more shifting of capital for low trade levels relative to high trade levels than in the baseline model. Consistent with that, negative transfers are also more pronounced. Since all this is a source of negative comovement, dynamic elasticity model implies more positive relation between trade and comovement.



(a) Transfer T (b) Foreign Investment (c) Home Investment

Figure 6: The effect of raising trade from median to 90th percentile on the impulses response to a positive productivity shock in the foreign country.

Notes: The figure shows the difference in impulse responses to foreign shock due to an increase in trade from median (p50) to 90th percentile country pair (p90), as is the case in our main quantitative exercise in Table 7. The models have been calibrated as described in Section 2.3.

3 Conclusions

We characterized the forces responsible for the trade-comovement puzzle and analyzed three candidate resolutions of the puzzle suggested by this analysis. We have shown that two modifications are particularly promising quantitatively: GHH preferences and low trade elasticity modeled as dynamic elasticity consistent with high long-run trade elasticity. Since these modifications do not fundamentally change model's transmission mechanism or its shock structure, and they come close to closing the gap between theory and data, we came to the conclusion that the trade-comovement puzzle is best interpreted as imposing empirically viable parametric and structural restrictions on the standard transmission mechanism rather than rejecting it outright. Lastly, while we focus here on productivity shocks, the forces we single out, the effect of trade on these forces, and our methodology can be applied to study other sources of business cycle fluctuations.

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Appendix

A.1. Omitted proofs

To derive (22)-(26), we used Mathematica's symbolic processor. Detailed derivations are available in the online Mathematica notebook and are omitted from here. Plugging in $\alpha = \psi = 1/3$ to (22)-(26), we obtain:

$$\hat{y}(\hat{A};\hat{p},\hat{T}) : = \frac{3}{2}\hat{A} + \frac{3}{2}\bar{x}\hat{p} - 3\sqrt{3}T, \tag{38}$$

$$\hat{p}(\hat{A},\hat{A}^*,T) : = -\frac{3}{6\bar{x}+4(1-\bar{x})\rho}(\hat{A}-\hat{A}^*) - \frac{1}{x}\frac{6\sqrt{3}}{6\bar{x}+4(1-\bar{x})\rho}T, \tag{39}$$

$$T(\hat{A},\hat{A}^*,\hat{p}) : = -\frac{2+\sigma}{6\sqrt{3}\sigma}(\hat{A}-\hat{A}^*) - \frac{1}{3\sqrt{3}\sigma}\hat{p}. \tag{40}$$

While we use a numeric value for α and ψ, the results generalize of just assuming $\alpha = \psi$. For later reference, we denote the functions that solve (39)-(40) by $\hat{p}(\hat{A},\hat{A}^*), T(\hat{A},\hat{A}^*)$ and note that by definition they obey the following identities:

$$\hat{p}(\hat{A},\hat{A}^*) : = -\frac{3}{6\bar{x}+4(1-\bar{x})\rho}(\hat{A}-\hat{A}^*) - \frac{1}{x}\frac{6\sqrt{3}}{6\bar{x}+4(1-\bar{x})\rho}T(\hat{A},\hat{A}^*), \tag{41}$$

$$T(\hat{A},\hat{A}^*) : = -\frac{2+\sigma}{6\sqrt{3}\sigma}(\hat{A}-\hat{A}^*) - \frac{1}{3\sqrt{3}\sigma}\hat{p}(\hat{A},\hat{A}^*). \tag{42}$$

Proof of Lemma 1 and 2 in text

Lemma 1 and Lemma 2 follow from the following sequence of lemmas.

Lemma 4 $\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0.$

Proof. Differentiating (41), we obtain:

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial \hat{p}(\hat{A},\hat{A}^*;T)}{\partial \hat{A}^*} + \frac{\partial \hat{p}(\hat{A},\hat{A}^*;T)}{\partial T}\left(\frac{\partial T(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{A}^*} + \frac{\partial T(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}\right),$$

and hence

$$\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}(1 - \frac{\partial \hat{p}(\hat{A},\hat{A}^*;T)}{\partial T}\frac{\partial T(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{p}}) \equiv \frac{\partial \hat{p}(\hat{A},\hat{A}^*;T)}{\partial A^*} + \frac{\partial \hat{p}(\hat{A},\hat{A}^*;T)}{\partial T}\frac{\partial T(\hat{A},\hat{A}^*,\hat{p})}{\partial \hat{A}^*},$$

Using (39)-(40), we sign the terms of this expression as follows:

$$(\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*}) \times (1 - (+) \times (-)) \equiv (+) + ((+) \times (+)),$$

Accordingly,

$$\frac{\partial p(A,A^*)}{\partial A^*}) > 0.$$

■

Lemma 5 *If* $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$ *and* $0 < \bar{x} \leq 1/3$, *then* $\frac{\partial T(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > 0.$

Proof. By a way of contradiction, suppose

$$\frac{\partial T(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} < 0.$$

Differentiating (41), we obtain

$$\frac{\partial p(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial p(\hat{A},\hat{A}^*;T)}{\partial \hat{A}^*} + \frac{\partial p(\hat{A},\hat{A}^*;T)}{\partial \hat{A}^*}\frac{\partial T(\hat{A},\hat{A}^*)}{\partial \hat{A}^*},$$

Since

$$\frac{\partial p(\hat{A},\hat{A}^*;T)}{\partial \hat{A}^*} = \frac{3}{6\bar{x} + 4(1-\bar{x})\rho} > 0,$$

we conclude by Lemma 4 and the hypothesis that

$$0 < \frac{\partial p(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} < \frac{\partial p(\hat{A},\hat{A}^*;T)}{\partial \hat{A}^*}.$$

Similarly, differentiating (42), we obtain

$$\frac{\partial T(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} \equiv \frac{\partial T(\hat{A},\hat{A}^*;T)}{\partial \hat{A}^*} + \frac{\partial T(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(\hat{A},\hat{A}^*)}{\partial \hat{A}^*},$$

and given

$$\frac{\partial T(\hat{A},\hat{A}^*;\hat{p})}{\partial \hat{p}} = -\frac{1}{3\sqrt{3}\sigma} < 0,$$

and Lemma 4, we have

$$\frac{\partial \hat{T}(A,A^*)}{\partial A^*} \equiv \frac{\partial \hat{T}(A,A^*;\hat{T})}{\partial A^*} + \frac{\partial \hat{T}(A,A^*;\hat{p})}{\partial \hat{p}}\frac{\partial p(\hat{A},\hat{A}^*)}{\partial \hat{A}^*} > \frac{\partial T(A,A^*;T)}{\partial A^*} + \frac{\partial T(A,A^*;\hat{p})}{\partial \hat{p}}\frac{\partial p(A,A^*;T)}{\partial A^*}.$$

It is now clear that, if

$$\frac{\partial \hat{T}(A,A^*;\hat{T})}{\partial A^*} + \frac{\partial \hat{T}(A,A^*;\hat{p})}{\partial \hat{p}}\frac{\partial p(A,A^*;\hat{T})}{\partial A^*} \geq 0, \tag{43}$$

then

$$\frac{\partial T(A,A^*)}{\partial A^*} > 0,$$

which is a contradiction. Plugging in from (39)-(40) to (43), the necessary and sufficient condition for the hypothesis of the lemma is:

$$\rho \ \geq \frac{3}{2}\frac{1-\bar{x}(2+\sigma)}{(1-\bar{x})(2+\sigma)}. \tag{44}$$

The condition in (44) is strictly decreasing in $\bar{x}$ and at $\bar{x}=0$ it boils down to the assumed condition $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$. ■

Lemma 6 *If $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$ and $0 < \bar{x} \leq 1/3$, then $\frac{\partial^2 p(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} < 0$ and $\frac{\partial^2 T(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$.*

Proof. Differentiating (40), we obtain:

$$\frac{\partial^2 T(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} = \frac{\partial T(A,A^*;\hat{p})}{\partial \hat{p}}\frac{\partial \hat{p}(A,A^*)}{\partial A^* \partial \bar{x}} = -\frac{1}{3\sqrt{3}\sigma}\frac{\partial \hat{p}(A,A^*)}{\partial A^* \partial \bar{x}}.$$

Consequently, we have

$$sign\left(\frac{\partial^2 T(\hat{A},\hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}}\right) = -sign\left(\frac{\partial \hat{p}(A,A^*)}{\partial A^* \partial \bar{x}}\right).$$

We next differentiate (39) to obtain

$$\frac{\partial \hat{p}(A,A^*)}{\partial A^*} = \frac{3}{6\bar{x}+4(1-\bar{x})\rho} - \frac{1}{x}\frac{6\sqrt{3}}{6\bar{x}+4(1-\bar{x})\rho}\frac{\partial T(A,A^*)}{\partial A^*},$$

and similarly differentiate (40) to obtain

$$\frac{\partial T(A,A^*)}{\partial A^*} = \frac{2+\sigma}{6\sqrt{3}} - \frac{1}{3\sqrt{3}\sigma}\frac{\partial \hat{p}(A,A^*)}{\partial A^*}.$$

Combining these two equations,

$$\begin{aligned}
\frac{\partial \hat{p}(A,A^*)}{\partial A^*}\left(1+\frac{1}{\bar{x}}\frac{6\sqrt{3}}{6\bar{x}+4(1-\bar{x})\rho}\frac{1}{3\sqrt{3}}\right) &= \frac{3\sigma}{6\bar{x}+4(1-\bar{x})\rho} + \frac{1}{\bar{x}}\frac{6\sqrt{3}}{6\bar{x}+4(1-\bar{x})\rho}\frac{2+\sigma}{6\sqrt{3}} \\
\frac{\partial \hat{p}(A,A^*)}{\partial A^*}(\bar{x}(6\bar{x}+4(1-\bar{x})\rho)+2) &= 3\bar{x}\sigma + \ 2+\sigma,
\end{aligned}$$

we have

$$\frac{\partial \hat{p}(A,A^*)}{\partial A^*} = \frac{3\bar{x}\sigma + \ 2+\sigma}{\bar{x}(6\bar{x}+4(1-\bar{x})\rho)+2}, \tag{45}$$

hence

$$\frac{\partial^2 \hat{p}(A,A^*)}{\partial A^* \partial \bar{x}} = \sigma\frac{3+3\bar{x}^2(2\rho-3)\sigma - 2\rho(2+\sigma) + 2\bar{x}(2\rho-3)(2+\sigma)}{2(1+\bar{x}(\bar{x}(3-2\rho)+2\rho)\sigma)^2}, \tag{46}$$

and

$$sign \frac{\partial^2 \hat{p}(A, A^*)}{\partial A^* \partial \bar{x}} = sign(3\bar{x}^2(2\rho - 3)\sigma \ - 2\rho(2+\sigma) + 2\bar{x}(2\rho-3)(2+\sigma)+3),$$

To complete the proof, we must establish that the following expression is negative:

$$E(\bar{x}, \rho) := 3\bar{x}^2(2\rho - 3)\sigma \ - 2\rho(2+\sigma) + 2\bar{x}(2\rho-3)(2+\sigma)+3.$$

This follows from the following properties: 1) $E(\bar{x}, \rho)$ is decreasing with respect to ρ for all $0 \leq \bar{x} \leq 1/3$. This follows from the fact that the partial derivative of E with respect to ρ,

$$\frac{\partial E(\bar{x}, \rho)}{\partial \rho} = 6\bar{x}^2 + 4\bar{x}(2+\sigma) - 2(2+\sigma),$$

is a quadratic function with roots given by:

$$\begin{aligned} \bar{x}_1 &= \frac{-2-\sigma-\sqrt{4+10\sigma+4\sigma^2}}{3\sigma} < 0, \\ \bar{x}_2 &= \frac{-2-\sigma+\sqrt{4+10\sigma+4\sigma^2}}{3\sigma} > \frac{-2-\sigma+\sqrt{4+8\sigma+4\sigma^2}}{3\sigma} = \frac{1}{3}. \end{aligned}$$

Accordingly, $E(\bar{x}, \rho)$ is a decreasing function of $\bar{x}$ for $0 \leq \bar{x} \leq \frac{1}{3}$.. 2) $E(\bar{x}, \rho) = 0$ at

$$\rho = \frac{9\bar{x}^2\sigma + 6\bar{x}(2+\sigma) - 3}{6\bar{x}^2\sigma - 2(2+\sigma) + 4\bar{x}(2+\sigma)}, \tag{47}$$

which is a strictly decreasing function of $\bar{x}$ – which can be verified by taking derivative with respect to $\bar{x}$ – and at $\bar{x} = 0$ the expression boils down to the assumed condition $\rho \geq \frac{3}{2}\frac{1}{2+\sigma}$. ■

Lemma 7 *If* $\bar{x} \leq \frac{1}{1+\sigma/2}$, *then* $\frac{\partial \hat{p}(\hat{A}, \hat{A}^*)}{\partial \hat{A}^*} + x\frac{\partial^2 \hat{p}(\hat{A}, \hat{A}^*)}{\partial \hat{A}^* \partial \bar{x}} > 0$.

Proof. Using (45) and (46), and simplifying, we obtain

$$\frac{\partial \hat{p}(A, A^*)}{\partial A^*} + x\frac{\partial^2 \hat{p}(A, A^*)}{\partial A^* \partial \bar{x}} = \frac{2+\sigma(1+\bar{x}(6+4\bar{x}(\rho(1+2\sigma)-3(2+\sigma))))}{2\left(1+\bar{x}(\bar{x}(3-2\rho)+2\rho)\sigma\right)^2}.$$

The sign of this expression depends on the sign of

$$Z(\bar{x}, \rho) := 2+\sigma(1+\bar{x}(6+\bar{x}(\rho(4+8\sigma)-3(2+\sigma)))),$$

and $Z(\bar{x}, \rho)$ is globally increasing in ρ. It thus suffices to establish that the expression is positive for $\rho = 0$. We solve for the critical value of ρ such that $Z(\bar{x}, \rho) = 0$,

$$\rho = -\frac{2+\sigma(1+3x(2-x(2+\sigma)))}{4x^2\sigma(1+2\sigma)}.$$

The expression

$$N(\bar{x}, \sigma) := 2+\sigma(1+3x(2-x(2+\sigma)))$$

is positive, which finished the proof. This follows from the following properties: 1) N is a quadratic equation with upward pointing ends. 2) One of its roots is positive and can be be bounded as follows:

$$x = \frac{3\sigma+\sqrt{3}\sqrt{\sigma(4+7\sigma+\sigma^2)}}{6\sigma+3\sigma^2} \geq \frac{1}{1+\sigma/2}.$$

2) The other root is negative because $N(\bar{x}=0, \sigma) = -6\sigma$. ■

Proof of Lemma 3 in text

The proof is a simple corollary to the proof of Lemma 5 and 6 above, since the cutoff value for ρ in equation (47) is always above the cut off value for ρ in (44). The effect for $\mathcal{L}_{\mathcal{I}}$ follows from its definition.

A.1. Remaining relations underlying decomposition in Section 1.2.1

The complete equilibrium system obtain in the first stage of decomposition (Section 1.2.1) are:

$$
\begin{aligned}
\hat{y} &= \frac{\hat{A}}{1-\alpha} + \bar{x}\frac{1-\psi+\alpha}{1-\alpha}\hat{p} - \frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\frac{1-\psi}{\psi}T \\
\hat{k} &= \frac{\hat{A}}{1-\alpha} + \bar{x}\frac{2-\psi}{1-\alpha}\hat{p} - \frac{\alpha^{\frac{\alpha}{\alpha-1}}(1-\psi)}{(1-\alpha)\psi}\hat{T} \\
\hat{l} &= \bar{x}\frac{1-\psi}{1-\alpha}\hat{p} - \frac{\alpha^{\frac{\alpha}{\alpha-1}}(1-\psi)}{(1-\alpha)\psi}\hat{T} \\
\hat{d} &= \frac{\bar{x}(-(1-\alpha)\rho+\alpha-\psi+1)}{1-\alpha}\hat{p} + \frac{\hat{A}}{1-\alpha} - \frac{\alpha^{\frac{\alpha}{\alpha-1}}(\alpha-\psi)}{(1-\alpha)\psi}\hat{T} \\
\hat{f} &= \frac{((1-\alpha)\rho(1-\bar{x})+\bar{x}(\alpha-\psi+1))}{1-\alpha}\hat{p} + \frac{\hat{A}}{1-\alpha} - \frac{\alpha^{\frac{\alpha}{\alpha-1}}(\alpha-\psi)}{(1-\alpha)\psi}\hat{T}
\end{aligned}
$$

Foreign analogs of the above can be obtained by replacing p by $-\hat{p}$, T by $-T$, d by f and f by d, while placing asterisk on all variables except for p and T. Detailed derivations of this system and other conditions from the analytic sections can be found in the online Mathematica files.